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ANALYSIS OF CHROMITE PROCESSING PLANT DATA BY FIRST ORDER AUTOREGRESSIVE MODEL

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Abstract: Many mineral processing data can be monitored by a time series model. This research presents results of analysis and simulations of a chromite processing plant data determined by time series model. The plant data obtained by shift to shift include feed grade, concentrate grade, tailing grade, Cr/Fe ratio in concentrate. All the chromite processing data were found stationary over time. The autocorrelation was high for feed grade and Cr/Fe ratio. Weaker autocorrelation was observed for concentrate grade and tailing grade. Autoregressive integrated moving average (ARIMA, 1,0,0) or first order autoregressive (AR, 1) model, was found to fit all data very well. The models obtained have been also shown to be used for the near future estimation of these data. The time constant which is an indicator of sampling frequency of the data sets were determined. It was found that sampling frequency was enough for concentrate and tailing grade and their original values can be used in process control charts for monitoring. On the other hand, the sampling frequency should be reduced for feeding grade and Cr/Fe ratio for the same aims hence ARIMA residual charts were more suitable to monitor their values.

Keywords: *time series, autoregressive model, time constant, process control, chromite processing*

Introduction

Although process data at any time are determined by process conditions, there are random or probabilistic data which can only be characterized by statistical methods (Gleit 1985). In mineral processing plants, many data are obtained over time. The performance of a mineral processing plant can be evaluated by analyzing these data. The analysis of the resulting data properly is a very important step in understanding plant's performance (Ketata and Rockwell, 2008). A set of observations in time sequence is defined as a time series (Ganguli and Tingling, 2001). The data from mineral processing plants may be evaluated by applying times series models since the data structure is identical to the time series form. In some processes, these observations are

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correlated. Many quality characteristic values are a good example of a time series that is correlated in time domain (Ganguli and Tingling, 2001).

Time series analysis models are very useful in modeling dynamic systems in science and engineering applications (Capodaglio, et al., 1992). This class of models is in fact able to represent the dynamic features of physical systems that are subject to often uncontrollable inputs with random components. Mineral processing plants can also be considered as examples of dynamic systems, with inputs (ore characteristics, feeding and flow, organic loads, etc.) that vary stochastically within more or less wide ranges. The use of stochastic models allows a more detailed representation of the dynamic nature of these systems, while retaining the degree of information contained in most deterministic models (Capodaglio, et al., 1992). A fundamental utility of these models is their ability to forecast the level of the process into the future, accounting for its recent history and the underlying stochastic nature (Ganguli and Tingling, 2001).

In quality control, the production achieved can be collected under two headings by taking into consideration the type and nature of it. If production can be measurable, can be weighted and can be expressed in a unit with one or more of these feature, it can be mentioned that the production is continuous. Otherwise, there is a discrete manufacturing process. However, continuous and discrete concepts in time series are determined according to whether the observation values are obtained from equal time intervals or not. In other words, if the observation values making time series are obtained from unequal time intervals, the series occurred are defined as continuous time series. If the observations are obtained at equal or certain time intervals, they are known as discrete time series (Kaya, 1995). Even where the observations carried out continuously, the observation for the specific time intervals or based on the total value or can be converted by means of sampling to discrete continuous series.

Trybalski and Cieply (2000) stated that most mineral processing unit operations are in principle continuous processes but the majority of continuous processes such as mineral processing plants can be regarded as discrete. Therefore, the processes of mineral processing can be regarded as discrete processes which can be described with time series (Trybalski and Cieply, 2000). Some applications of time series models for mineral/metal processing plants have been reported in the literature for different aims. Examples can be given such as the investigation of dynamic characteristics of the flotation circuits (O'Keefe et al., 1981), the coal data from preparation plants (Cheng at al., 1982), the SO_2 stack emissions from coal boilers (Gleit, 1985), modeling of daily data of metal grade or recovery (Napier-Munn and Meyer, 1999), estimation of world copper production (Kutlar and Elevli, 1999), the copper ore flotation (Trybalski and Cieply, 2000), analyzing and modeling the behavior of operators of ilmenite reduction furnaces (Bazin et al., 2000), the coal segregation control (Ganguli and Tingling, 2001) and the variables of stream materials and sampling errors (Ketata and Rockwell, 2008). The time series models have been also used for the statistical process control charts of autocorrelated data of mineral processing/mining applications (Samanta and

Bhattacherjee, 2001; Bhattacherjee and Samanta, 2002; Samanta, 2002; Elevli et al., 2009; Taşdemir, 2012)

The autoregressive integrated moving average, called as ARIMA time series models in short, are perhaps the most popular methods to evaluate a process variable and to make estimation for the future. The ARIMA time series models capture the stochastic characteristics of the fluctuations in a quality levels over time (Ganguli, and Tingling, 2001).

The purpose of this study was threefold. First aim was to determine if the first order autoregressive time series model, ARIMA(1,0,0) or AR(1), can be used an appropriate time series model in monitoring and analyzing the four types of data obtained from a chromite processing plant and to test the performance of models in the near future forecasting. The next purpose was to determine the time constants of data sets obtained from AR(1) models to test adequacy of sampling frequencies of data. The last aim was to show how weak and moderate autocorrelation of datasets affect the Shewhart charts of individual observations for the process control of these data sets.

Autoregressive (AR) time series models

Autoregressive (AR) time series models are called according to referring to the past period by the number of observations containing the value in the AR models. If AR model contains one observation values in the past period it is called as first order AR model. If it contains two value of the past period of observation, it is called the second order AR model. In general, *p*-order AR model contains *p* historical value of the observation period in question. Let X_1, $X_2 \ldots X_t$ be stationary time series (such as feeding Cr_2O_3 grade, tailing Cr_2O_3 grade, concentrate Cr_2O_3 grade etc.) The object of this modeling approach is to derive an expression for X_t, the value of the series at time *t*, in terms of values of the series from the past, i.e at times *t*–1, *t*–2, etc. (Cheng at al., 1982). There must be some totally random shocks a_t entering the model at time *t* due to the random fluctuations in the series. General expressions of the AR (*p*) model are as follows (Montgomery et al., 2008):

$$X_t = \delta + \varphi_1 X_{t-1} + \varphi_2 X_{t-2} + \ldots + \varphi_p X_{t-p} + \varepsilon_t \quad (1)$$

X_t , X_{t-1} , X_{t-2},... , X_{t-p} are the observation values. The φ_1, φ_2, ...,φ_p are the model parameters which are termed the autoregressive constants $\delta = \left(1 - \sum_{i=1}^{p} \phi_i\right)\mu$ with μ denoting the process mean, *p* is the order of model, ε_t is a random shock which is independent error term which reflects the amount of variation in the data which is not explained by the AR model and is assumed to follow a normal distribution with mean zero and variance, σ_e^2, which is termed white noise variance (WNV). A random shock is a random variable that is independent of all past history (Gleit, 1985). It accounts for any inherent variance in the data.

In practice, the first and second-order AR models commonly used as models and they are shown as AR (1) and AR (2) in short respectively. In AR (1) model, an observation value at t period of a time series is explained by observation value of X_{t-1} at t–1 period of time series and an error term. The variability associated with a chromite property, X, is subject to correlated and random elements. This stochastic process is modeled by autoregressive model of order one (i.e., an AR(1) process) with the equation:

$$X_t = \delta + \varphi X_{t-1} + \varepsilon_t \tag{2}$$

where: X_t – measurement at time t,
X_{t-1} – measurement at time t–1,
δ – $(1 - \varphi)\ \mu$, μ is the process mean
φ – autocorrelation coefficient ($-1 < \varphi < 1$),
ε_t – error term, normal random shock at time t.

The AR (1) model implies that each observation depends on the previous one to an extent defined by φ (Napier-Munn and Meyer, 1999). The autoregressive parameter, φ, is a measure of the autocorrelation between the past data point X_{t-1} and the current data point of X_t.

Autocorrelation

Autocorrelation in a time series, meaning the correlation between current observations (X_t) and observation from p periods before the current one (X_{t-p}) (Montgomery et al., 2008). In a given series, the autocorrelation at lag p which ranges from –1 to +1 is the correlation between the X_t, and X_{t-p} pairs and is given by:

$$r_p = \frac{\sum_{t=1}^{n-p}(X_t - \bar{X})(X_{t+p} - \bar{X})}{\sum_{t=1}^{n}(X_p - \bar{X})^2}. \tag{3}$$

Theoretical autocorrelation functions (ACFs) and partial autocorrelation functions (PACFs) (autocorrelations versus lags) are available for the various models chosen. Many important conclusions about a time series can be made according to the correlograms which a plot of sample ACFs/PACFs versus lags obtained for a data set when choosing an appropriate time series model. More details can be found in Montgomery et al (2008). For example, ACF of a characteristic AR model slowly approaches zero and its PACF spikes at lag p. Therefore, the model is most probably characterized by ARIMA (1,0,0) or AR(1) when there is a significant spike only at lag 1 of the PACF (partial autocorrelation between X_t and X_{t-1}) and the ACF slowly declines.

The Time Constant (*T*)

Rius and Callao (2001) and Callao and Rius (2003) used the time constant (*T*) that is related to the behavior of AR(1). The *T* parameter is calculated by the following equation for the AR(1) model:

$$T = -\frac{1}{\ln \phi_1}. \tag{4}$$

The time constant is an indicator which controls the appropriateness of sampling frequency. It provides a time constant for eliminating autocorrelation if the frequency of analysis can be decreased. When it equals to 1, we can conclude that the sampling frequency might be enough. If *T* is equal to two, the sampling frequency can probably be cut by half. In case of its value 3, it means that it can probably be cut by a third. The data autocorrelation disappears when the sampling frequency decreases (Rius and Callao, 2001; Callao and Rius, 2003). The time constant of the system indicates the sampling frequency that must be reduced to apply control charts to the original data. In other words, the time constant of time series obtained by Eq. 4 is used to determine whether sampling frequency is correct (Rius and Callao, 2001).

Methodologies

The data sets were obtained from a chromite processing plant in Turkey to monitor chances shift by shift, in four dataset characteristics in 30 days time period. These are Cr_2O_3 per cent of feeding ore, concentrate Cr_2O_3 content, tailing Cr_2O_3 content and Cr/Fe ratio of concentrate. Three shifts in a day are applied at the plant. Therefore, 93 observations in total for each data set which was collected from December 1 to December 31, 2011 were obtained and used in the study.

Since the observation values obtained from chromite production process have occurred from the measurements in each shift, these observations have been obtained at equal time intervals. Therefore, the time series obtained was a series of discrete time series. However, the observation values that the continuous observation characteristic features carry measurable. The ARIMA model data sets can be used.

For each data set, the model parameters of time series were estimated by applying ARIMA time series model (known as Box-Jenkins model). Software of Minitab 16 and trial version of Statgraphics Centurion XVI were used for the statistical analyses of data sets. The time series models which have the lowest AIC (Akaike Information Criterion) values are selected for representing the best model for each data set. The residuals of the models were evaluated by residual analysis. The near future forecasting performances of the models were also performed by comparing the real data with estimated data. In addition, effect of Shewhart charts of individual observations were compared with the X control charts based on ARIMA residuals to examine the effect of autocorrelation on the performance of the Shewhart chart. The time constants of

data sets required for the estimation of sampling frequency to reduce the autocorrelation for the usage of original data with control charts were also determined.

Results and discussions

Autocorrelation and Time Series Models

The time series of four data sets are shown in Fig. 1. Mineral processing plant production data often exhibit great variability with time (Napier-Munn and Meyer, 1999). Napier-Munn and Meyer (1999) stated that most mineral processing data moves around, both from day to day in an apparently random fashion and in short or long-term trends or cycles, even if over a long period the mean remains approximately constant. We can see all these statements on the time series plots presented in Fig. 1. There are two major kinds of time series. The one is the stationary time series which both the mean and variance of the values remain stable over time. The other one is the non-stationary time series which the mean or variance, or both, change with time. By applying a difference process, a non-stationary series can make a stationary series (Huang et al., 2002). As it can be seen clearly, all chromite data sets exhibit a stationary behavior over time and there is no trend and no need to a difference process to the data.

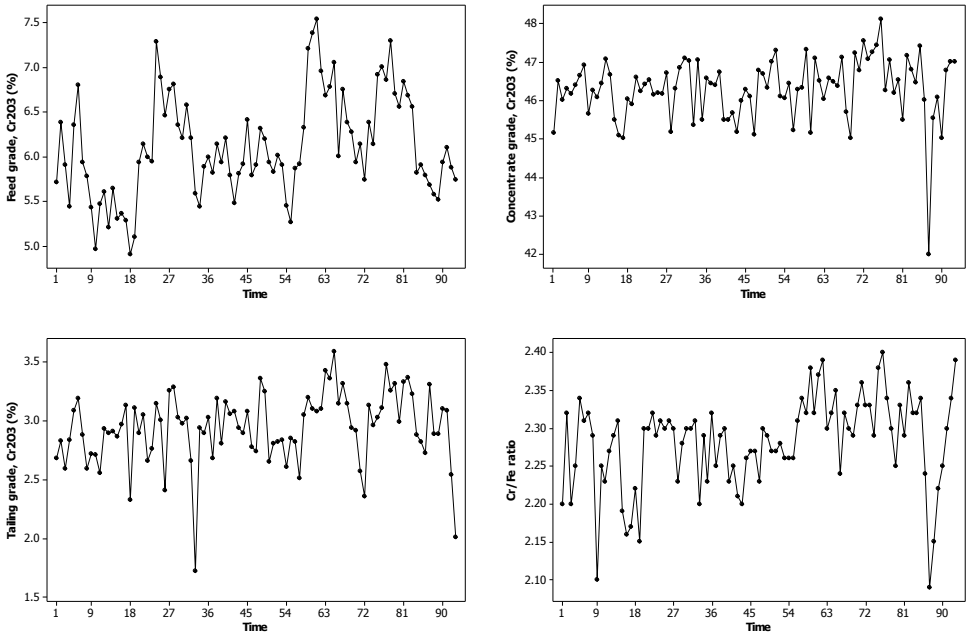

Fig. 1. Time series plots of four data sets of chromite processing plant

For the stationary control of the time series of four chromite data, their ACF plots are also generated and presented in Fig. 2. It is seen that these ACF patterns of data sets show typical stationary time series, because they are cutting off or tailing off near zero after a few lags. There are significant autocorrelations at first lag for feeding

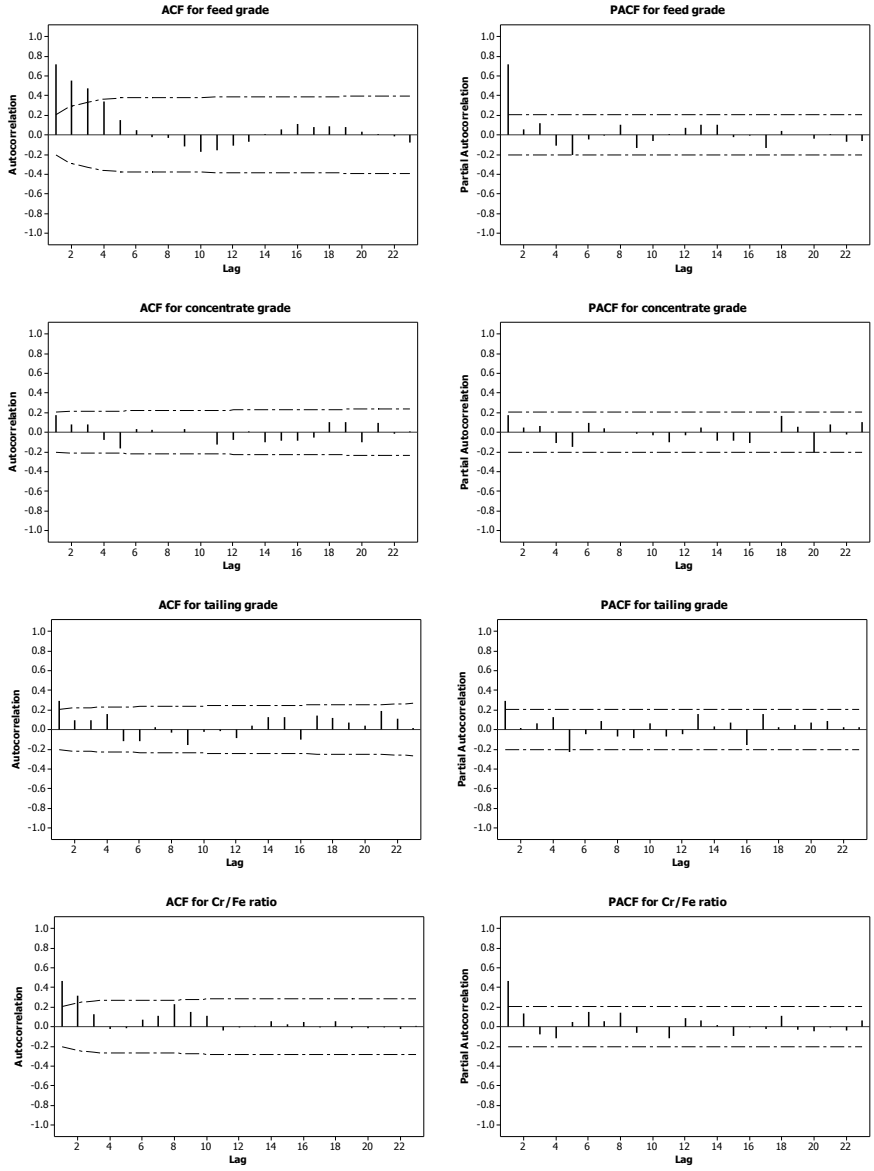

Fig. 2. Autocorrelation (ACF) and partial autocorrelation (PACF) functions for four chromite data sets with 5% significance limits

grade (r_p = 0.728) and Cr/Fe ratio (r_p = 0.493). The tailing grade (r_p = 0.327) and concentrate grade (r_p = 0.178) exhibited weaker autocorrelation compared to other two datasets. Moreover, ACF values of data sets decrease or decay very quickly (Fig. 2). In contrary, a non-stationary time series decay very slowly and exhibits sample autocorrelations that are still father large even at long lags (Montgomery et al., 2008). Therefore, it can be concluded that they are all may be considered as stationary time series. There exists serial autocorrelation of chromite processing data over one shift time intervals.

According to the PACF plots of datasets in Fig. 2, they all followed the AR(1) model since they all, except concentrate grade, show a significant spike (significant autocorrelation) at lag 1 followed by no apparent pattern meaning that all the higher-order autocorrelations are effectively explained by the lag 1 autocorrelation. When the PACF displays a sharp cutoff while the ACF decays more slowly as in the Fig. 2, the series displays an AR signature and the autocorrelation can be explained more easily by AR terms (Montgomery et al., 2008). The statistical test results also confirmed that the data sets except concentrate grade data, are best described by the AR(1) model according to the automatic time series model selection module of Statgraphics software taking into lowest AIC (Akaike Information Criterion) value.

The AIC is a function of the variance of the model residuals, penalized by the number of estimated parameters. In general, the model that minimizes the mean squared error without using too many coefficients is selected. As seen in Fig. 2, the concentrate grade data has little autocorrelation value which can be ignored. It is found to be best described by constant mean or ARIMA (0,0,0) time series model having an AIC value of –0.3942. On the other hand, the AR (1) model which has an AIC value of –0.3936 was found the second best model for this data set. Considering the very small differences between the two AIC values, the AR(1) model which was the best for other data sets was also used in this study for the concentrate grade data. The parameters of the AR(1) models for the four datasets are summarized in Table 1. According to the *t* values and corresponding *p* values, all data sets have *p*-values smaller than 0.05 indicating the suitability of model parameters. Only *p* value of autocorrelation for concentrate grade was 0.089 but it was also accepted to use the AR(1) model to evaluate the results according to the reasons explained above.

We obtained the following AR(1) models for the four data sets in the form:

$$X_t = 0.726X_{t-1} + 1.669 \text{ (feed grade)},$$

$$X_t = 0.176X_{t-1} + 38.131 \text{ (concentrate grade)},$$

$$X_t = 0.325X_{t-1} + 1.981 \text{ (tailing grade)},$$

$$X_t = 0.485X_{t-1} + 1.175 \text{ (Cr/Fe ratio)}.$$

Table 1. Parameters in AR(1) models for four datasets

Feed grade, Cr_2O_3 %				
Parameter	Estimate	Stnd. Error	*t*	*p*-value
AR(1)	0.726	0.072	10.105	0.000
Mean, μ	6.101	0.145	42.109	0.000
Variance, σ	0.329			
WNV, σ_e^2	0.158			
Constant, δ	1.669			
Time constant, *T*	3.123			
Box-Pierce Test based on first 36 autocorrelations				0.949
Concentrate grade, Cr_2O_3 %				
Parameter	Estimate	Stnd. Error	*t*	*p*-value
AR(1)	0.176	0.102	1.719	0.089
Mean, μ	46.297	0.100	460.308	0.000
Variance, σ	0.659			
WNV, σ_e^2	0.647			
Constant, δ	38.131			
Time constant, *T*	0.576			
Box-Pierce Test based on first 36 autocorrelations				0.609
Tailing grade, Cr_2O_3 %				
Parameter	Estimate	Stnd. Error	*t*	*p*-value
AR(1)	0.325	0.105	3.097	0.003
Mean, μ	2.933	0.044	67.255	0.000
Variance, σ	0.092			
WNV, σ_e^2	0.084			
Constant, δ	1.981			
Time constant, *T*	0.889			
Box-Pierce Test based on first 36 autocorrelations				0.114
Cr/Fe ratio				
Parameter	Estimate	Stnd. Error	*t*	*p*-value
AR(1)	0.485	0.093	5.234	0.000
Mean, μ	2.282	0.010	221.911	0.000
Variance, σ	0.004			
WNV, σ_e^2	0.003			
Constant, δ	1.175			
Time constant, *T*	1.382			
Box-Pierce Test based on first 36 autocorrelations				0.930

WNV – White noise variance

Residual Analysis of AR(1) Models

Diagnostic checks of the residuals of four data sets through sample ACF plots and residuals plots are presented in Fig. 3 and in Fig. 4, respectively. Plots imply that we have a good fit for all the data sets. The Box-Pierce (*Q*-statistics) of the residuals for the data sets in Table 1 is based on the sum of squares of the first lag 36 autocorrelation coefficients. Since the *p*-values for this test are greater than 0.05 for all data sets,

we can conclude that the residual series are random at the 95% level. These results are confirmed by the ACF graphs of residuals for four data sets as shown in Fig. 3 since the examinations of ACF graphs of the residuals did not differ from the conclusions of Box-Pierce tests.

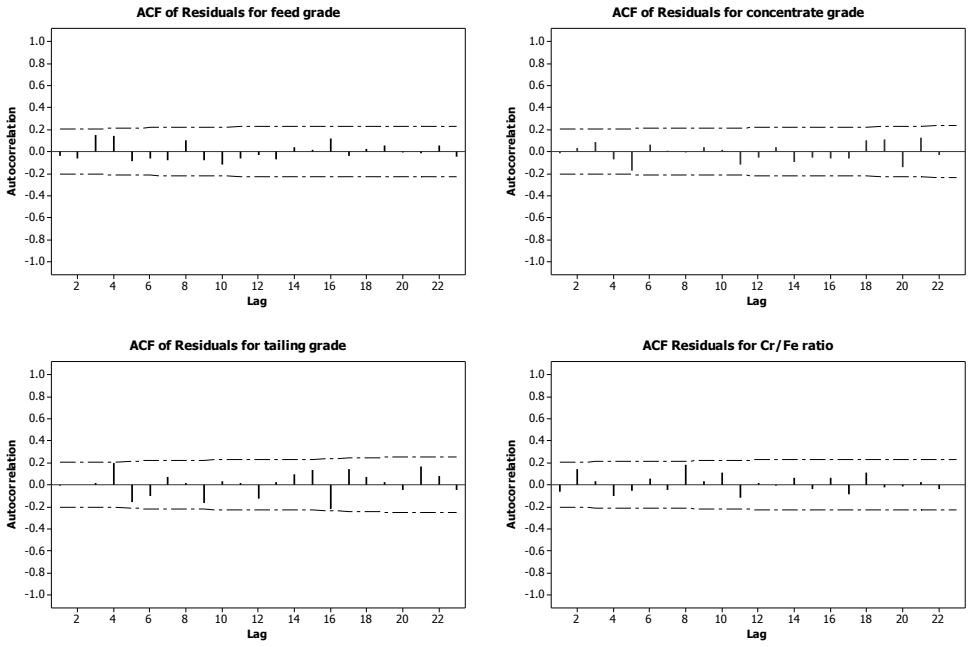

Fig. 3. Autocorrelation functions (ACF) for the residual values obtained from AR(1) models with 5% significance limits

Figure 4 plots the residuals for the time series models that fit the four chromite data. In these plots, four types of residual presentation format obtained by Minitab software are given. The plots in the upper left-hand portion of the display are a normal probability plot of the residuals. The residuals of data sets lie generally along a straight line, so the normality assumptions are satisfied properly. According to the histograms of the residuals presented in the lower left plots in Fig. 4, they do not give any serious indication of nonnormality. The upper right plots are the residuals versus the fitted values. These plots indicate the ideal patterns with essentially random scatter in the residuals. If these plots had exhibited a funnel shape, it could be indicate problems with the equality of variance assumption (Montgomery et al., 2008). The lower right plots are the plots of the observations in the order of the datasets. If these were of the order in which the data were collected, or if the data were a time series, this plot could reveal information about how the data may be changing over time.

According to the all results regarding to the residuals of the AR(1) models, the models can be used for the forecasting and process control charts.

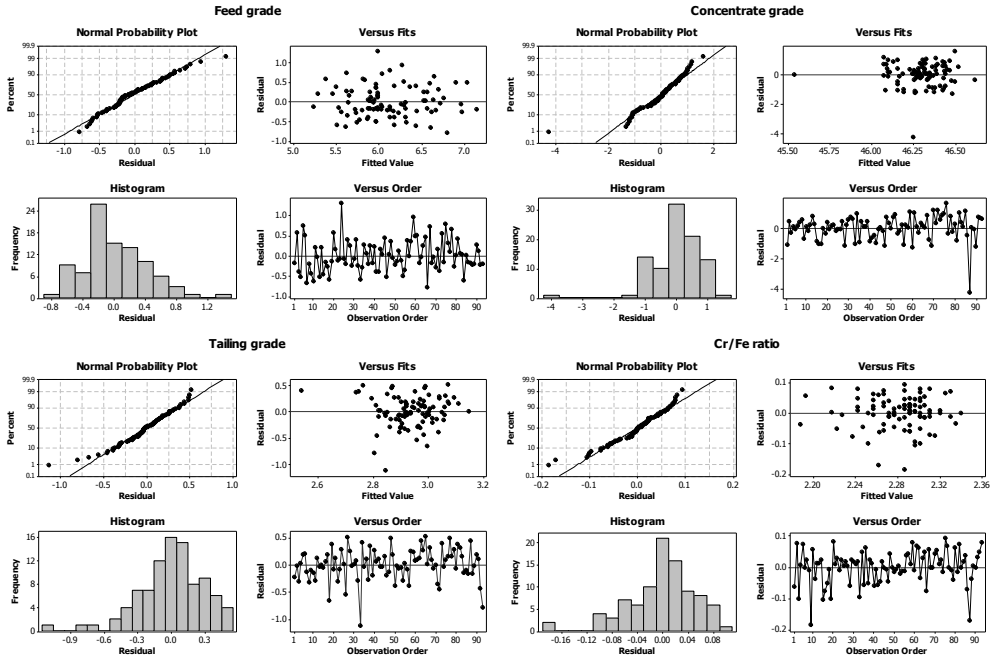

Fig. 4. Residual plots from AR(1) models for four chromite datasets

AR(1) Model Fits to Datasets and Forecasting Performances

The AR(1) time series models to capture the stochastic characteristics of the fluctuations in the four chromite data sets over time and their forecast values using these models are given in Fig. 5. As it can be seen in the actual versus fits plots, the selected AR(1) models of datasets follow the actual data closely. As a result, the plots do not reveal any problems with AR(1) model fits to the original data of chromite data sets. These results are consistent with literature. Some mineral and mineral processing data have been shown to be modeled by simple a AR(1) model. For example, Napier-Munn and Meyer (1999) showed that mineral processing plant performance data of daily metal recovery or concentrate grade followed a first order autoregressive time series model, AR(1), for a zinc flotation plant. Meyer and Napier-Munn (1999) have shown that dependence in the daily gold feed grade and gold recovery data can be described by the AR(1) model. Elevli et al (2009) has also found recently that the AR(1) model was a suitable model for the contents of B_2O_3% at the two colemanite concentrator plants in Turkey. Similarly, Bhattacherjee and Samanta (2002) have shown that Al_2O_3% and SiO_2% constituents of a bauxite ore can be estimated by a simple AR(1) model. However, it does not mean that all mineral processing data can only be modeled by an universal AR(1) model since other time series models such as ARMA for flotation (Trybalski and Cieply, 2000), ARMA for SO_2 emissions (Gleit, 1985) and ARMA and ARIMA models for coal data (Taşdemir, 2012) have been also reported

depending on the nature of the data. Several coal data sets were found to fit often AR(1) models but not always (Cheng et al., 1982).

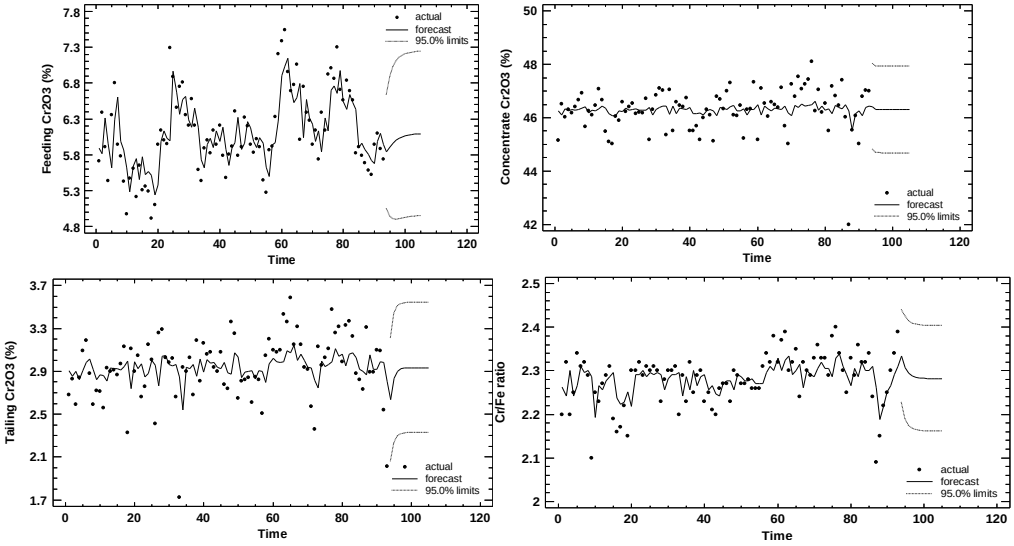

Fig. 5. Actual values versus fits plots by AR(1) models with four days estimation within 95% confidence limit

To test the performances of AR(1) models, the real data of last twelve shifts data values (observation values from 82 to 93 shifts) were estimated by the models for each data set and these results were compared to real data values. Shown in the plots of Fig. 6 are the results obtained within 95% confidence limits. As seen clearly from these plots, the results are very good for feed grade and tailing grade since most estimated data points are near to real values and all estimated points are within the 95% limits. The results can be also considered well for the other data sets. However, there is an unusual point in these plots exceeding the 95% limits. The unusual point at shift 87 is below the control limit for concentrate grade and Cr/Fe ratio. Since the variability of the chromite feeding grade does not change and within the confidence limits in the twelve shifts, this unusual point may be attributed to the measurement/analysis errors or unusual plant working conditions of plant at 87^{th} shift rather than variability of the chromite.

In the plots in Fig. 5, we can also see the forecasting lines drawn from estimation points with their 95% confidence intervals for future 4 days (12 shifts) after 30 days (after shift 93). According to the above results, the estimation of near future values of four variables can be made conveniently by applying their AR(1) models. Therefore, it is possible to make any preventive and corrective actions for the quality characteristics of chromite by using AR(1) models since these models can also forecast for the near

future of any data using past data of the processes. Estimation for the future values of data sets can reduce the operating costs.

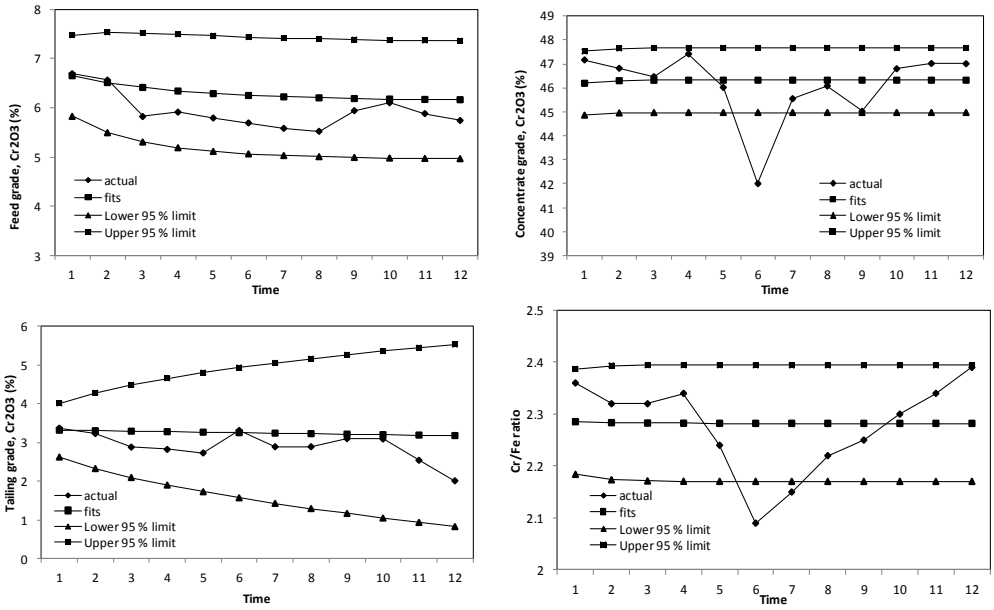

Fig. 6. Estimation of dataset values by AR(1) models versus actual values for the 82–93 shifts

Time Constant and Individual Control Charts of Original Data and ARIMA Residuals

The time constant (*T*) was found to be 3.123 for the feed grade implying that the frequency of measurements need to be reduced by dividing it by 3 to eliminate the autocorrelation. Other *T* values were found as 1.38, 0.889 and 0.576 for the Cr/Fe ratio (Table 1), tailing grade and concentrate grade respectively. These results revealed that sampling frequency needs to be reduced by dividing approximately by 1.5 for the Cr/Fe ratio. Since the *T* values are smaller than 1, there is no need to reduce sampling frequency for tailing grade and concentrate grade. For fast fluctuating processes as in the concentrate grade and tailing grade, *T* is small and simple ACF decays quickly to zero (Callao and Rius, 2003). However, fast fluctuations are more common in the concentrate grade data which has the smallest time constant.

The effect of autocorrelation may cause wrong decisions for the monitoring of data by statistical process control charts (SPCs). Its effect has been shown also important for many mineral processing/mining applications of SPCs (Samanta and Bhattacherjee, 2001; Bhattacherjee and Samanta, 2002; Samanta, 2002; Elevli et al., 2009; Taşdemir, 2012). Figure 7 compares the individual charts of Shewhart and ARIMA residuals (special cause charts) with additional Western Electric rules which are applied to improve the efficiency of control charts for small shifts. The number of West-

ern Electric rules applied was four as the same in our previous study (Taşdemir, 2012). The details of the method can be found elsewhere (Montgomery and Runger, 2011). The values of data sets were presented as normalized on the *y* axes these plots to make the comparisons easily. Since our data sets had different autocorrelation degree, this analysis showed the efficiency of the Shewhart charts of individual observations on the four data sets of chromite processing plant having from weak to moderate autocorrelations.

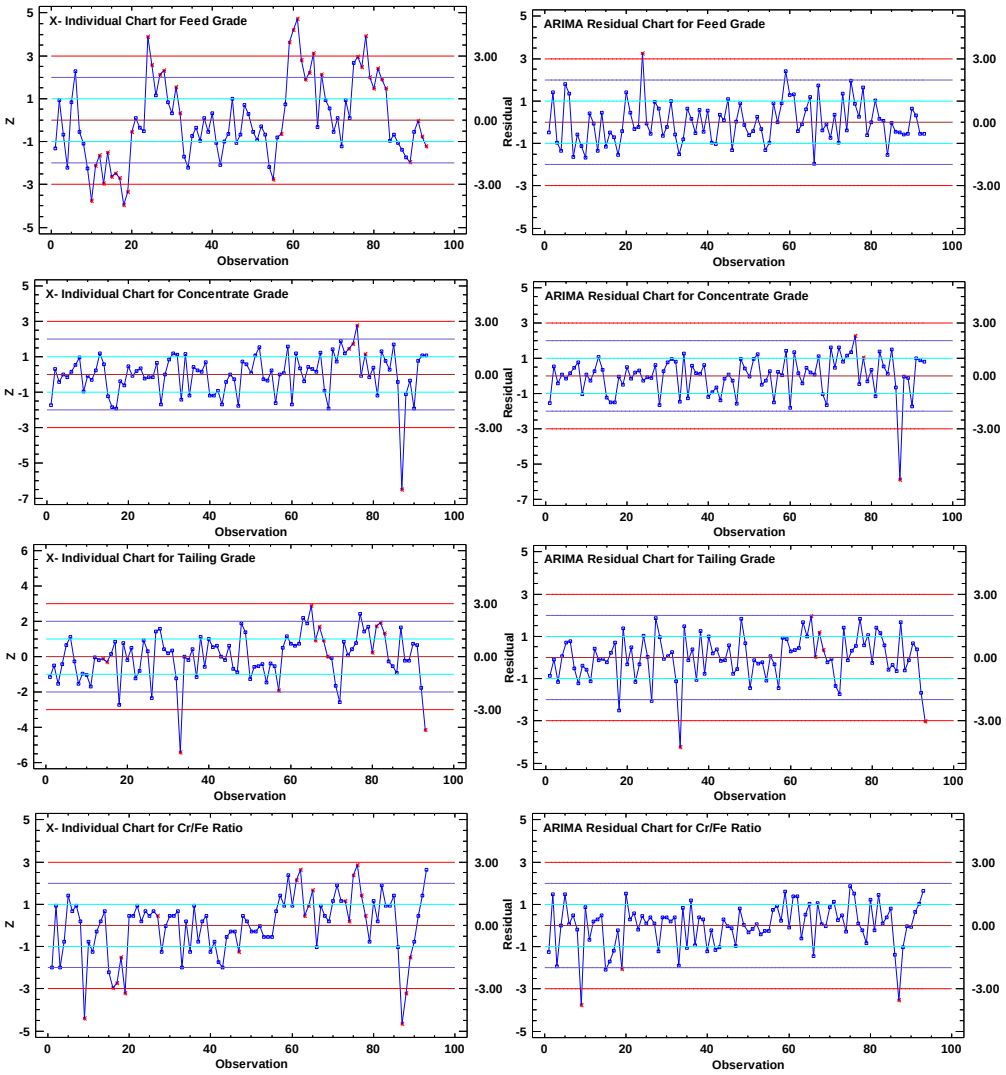

Fig. 7. Comparison of individual and ARIMA residual control charts of four chromite data sets

The Shewhart individual chart performs better when the datasets exhibit weak autocorrelation as in the case of concentrate grade and tailing grade. It shows almost the same results with ARIMA residual charts of individuals for these two data. However, when the autocorrelation increases, the number of wrong out of control points increases in the charts of original data compared to their ARIMA residual charts. These effects can be seen clearly on comparative charts of Cr/Fe ratio and feed grade variables. The Shewhart individual chart also performed weaker for Cr/Fe ratio data which has moderately autocorrelated data to catch the right out of control points. It showed the worst performance for the feed grade data which has the highest autocorrelation between the data sets when it is compared with its ARIMA residual chart. Only one point was out of control according to the residual chart while this number in original data was 39 points with Western Electric rules applied. This would cause wrong decisions about the homogeneity of chromite ore feed to plant. Almost the same wrong conclusion can be given when the Cr/Fe ratio with original data was evaluated by the Shewhart individual chart. The number of out of control points was 3 in the ARIMA residual chart of individual although it was 21 in the individual chart. The performance of the Shewhart individual chart has been shown better for weak positive and negative autocorrelation and got worse with increasing autocorrelation (Karaoğlan and Bayhan, 2011). These results clearly indicate and reconfirm that autocorrelation must be taken into account when applying Shewhart individual chart whether the process is under control or not since wrong decisions can be given for the process control applications.

Meyer and Napier-Munn (1999) confirmed that the data dependence or autocorrelation can be reduced in mineral processing plants by increasing sampling frequency or sampling intervals. Reversely, the autocorrelation between the data obtained can be increased by decreasing the sampling frequency or sampling intervals. In this situation, time constants which are obtained by the AR(1) models may provide a tool for eliminating autocorrelation which is something that must be considered if we can decrease the frequency of the analysis (Rius and Callao, 2001). It has been suggested by the Rius and Callao (2001) and Callao and Rius (2003) that time constants can be used to reduce the autocorrelation for applying the Shewhart individual chart to the original data. It is seen that the original data of concentrate grade and tailing grade can be used for control charts. It was determined that sampling frequencies needs to be reduced for feed grade and Cr/Fe ratio variables by cutting 3 and 1.5, respectively, to use their original values with control chart. Rius and Callao (2001) concluded that sampling frequency is decreased in order to apply control charts to the original data. They also stated that the decrease in the sampling frequency has the advantage since there may not need to use time series models once the optimum frequency has been established. However, they also took attention that a drawback would be detected later. This finding is especially right for the mineral processing systems since there are many causes of the data variability including the error in sampling and grade and real variation in performance due to changes in ore mineralogy. Small changes in ore mineralogy of

feed material may not be determined by using long sampling intervals. As a result, reducing the sampling frequency may cause a drawback due to the inefficiency in homogeneity control of the feed grade properly since the feeding is made as a mixture of ores from different chromite mines. In conclusion, usage of autocorrelated data and AR(1) residuals seem to be a more suitable approach to control the homogeneity in the feed grade with control charts.

Conclusions

In this study, an analytical technique, time series, was used to quantify the correlated and random components of the variability in four variables of chromite data. The data of feed grade and Cr/Fe ratio values were found more moderately autocorrelated than the other data sets over time. The data from the concentrate grade exhibited weakest autocorrelation than the other data sets. The ARIMA (1,0,0) or AR (1) models were found to fit well for all data sets obtained from a chromite processing plant. The sample ACF plots as well as further residual plots of the AR(1) models of data sets revealed that no autocorrelation was left in the data and the models give reasonable fits for all data sets. AR(1) is the first order model indicating that their values are strongly dependent on their previous measurements. The model considers the autocorrelation between measurements and random shock error term that cannot be explained by the model.

The AR (1) time series models can accommodate the autocorrelated nature of chromite dataset levels when estimating parameters to characterize the process. It is shown that the AR(1) models can be used to forecast the near future estimation of data sets investigated. Only one point was out of 95% confidence limits on the actual and estimated plots for concentrate grade and Cr/Fe ratio. The reasons may be attributed to the operating conditions and to the measurement errors since the homogeneity of feed grade was supplied by the plant for this point. Moreover, they also provide forecasting capability to take preventive actions that will be useful in process control.

The AR(1) model have been also shown a potential applications to be used for homogeneity control of feed grade, concentrate grade, tailing grade and Cr/Fe ratio in a chromite processing plant. Whether original data can be used or not may be determined by the degree of autocorrelation during the application of the Shewhart charts of individual to detect the right out of control points. The number of out of control points increases with increasing autocorrelation and this causes no suitability usage of original values with Shewhart charts. Consistent with literature, when time constant obtained by the autocorrelation of AR(1) model is smaller than 1, we can use original data, otherwise it is suitable to use the ARIMA residual charts to detect right out of control points for the data sets having higher time constants.

References

BAZIN C., GIRARD B., HODOUIN D., 2000, *Time Series Analysis: A Tool for Operators Training? Application to the Direct Reduction of Ilmenite Concentrate*, Powder Technology, 108, 155–159.

BHATTACHERJEE A., SAMANTA B., 2002, *Practical Issues in the Consruction of Control Charts in Mining Applications*, The Journal of The South African Institute of Mining and Metalllurgy, 173–180.

CAPODAGLIO A.G., NOVOTNY V., FORTINE L., 1992, *Modelling wastewater treatment plants through time series analysis*, Environmetrics, 3(1), 99–120.

CALLAO M.P., RIUS A., 2003, *Time Series: A Complementary Technique to Control Charts for Monitoring Analytical Systems*, Chemometrics and Intelligent Laboratory Systems, 66, 79–87.

CHENG B.H., WOODCOCK B., SARGENT D., GLEIT A., 1982, *Time Series Analysis of Coal Data from Preparation Plant*, Journal of Air Pollution Control Association, 32(11), 1137–1141.

ELEVLİ S., UZGÖREN N., SAVAS M., 2009, *Control Charts for Autocorrelated Colemanite Data*, Journal of Scientific & Industrial Research, 68: 11–17.

GLEIT A., 1985, *SO_2 Emissions and Time Series Models*, Journal of Air Pollution Control Association, 35(2), 115–120.

GANGULI R., TINGLING J.C., 2001, *Algorithms to Control Coal Segregation Under Non–Stationary Conditions.* Part II: *Time Series Based Methods*, International Journal of Mineral Processing, 61, 261–271.

HUANG Z., MUMAR R., YINGLING J., SOTTILE J., 2002, *Coal Segregation Control for Meeting Homogenity Standards*, International Journal of Mineral Processing, (submitted manuscript, (http://home.engineering.iastate.edu/~rkumar/PUBS/coal2.pdf).

KARAOĞLAN A.D., BAYHAN G.M., 2011, *Performance Comparison of Residual Control Charts for Trend Stationary First Order Autoregressive Processes*, Gazi University Journal of Science, 24(2):329–33.

KAYA A., 1995, *Outlier Analysis in Time Series Used for Statistical Process Control*, MSc Thesis in Statistics of Ege University (in Turkish).

KETATA C., ROCKWELL M.C., 2008, *Stream Material Variables and Sampling Errors*, Mineral Processing & Extractive Metall. Rev., 29, 104–117.

KUTLAR A., ELEVLİ S., 1999, *Estimation of World Copper Production with Linear Time Series*, Madencilik, 38(4), 43–55 (in Turkish).

MEYER D., NAPIER-MUNN T.J., 1999, *Optimal Experiments for Time-Dependent Mineral Processes*, Austral. & New Zeland J. Statist. 41(1), 3–17.

MONTGOMERY D.C., JENNINGS C.L., KULAHÇI M., 2008, *Introduction to Time Series Analysis and Forecasting*, Wiley Series in Probability and Statistics.

MONTGOMERY D.C., RUNGER G.C., 2011, *Applied Statistics and Probability for Engineers*, Fifth Edition, John Wiley&Sons, Inc.

NAPIER-MUNN and MEYER D.H., 1999, *A Modified Paired t-Test for the Analysis of Plant Trials with Data Autocorrelated in Time*, Minerals Engineering, 12(9), 1093–1100.

O'KEEFE K., SELBY W., SUTHERLAND D.N., 1981, *An Analysis of the Dynamic Characteristics of Flotation Circuits by Time-Series Analysis*, International Journal of Mineral Processing, 8(2), 147–163.

RIUS A., CALLAO M.P., 2001, *Application of Time Series Models to the Monitoring of a Sensor Array Analytical System*, Trends in Analytical Chemistry, 20(4), 168–177.

SAMANTA B., BHATTACHERJEE A., 2001, *An Investigation of Quality Control Charts for Autocorrelated Data*, Mineral Resources Engineering, 10: 53–69.

SAMANTA B., 2002, *Multivariate Control Charts for Grade Control Using Principal-Component Analysis and Time Series Modelling*, Trans. Inst. Min. Metall. (Sect A: Min Technol), 307: 149–157.

TAŞDEMİR A., 2012, *Effect of Autocorrelation on the Process Control Charts in Monitoring of a Coal Washing Plant*, Physicochemical Problems of Mineral Processing, 48(2), 495–512.

TRYBALSKI K., CIEPLY J., 2000, *ARMA Type Model for Copper Ore Flotation*, Proceedings of the XXI International Mineral Processing Congress, C3(72–78).