Aspirant Natalia IARETSKA

Khmelnitsky National University, Faculty of Computer Programming and Locomotive Systems, Department of Higher Mathematics, Ukraine *(PhD Student)*

Chmielnicki Narodowy Uniwersytet, Wydział Programowania Komputerowych Systemów Lokomocyjnych, Katedra Matematyki Wyższej, Ukraina *(Doktorantka)*

DIMENSIONAL CONTACT PROBLEM FOR THE PRESTRESSED CYLINDRICAL PUNCH AND LAYER LYING WITHOUT FRICTION ON A RIGID FOUNDATION

Abstract

Introduction and aim: The article deals with the problem of the elastic cylindrical die pressure on the layer with initial stresses within the framework of linearized elasticity theory. In general, the research was carried out for the theory of great initial deformations and two variants of the theory of small initial deformations with arbitrary structure of elastic potential.
Material and methods: The mode of deformation in the elastic layer with initial stresses is defined with the help of harmonic functions by way of Henkel integrals. It reduces the task to Fredholm equations and the method of consecutive approximations**.**
Results: We obtained a correlation between the components of potential vector and tensor of deformations in the case of equal roots**.** The solutions are defined by way of lines with the help of infinitive system of constants, derivated from the regular and linear algebraic system.
Conclusion: The research investigates the influence of initial stresses on the law of distribution of contact stresses in the layer and punch with initial stresses.

Keywords: The theory of linear elasticity, initial tension, Henkel integrals, Fredholm equations, the method of consecutive approximations.

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PRZESTRZENNE ZADANIE BRZEGOWE DLA WSTĘPNIE NAPIĘTEJ CYLINDRYCZNEJ STEMPLI I WARSTWY LEŻĄCEJ BEZ TARCIA NA SZTYWNEJ PODSTAWIE

Streszczenie

Wstęp i cel: *W ramach liniowej teorii sprężystości, należy rozważyć zadania o ciśnieniu sprężystego cylindrycznego stempla na warstwę z początkowym napięciem. Badanie wykonane w sposób ogólny do teorii znacznych odkształceń i dwóch wariantów teorii początkowych małych odkształceń przy dowolnej strukturze sprężystego potencjału.*
Materiał i metody: *Naprężenie-odkształcenie stan w elastyczne warstwie z początkowym napięciem określamy poprzez harmoniczne funkcje w postaci całki Henkelego, które pozwalają sprowadzić zadanie do całkowania równań typu Fredgolma i metody kolejnych przybliżeń.*
Wyniki: *Otrzymano korelację pomiędzy współrzędnymi wektora przemieszczeń i tensora naprężeń w przypadku równych stopni. Rozwiązanie przedstawione w postaci szeregu przez nieskończony system stałych. Te stałe są zdefiniowane z układem regularnych równań liniowych.*
Wniosek: *Zbadana kwestię wpływu naprężeń początkowych na prawo dystrybucji kontaktowych w warstwie i stempli z naprężeniami początkowymi.*

Słowa kluczowe: *Liniowa teoria sprężystości, próbne naprężenia, całki Henkelego, równania Fredholma, metoda kolejnych przybliżeń.*

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1. Introduction and aims

One of the important and topical areas of mechanics of deformed solid body is to study the problem of load transfer in car designs and parts by means of their contact interaction, taking into account the initial (residual) stress. They may occur as a result of such deformations as: plasticity, creep, structural changes in the material, aggregation state changes, physico-mechanical, chemical and technological processes, construction, creation, and after stress in elasto-plastic bopunchs. Also in the earth's crust initial (residual) stress arise due to the action of gravitational forces, technical processes etc. They cause deformations, fractures, increasing tendency in a loss of stability and internal friction, acceleration of phase transitions, corrosion and other.

The investigation of this problem is constantly stimulated by the growing needs of engineering practice, for which more precise calculation methods are required for the adoption of a new technological solutions in the application of new materials, to increase their strength (in the framework of the economic and engineering feasibility) and to reduce material capacity of designs at the same time.

The study of the problems of contact interaction of prestressed bodies appeared rather recently, in the theory of elasticity and in a limited number. However the mission of the classical theory gained a huge development in our and foreign countries. This explains the fact that the linear theory of elasticity ignores the effect of the initial (residual) stresses. But they should be taken into consideration in the framework of the linearized theory of elasticity, which was developing simultaneously in two directions, namely: taking into account the specific form of the elastic potential (V. M. Aleksandrov, N. H. Harutyunyan, V. S. Poroshin and others) [1], and arbitrary structure of the elastic potential theory for large initial deformations and two variants of the theory of small initial deformations for compressible (incompressible) bopunchs (academician of NAS of Ukraine A. N. Guz, S. Yu. Babych, V. B. Rudnitsky) [2]-[5]. In the General case objectives such requires the involvement of the staff of the non-linear theory of elasticity, but, at high initial (residual) stress can be limited, and its linearized option [5].

Today, increasing reliability and durability of structures and machines is one of the urgent problems in many industries, such as construction, mechanical engineering, aircraft building, shipbuilding and others. And this requires strict compliance with the terms of durability, reliability, cost effectiveness structures and simultaneously low material. And the successful decision of such tasks great extent with the help of scientific research in the field of mechanics of contact interaction of prestressed deformable solid phone. Therefore their solution entails much attention of scientists.

Despite the existing achievements in this direction, there is still insufficiently developed a number of points. Among which the problem of the contact interaction of prestressed bopunchs, namely: elastic pressure of a stamp and a layer with initial (residual) stresses. Because one of the aspects of transferring the load associated with the account residual stresses in bopunchs on the law of the distribution of effort in their places of contact of the calculation and the creation of significant elements of designs more efficient to consider the durability of the materials by the correct assessment of the reserves of strength and enough to reduce consumption of materials, keeping the overall desired functionality.

Works of the contact interaction of the elastic stamps with a layer or a half-space even in the framework of linear theory of elasticity quite a few. This is explained by the fact that their stupunchs are reduced to one of the most difficult equalizations of mathematical physics and the solution which is associated with considerable mathematical difficulties.

Influence of initial stresses on the distribution of contact forces in an elastic half-plane and a half-space with their contact interaction with elastic stamps investigated in the papers [4], [3], where is wrote the General method of solution.

The task about pressure of a rigid punch into an elastic layer with initial stresses considered in [2], and for a particular type of elastic potential of incompressible bopunchs in [1].

In the paper [5] described a General approach to solving the problems of contact interaction of elastic bopunchs with initial stresses in the framework of the linearized theory of elasticity, where were considered cases of deformations of the infinite elastic half-space (layer) under the action of stamps of different forms.

But, although the number of stupunchs devoted to the contact interaction of bopunchs with pre-strained state is constantly increasing, the task about pressure the elastic layer and stamp with initial (residual) voltages the authors have not been investigated here in the framework of the linearized theory of elasticity.

Because this work is great practical and theoretical value, and accounting for initial (residual) stress is one of actual problems of load transfer in designs and details of cars.

The purpose of the work is the solution of the axi-symmetric static task about pressure the elastic cylindrical punch into an elastic layer with initial (residual) stress within the linearized elasticity theory in a General form for the theory of large (finite) initial deformations and two variants of the theory of small initial deformations with an arbitrary structure of the elastic potential.

Research conducted in the coordinates of the initial strain state Oy_i, associated with the Lagrangian co-ordinates the following ratios: $y_i=\lambda_i x_i$ $(i=\overline{1,3})$, where λ_i - coefficients lengthening determining movement of the initial state.

We consider the isotropic elastic body (compressible or incompressible) with an arbitrary form of the elastic potential. In the case of orthotropic bodies, we assume that the elastic-equivalent directions coincide with the direction of the coordinate axes in deforming condition.

Let assumption that the initial state of the layer and stamp uniform and equal, and elastic potentials - is twice continuously differentiated functions of the algebraic invariants of the deformation tensor of Green [5]. In addition, the action of stamp calls in a layer of a small perturbation of the main stress state, for which the conditions

$$S_0^{11} = S_0^{22} \neq 0; \quad S_0^{33} = 0; \quad \lambda_1 = \lambda_2 \neq \lambda_3$$

It is assumed that the elastic cylindrical stamp and layer made of various isotropic, transversally-isotropic or composite materials and interact on the square in one of the foundations of a stamp.

2. Statement of problem

We will consider the elastic cylindrical stamp (Fig. 1) radius R and height H with initial (residual) voltages which pressed into an elastic layer under the action of the force P after the occurrence of there initial stresses (which appear to contact). The symbol h_1 is the thickness of the layer initial strain state that is associated with a thickness of h_2 in the not-strain state the following relation: $h_1=\lambda_3 h_2$.

Power is applied to the elastic stamp so that its free end face is deformed in the direction of the axis Oy_3 on the same value ε, and the surface outside the contact area are free from the stress.

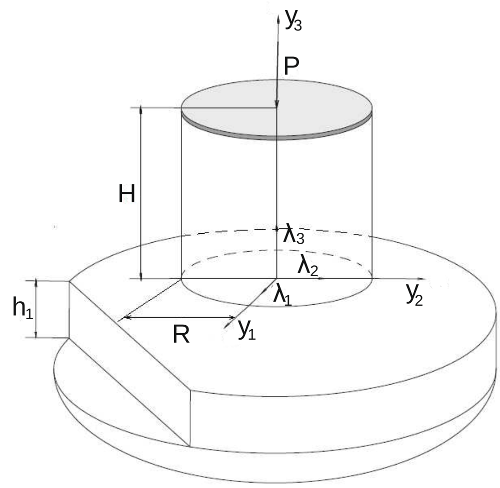

Fig. 1. Cylindrical stamp and a layer with initial (residual) voltages
Source: Elaboration of the Author

In the system of circular cylindrical coordinates $(r,\ \theta,\ z_i)\,(i=\overline{1,2})$ we get such boundary conditions:

- At the end of the elastic stamp $z_i = Hv_i^{-1}$, where $v_i = \sqrt{n_i}$, $(i=\overline{1,2})$:

$$u_3^{(1)} = -\varepsilon; \quad \tilde{Q}_{3r}^{(1)} = 0 \quad (0 \le r \le R)\,; \tag{1}$$

- On the boundary of an elastic layer in the contact region $z_i = 0$:

$$u_3^{(1)} = u_3^{(2)}; \quad \tilde{Q}_{33}^{(1)} = \tilde{Q}_{33}^{(2)} \quad \tilde{Q}_{3r}^{(1)} = \tilde{Q}_{3r}^{(2)} = 0 \quad (0 \le r \le R)\,; \tag{2}$$

- On the boundary of an elastic layer outside of the contact area $z_i = 0$:

$$\tilde{Q}_{33}^{(2)} = 0 \quad \tilde{Q}_{3r}^{(2)} = 0 \quad (R \le r < \infty)\,; \tag{3}$$

- On the lateral surface of the elastic stamp $r=R$:

$$Q_{rr}^{\prime(1)} = 0, \quad Q_{3r}^{\prime(1)} = 0, \quad (0 \le z_i \le Hv_i^{-1}) \tag{4}$$

- On the bottom surface layer lying on the hard ground, and United with the basis,

$$z_i = -\lambda_3 h_2 v_i^{-1} = -Hv_i^{-1}\ (i=\overline{1,2})$$

$$u_3^{(2)} = 0 \quad \tilde{Q}_{3r}^{(2)} = 0 \quad (0 \le r < \infty)\,, \tag{5}$$

$$u_3^{(2)} = 0 \quad u_r^{(2)} = 0 \quad (0 \le r < \infty). \tag{6}$$

where $z_i = y_3 v_i^{-1}, \quad (i=\overline{1,2})$; n_i – the roots of allowing equation [5, (2.19)].

From the equilibrium condition, we establish the relationship between the draft and the resultant load P

$$P = -2\pi\pi^2 \int_0^1 \rho Q_{33}^{(2)}(0,\rho)d\rho. \tag{7}$$

Note that all values related to the elastic stamp marked by an upper index «(1)», to the layer «(2)», and to the base «(3)».

For determining the stress-strain state in elastic cylinder with initial stresses, which is pressed into the elastic pre busy layer, use the expressions, which is provided in [6] for the components of the displacement vector and stress tensor for compressible and incompressible bo-punchs in case of equal roots allowing equations [5].

Stress-strain state in an elastic layer with initial stresses write through harmonic functions in the form of integrals Henkel. Satisfying the boundary conditions (1) - (6), after a number of transformations we obtain:

$$u_3^{(2)} = \theta_3\left(\int_0^\infty \frac{F(\eta)}{\eta} J_0(\eta\rho)d\eta - \int_0^\infty \frac{F(\eta)}{\eta} G(\eta h) J_0(\eta\rho)d\eta\right),\ Q_{33}^{(2)} = \theta_1 \int_0^\infty F(\eta)J_0(\eta\rho)d\eta,\ Q_{3r}^{(2)} = 0 \quad (8)$$

where η - dimensionless value, $F(\eta)$ - unknown function,

$$G(t) = \frac{t - e^{-t} + 1}{sht} P(t), \qquad P(t) = \frac{\kappa \cdot sht}{t + \kappa \cdot sht}, \qquad \kappa = s_0 - s, \qquad t = \frac{2h\eta}{v_1}, \qquad \theta_1 = C_{44} l_1 (1 + m_1)\kappa,$$

$$\theta_3 = \frac{m_1(s_1 - s_0)}{v_1}, \quad h = h_1 / R, \quad \varphi_i = 2\eta \frac{h}{\sqrt{n_i}}, \quad s_0 = (1+m_2)(1+m_1)^{-1}, \quad s_1 = (m_1 - 1)m_1^{-1}, \quad s = s_0 l_2 l_1^{-1},$$

$$C_{44} = \begin{cases} \omega'_{1313}, \\ \kappa'_{1313}; \end{cases} \qquad m_1 = \begin{cases} (\omega'_{1111}n_1 - \omega'_{3113})(\omega'_{1133} + \omega'_{1313})^{-1}, \\ \lambda_1 q_1 n_1 (\lambda_3 q_3)^{-1}; \end{cases}$$

$$l_1 = \begin{cases} \omega'^{-1}_{1313}(\omega'_{1331} + (\omega'_{1313} - \omega'_{1331})(\omega'_{1133} + \omega'_{1313})(\omega'_{1111}n_1 + \omega'_{1133})^{-1}), \\ \kappa'^{-1}_{1313}(\kappa'_{1331} + \lambda_3 q_3 (\kappa'_{1313} - \kappa'_{1331})(\lambda_3 q_3 + \lambda_1 q_1 n_1)^{-1}); \end{cases}$$

$\omega'_{\alpha\beta ab}$, $\kappa'_{\alpha\beta ab}$ – components of voltage, respectively for compressible and incompressible bo-punchs, $J_v(x)$, $I_v(x)$ – the Bessel functions of real and imaginary argument.

3. Method of solution

Using solutions for cylinder [6] and satisfying the boundary conditions (1) - (6), we find the eigenvalues tasks:

$$\gamma_k = 2\pi k H^{-1},\ \alpha_k = \mu_k R^{-1}.$$

Not stopping at the side, say that the unknown function of (8) is determined as a result of reduction of a problem to paired integral equations of Fredholm type of the second kind [6] using the method of successive approximations [5], [6] when $h > 1, \lambda_1 > \lambda_{kr}$. This method is convergent, noting the research carried out in [5]. Therefore, the solution is presented in the form of series through the infinite system of constants. These constants are determined from a system of regular [5] linear algebraic equations

$$\vartheta_k \chi_k + \sum_{n=0}^{\infty} \vartheta_{kn} \chi_n = \varpi_k, \quad (k = 0,1,2,..) \qquad (9)$$

where the coefficients of which is provided in [6].

Defining unknown constants $\chi_i \quad (i = 0,1,2,..)$ from (9), we can calculate the force P

$$P = 8\pi\varepsilon E \theta_1 (\kappa\theta_2 l R)^{-1} \chi_0,$$

moves and stresses in an elastic stamp [6] and the layer with the initial (residual) voltages. When calculating stresses and displacements for the layer with initial stresses the majority of the integrals in the end, are not evaluated, given the complexity of functions G(η), F(η). Therefore, starting from the second approximation of the integral functions in (8) lay in a series in powers of the h^{-1}, that allows us to calculate the coefficients of (9) approximations by writing them in the form of series [6].

Therefore, displacements and stresses in an elastic layer with initial stresses determined by the following formulas:

$$u_r'^{(2)} = \varepsilon(\pi\theta_3)^{-1}\tilde{T}^1(\Omega_+^1; S_1^1; N_0^1; K_0^1; 1; 1-s_0),\ u_3'^{(2)} = m_1\varepsilon(\pi\theta_3 v_1)^{-1}\tilde{T}^1(\Omega_-^1; S_1^0; N_0^0; K_0^0; s_1; s_1 - s_0)$$

$$Q_{33}'^{(2)} = (1+m_1)\varepsilon l_1 C_{44}(\pi\theta_3 R)^{-1}\tilde{T}^1(\Omega_+^1; S_2^0; N_1^0; K_1^0; s; s-s_0)$$

$$Q_{3r}'^{(2)} = -(1+m_1)\varepsilon C_{44}(\pi\theta_3 R v_1)^{-1}\tilde{T}^1(\Omega_-^1; S_2^1; N_1^1; K_1^1; s_0; 0)$$

where $S_n^m(\rho; z) = \int_0^\infty \eta^{n-2}\sin\eta e^{-z\cdot\eta} J_m(\eta\rho)d\eta$, $K_n^m(\rho; \mu_k; z) = \int_0^\infty \eta^n \psi_0(\eta, \mu_k)e^{z\cdot\eta}J_m(\eta\rho)d\eta$,

$$\tilde{T}^1(\Omega_\pm^l; S_{m_1}^n; N_{m_2}^n; K_{m_3}^n; \mathrm{k}; \mathrm{a}) = (1+a_0)\Big\langle (1-\chi_0)\Omega_\pm^l\left(S_{m_1}^n; 0; \mathrm{k}; \mathrm{a}; 0\right) - \frac{\theta_3}{\varepsilon}\sum_{j=0}^\infty C_j^{**}\Omega_\pm^l\left(S_{j+m_1}^n; 0; \mathrm{k}; \mathrm{a}; 0\right) -$$

$$-2(m_2-1)R^2\theta_2^{-1}\chi_0\Omega_\pm^l\left(N_{m_2}^n; 0; \mathrm{k}; \mathrm{a}; 0\right) + \theta_4\sum_{k=1}^\infty \chi_k\Omega_\pm^l\left(K_{m_3}^n; \mu_k; \mathrm{k}; \mathrm{a}; 0\right) +$$

$$+0,5(m_2-1)R^2\sum_{k=1}^\infty b_1^{(k)}\chi_k\Omega_\pm^l\left(K_{m_3}^n; i\gamma_k v_1 R; \mathrm{k}; \mathrm{a}; 0\right)\Big\rangle + \sum_{\tau=1}^\infty a_\tau\Big\langle (1-\chi_0)\Omega_\pm^l\left(S_{m_1}^n; 0; \mathrm{k}; \mathrm{a}; v_1\tau\right) -$$

$$-\theta_3\varepsilon^{-1}\sum_{j=0}^\infty C_j^{**}\Omega_\pm^l\left(S_{j+m_1}^n; 0; \mathrm{k}; \mathrm{a}; v_1\tau\right) - 2(m_2-1)R^2\theta_2^{-1}\chi_0\Omega_\pm^l\left(N_{m_2}^n; 0; \mathrm{k}; \mathrm{a}; v_1\tau\right) +$$

$$+\theta_4\sum_{k=1}^\infty \chi_k\Omega_\pm^l\left(K_{m_3}^n; \mu_k; \mathrm{k}; \mathrm{a}; v_1\tau\right) + 0,5(m_2-1)R^2\sum_{k=1}^\infty b_1^{(k)}\chi_k\Omega_\pm^l\left(K_{m_3}^n; i\gamma_k v_1 R; \mathrm{k}; \mathrm{a}; v_1\tau\right)\Big\rangle,$$

$$\Omega_\pm^1(\hat{\mathrm{L}}_m^n, \mu, \mathrm{k}, \mathrm{a}, \theta) = (A_1^{01} + kA_2^{01} + \mathrm{a})\,\hat{\mathrm{L}}_m^n\left(\rho, \mu, \frac{z_1}{R} - \theta\right) + \left(A_1^{11} + \frac{z_1}{R}(A_2^{01}+1)\right)\hat{\mathrm{L}}_{m+1}^n\left(\rho, \mu, \frac{z_1}{R} - \theta\right) +$$

$$+(A_1^{21} + A_2^{11})\,\hat{\mathrm{L}}_m^n\left(\rho, \mu, \frac{2h_1}{Rv_1} + \frac{z_1}{R} - \theta\right) + \left(A_1^{31} + \frac{z_1}{R}A_2^{11}\right)\hat{\mathrm{L}}_{m+1}^n\left(\rho, \mu, \frac{2h_1}{Rv_1} + \frac{z_1}{R} - \theta\right) \pm (A_1^{01} + kA_2^{01} + \mathrm{a})\times$$

$$\times\hat{\mathrm{L}}_m^n\left(\rho, \mu, -\frac{z_1}{R} - \theta\right) \pm \left(A_1^{11} - \frac{z_1}{R}(A_2^{01}-1)\right)\hat{\mathrm{L}}_{m+1}^n\left(\rho, \mu, -\frac{z_1}{R} - \theta\right) \pm (A_1^{21} + A_2^{11})\,\hat{\mathrm{L}}_m^n\left(\rho, \mu, \frac{2h_1}{Rv_1} - \frac{z_1}{R} - \theta\right) \pm$$

$$\pm\left(A_1^{31} - \frac{z_1}{R}A_2^{11}\right)\hat{\mathrm{L}}_{m+1}^n\left(\rho, \mu, \frac{2h_1}{Rv_1} - \frac{z_1}{R} - \theta\right),$$

$$A_1^{01} = \frac{s_0(s_0 - s_1)}{s_0 + s_1},\ A_1^{11} = \frac{2h_0 s_0}{v_1 R(s_0 + s_1)},\ A_1^{31} = \frac{h_1}{s_0 v_1 R}A_1^{21},\ A_1^{21} = \frac{2s_0(s_0 - s_1)(s_0 + 2s_1)}{(s_0 + s_1)^2},$$

$$A_1^{13} = \frac{h_0(\beta k(s_1 - 2 - s_0)s_0 - 1 + s + s_1) + \alpha k_1(2s_0 s - s_0^2 - s + 1))(s_0 + 1)}{\alpha k_1 v_1 R(s - s_0 - 1)^2},\ A_1^{03} = \frac{s}{s - s_0 - 1},$$

$$A_2^{03} = \frac{s}{s_0 + 1 - s},\ A_2^{13} = \frac{h_1(\beta k(s + s_1 - 2s_0 - 1 + s_0 s_1 - s_0^2) + \alpha k_1(1 + 2s_0 s - s_0^2 - s^2))}{-\alpha k_1 v_1 R(s - s_0 - 1)^2},$$

$$N_n^m(\rho; z) = \int_0^\infty \eta^n \psi_1(\eta, 0)e^{z\cdot\eta}J_m(\eta\rho)d\eta,\ \psi_n(x, y) = \int_0^1 t^n \cos xt \cos yt dt,$$

$$m_2 = \begin{cases} (\omega'_{1133} - \omega'_{1313})(\omega'_{1133} + \omega'_{1313})^{-1}, \\ 1, \end{cases} \quad \theta_2 = E\left(\frac{8m_1(1+H)}{n_1} - \frac{4H}{v_1} + \frac{(1-m_2)R^2}{H}\right),$$

where $b_1^{(k)}$ is expressed with the boundary conditions (1)-(6), a_j, C_j^{**} – the coefficients of expansions in series of functions G(η), F(η), respectively, χ_k – desired constants of an infinite system of linear equations (9).

4. Results and discussion

The system [6] was solved by reduction with the following values of parameters: k=32, E=3,92; *v*=0,5. And the influence of the initial (residual) stress on the distribution of the contact forces for the task about pressure the elastic cylindrical punch into an elastic layer with initial stresses that lies without friction on the hard ground, consider for the example of a harmonic potential (the case of compressible bopunchs) what is portrayed in Fig. 2 – Fig. 5.

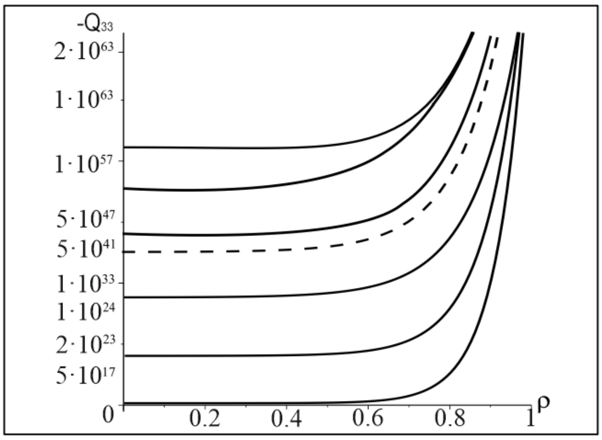

Fig. 2. Contact stresses for an elastic cylinder with initial stresses
Source: Elaboration of the Author

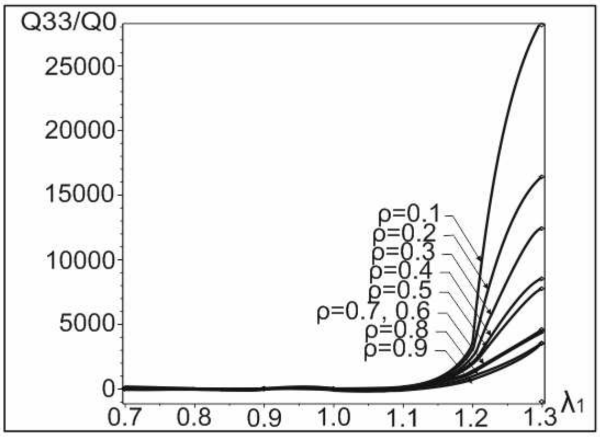

Fig. 3. Influence of initial stresses on the distribution law
Source: Elaboration of the Author

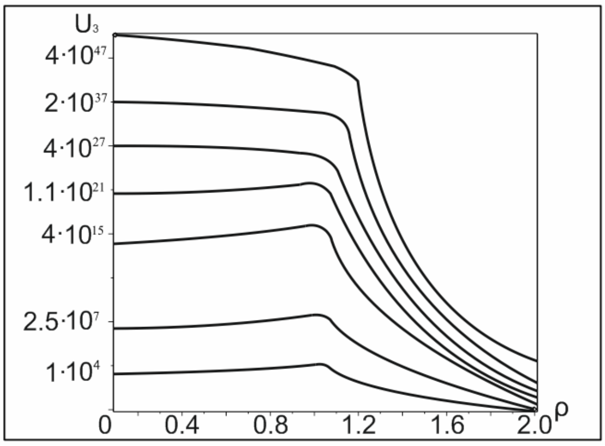

Fig. 4. Displacement U_3 in an elastic layer with initial stresses
Source: Elaboration of the Author

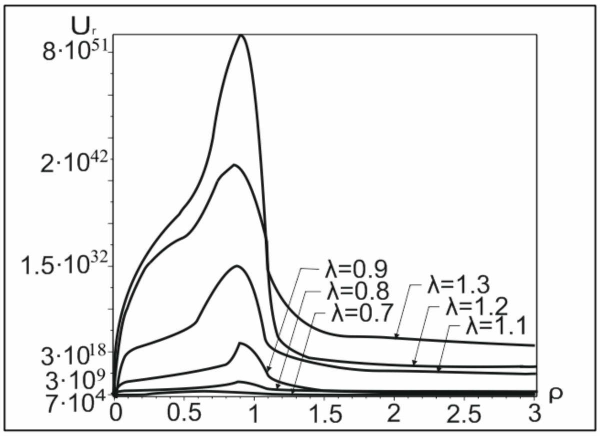

Fig. 5. Displacement U_r in an elastic layer with initial stresses
Source: Elaboration of the Author

On the basis of permanent found χ_j (j=0,1,2,...) from the system (9) identified the characteristics of stress-strain state of cylindrical punch and the layer with the initial (residual) stresses. All the presented charts depict the curves that correspond to the next values λ_1=0,7; 0,8; 0,9; 1; 1,1; 1,2; 1,3 where λ_1 - determines the initial state of the layer and the cylinder. For the value λ_1=1 (condition without the initial stress) will the graphs denote the shaded line. If all calculations put h=∞, we can get solutions for half-space with initial stresses [5].

Of analytical and graphical results of the work revealed that:

- if $\lambda_1 \to \lambda_{кр}$ influence of initial stresses in an elastic layer is rapidly decreasing, and in elastic cylinder tends to zero. That is, as in [5], revealed the mechanical effect, which is that the elastic cylindrical stamp in this case is in a state of elastic equilibrium and almost no effect on the layer,

- initial stress in compression lead to reduction of power voltages in the cylinder, tensile strength - to increase and for the movements of the other way around,
- the greatest impact of initial stresses marked on the side of stamp,
- for the elastic layer of influence of initial stresses similar, and on character of action of initial stresses its thickness does not affect, and affect only on their values.

All the conclusions that were made in the work consistent with the conclusions of the impact of initial stresses on the distribution of the contact forces, stupunchs in [4], [5].

5. Conclusions

Based on the findings, we propose the following recommendations for practical use of the obtained results:

- Initial stress in compression lead to a reduction of force of stresses in the cylindrical stamp and a layer, tensile strength - to increase, in the case of movements of the other way around. That is, the presence of pre-stressed state at the time of contact interaction of elastic bopunchs gives the possibility to adjust the contact stresses and move in calculating the strength of parts of machines and structures. Moreover, the contact stresses are dangerous initial stresses in case of a strain to move - in case of compression.
- The greatest impact of initial stresses found on the side of the punch. And thickness of the layer does not affect the nature of the action of initial stresses, it only affects the value.
- More significantly, in quantitative terms, initial stresses operate in the highly elastic materials compared with more stringent, but qualitatively their influence remains.
- Dangerous is the situation when the initial voltage closer to the values of the surface instability, because the contact efforts and move sharply change their values.

Thus, in this work we obtained dependencies reflecting the effect of the initial stress on stress-deforming the state of a system of elastic cylinder and the layer with the initial stresses that lies without friction on the hard ground. This impact is significant as for compressible and as for not compressible bopunchs and should be considered in engineering calculations.

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Literature

[1] Aleksandrov V.M., Poroshin V.S.: *Contact problem for the prestressed physically nonlinear elastic layer.* English Journal «Solid Mechanics», Moscow, 1984, № 6, pp. 79-85.

[2] Babich, S. Yu.: *Contact problem of elasticity theory for a layer with initial stresses.* Applied Mechanics, Kyiv, 1984, 20, № 6, pp. 34-40.

[3] Guz A.N., Rudnitsky V.B.: *Contact problem of elastic pressure of a stamp on an elastic half-space with initial stresses.* Applied Mechanics, Kyiv,1984, 20, № 8, pp. 3-11.

[4] Guz A.N., Rudnitsky V.B.: *Contact problems for elastic bopunchs with initial stresses applied to the rigid and elastic punches.* Applied Mechanics, Kyiv, 2004, 40, № 7, pp. 41-69.

[5] Guz A.N., Rudnitsky V.B.: *Fundamentals of the theory of contact interaction of elastic bopunchs with initial (residual) stresses.* ed. PP Melnik, Khmelnitsky, 2006, p. 710.

[6] Iaretska N.A.: *Mixed static contact problem for an elastic layer and elastic cylindrical punch with initial (residual) stresses.* Scientific Journal of the Ternopil National Technical University, Ternopil, 2013, № 2 (70), pp. 63-73.