# ENSURING IDENTIFICATION ACCURACY OF NAVIGATIONAL ACCELEROMETER'S CONVERSION FUNCTION PARAMETERS IN THE GRAVITY FIELD OF THE EARTH

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#### Abstract

The problems of determining instrumental errors of navigation accelerometer metrological model's coefficients determination by its calibration on an uniaxial swivel stand and ensuring the specified accuracy of the calibration were considered. Analytical expressions for calculating appropriate identification errors and conditions for the calibration equipment the implementation of which provides the desired accuracy of the accelerometer calibration were gained. Adequacy of those expressions and conditions by calibrating a navigation accelerometer of a real inertial navigation system was experimentally confirmed. Adequacy of those expressions and conditions by calibrating a navigation system was confirmed experimentally. Keywords: navigation accelerometer, calibration, identification, metrology model, instrumental errors, specified accuracy.

## **1. INTRODUCTION**

Navigation pendulous accelerometers (NA) are the sensors of the primary information of practically all contemporary strapdown intertial navigation systems (SINS) and orientation systems (SSO). It is a well-known fact that accelerometer's drifts seriously affect errors in tasks solved by SINS and SSO.

By accelerometer's metrological model (MM) we understand the mathematical formula for estimating the projection of the apparent linear acceleration value by measuring the accelerometer's output signals meaning. Coefficients of this metrological model are the individual certificated coefficients of NA, which are identified by the NA's calibration.

In [1] is described a metrological model of a navigational accelerometer unit that considers NA's cross sensitivity components. It also shows the algorithm of this model's coefficients identification.

In [2] is described a nonlinear (second order) MM of uniaxial NA and model of its coefficients identification that was gained by an approximate solution of nonlinear equations set that caused appearance of methodic errors of MM's coefficients identification. In [3] was described the algorithm of identification of the nonlinear (third order) MM's NA coefficients [4]. Numerical values of this metrological model's coefficient identified by NA's calibration according to this algorithm do not contain methodic errors because those values are calculated by expressions that were gained by authors without any approximation.

However, the problem that still remains unsolved is the problem of instrumental drifts of nonlinear MM's coefficients determination and the problem of assigned accuracy of identification by using a stand equipment with required tolerance that is used for calibration.

## 2. PROBLEM STATEMENT

The purpose of this article is to solve the following problems:

 developing a mathematical model of instrumental errors of navigation accelerometer nonlinear metrological model's coefficients identification;

- ensuring the accuracy of nonlinear MM's coefficients identification by making demands on the on the stand equipment that is used for its calibration.

## 3. METROLOGICAL MODEL OF NA AND EXPRESSIONS FOR DETERMINATION OF ITS COEFFICIENTS

A possible solution of the stated problems for metrological model defined in [4] for pendulous NA is shown in figure1, where: 1) is NA's housing; 2) are housing elements which define NA's basic mounting surface A; 0XYZ is a coordinate associated with surface A and OX is the pendulous axis (PA), OY is output axis (OA); OZ is the input axis (IA) orthogonal to the surface A.



Fig. 1. Uniaxial navigation accelerometer [Chernyak, Baranovska, Terokhin, 2015]

This model in the units of input acceleration can be represented as follows:

$$\begin{aligned} a_{ip} &= \hat{a}_i - k_{0\Sigma} - 0.5k_{1A}\hat{a}_i sign\hat{a}_i - k_2 \hat{a}_i^2 - k_3 \hat{a}_i^3 - \delta_o \hat{a}_p - \delta_p \hat{a}_o - \delta_{ip} \hat{a}_i \hat{a}_p , [g], \\ k_{1A} &= \left(K_{1+} - K_{1-}\right) K_1^{-1}, \end{aligned}$$
(1)

where  $a_{iP}$  – calculated after NA's metrological model value of input acceleration;  $a_0, a_p$  – projections of apparent acceleration on output (OA) and pendulous (PA) axis of NA;  $\hat{a}_j = \hat{Y}_j / K_1$  – estimation

of the tested NA's output signal in input acceleration units;  $\hat{a}_{o(p)} = \hat{Y}_{o(p)} / K_1$  – estimation of the output signal of other accelerometers of navigation system whose IAs oriented along the OA  $(\hat{a}_o)$  and PA  $(\hat{a}_p)$  of the test NA, in input acceleration units;  $K_1$  – scale factor (SF) of the accelerometer;  $K_{1+}, K_{1-}$  – real scale factors when  $a_i > 0$  and  $a_i < 0$ ;  $k_{1A}$  – certificated factor of SF asymmetry;  $k_{0\Sigma}$  – certificated zero offset factor;  $k_2, k_3$  – certificated nonlinearity factors;  $\delta_p, \delta_o$ , – certificated factors of additive cross sensitivity;  $\delta_{ip}$  – certificated factor of multiplicative cross sensitivity.

According to the [4], MM's coefficients are determined by method of NA test-positioning in terrestrial gravitation field described in [3]. The method lies in placing the accelerometer into 8 test positions (TP) relatively to the horizon plane (HP) with the help of a precise uniaxial swivel stand (for example, the optical index head (OIH)). Each position is formed by a rotation angle of NA relatively to the HP  $\varphi_j$ ,  $(j = \overline{1,8}, , \text{ where } j - \text{the test position number that begins with } \varphi_1 = 0^\circ \text{ with } 45^\circ \text{ step}$ ), defined by the following formula:

$$\varphi_{j+1} = \varphi_j + 45^\circ, \ (j = 1, 7).$$
 (2)

In each testing position output signals rates of NA  $Y_j$  are measured and later they are used for a calculation of numeric values of appropriate MM's coefficients according to the following expressions:

$$K_{1-} = \frac{1}{-3g} [Y_5 + \sqrt{2}(Y_4 + Y_6) - 0.5(1 + 2\sqrt{2})(Y_3 + Y_7)], [B / g];$$

$$K_{1+} = \frac{1}{3g} [Y_1 + \sqrt{2}(Y_2 + Y_8) - 0.5(1 + 2\sqrt{2})(Y_3 + Y_7)], [B / g];$$

$$K_1 = 0.5(K_{1+} + K_{1-}), [B / g]; \quad k_2 = \frac{1}{2K_1g^2} [Y_2 + Y_4 + Y_6 + Y_8 - 4K_1k_{0\Sigma}], [1 / g];$$

$$k_{0\Sigma} = \frac{(Y_3 + Y_7)}{2K_1}, [g]; \quad k_3 = \frac{\sqrt{2}}{K_1g^3} [Y_6 - Y_2 + \sqrt{2}gK_1(1 - \delta_o)], [1 / g^2];$$

$$\delta_{o(p)} = \frac{Y_7 - Y_3}{2K_1g}, [1]; \quad \delta_{ip} = \frac{1}{2K_1g^2} [Y_8 + Y_4 - Y_2 - Y_6], [1 / g].$$
(3)

## 4. MATHEMATICAL MODEL OF INSTRUMENTAL ERRORS OF NAVIGATION ACCELEROMETERS METROLOGICAL MODEL'S COEFFICIENTS IDENTIFICATION

Authors of the article [4] developed formulas (3) analytically without any approximations or numerical solving of equations set. Therefore, values of appropriate coefficients defined by those formulas do not contain methodic errors. In this case, only instrumental errors will appear during the coefficient identification with the help of expressions (3). The causes of these errors are drifts of calibration equipment. According to the NA test-positioning method [2,3], there are only two sources of sought instrumental errors: error of NA positioning relatively to the HP and error of NA's output signal measurement. Total influence of both those errors causes the effect when practical values of NA's output signals in each position differs from the ideal (when errors of positioning NA and measuring of its output signals are absent) ones on the value of  $\Delta Y_j$ . The formulas (3) took into consideration the fact that:

$$\begin{split} K_{1-\phi} &= \frac{1}{-3g} [ \left( Y_5 + \Delta Y_5 \right) + \sqrt{2} (Y_4 + Y_6 + \Delta Y_4 + \Delta Y_6) - 0.5(1 + 2\sqrt{2}) \left( Y_3 + Y_7 + \Delta Y_3 + \Delta Y_7 \right) ] = \\ &= K_{1-} - \frac{1}{3g} [ \Delta Y_5 + \sqrt{2} (\Delta Y_4 + \Delta Y_6) - 0.5(1 + 2\sqrt{2}) \left( \Delta Y_3 + \Delta Y_7 \right) ]; \\ K_{1+\phi} &= K_{1+} + \frac{1}{3g} [ \Delta Y_1 + \sqrt{2} (\Delta Y_2 + \Delta Y_8) - 0.5(1 + 2\sqrt{2}) \left( \Delta Y_3 + \Delta Y_7 \right) ], \left[ B / g \right]; \\ k_{0\Sigma\phi} &= k_{0\Sigma} + \frac{\Delta Y_3 + \Delta Y_7}{2K_1}, \left[ g \right]; \quad k_{3\phi} = k_3 + \frac{\sqrt{2}}{K_1 g^3} \left[ \Delta Y_6 - \Delta Y_2 \right], \left[ 1 / g^2 \right]; \\ \delta_{o\phi}(p\phi) &= \delta_{o(p)} - \frac{\Delta Y_7 - \Delta Y_3}{2K_1 g}, \left[ 1 \right]; \quad k_{2\phi} = k_2 + \frac{1}{2K_1 g^2} \left[ \Delta Y_2 + \Delta Y_4 + \Delta Y_6 + \Delta Y_8 \right], \left[ 1 / g \right]; \\ \delta_{ip\phi} &= \delta_{ip} + \frac{1}{2K_1 g^2} \left[ \Delta Y_8 + \Delta Y_4 - \Delta Y_2 - \Delta Y_6 \right], \left[ 1 / g \right]. \end{split}$$

$$\tag{4}$$

Every expression of the set (4) consists of two parts. One part matches expressions (3) and the other one is an additional part that depends on the added errors  $\Delta Y_j$ . Those parts will determine the sought errors of MM's coefficients identification. They can be represented with the help of the following expressions:

$$\delta_{K1} = \frac{(K_{1+\phi} - K_{1+}) + (K_{1-\phi} - K_{1-})}{2K_1} = \frac{1}{12K_1g} \Big[ \Delta Y_1 - \Delta Y_5 - \sqrt{2} \Big( \Delta Y_2 + \Delta Y_8 - \Delta Y_4 - \Delta Y_6 \Big) \Big], [1];$$
  

$$\delta_{K2} = \frac{1}{2K_1k_2g^2} \Big[ \Delta Y_2 + \Delta Y_4 + \Delta Y_6 + \Delta Y_8 \Big], [1]; \quad \delta_{Mip} = \frac{1}{2K_1\delta_{ip}g^2} \Big[ \Delta Y_8 + \Delta Y_4 - \Delta Y_2 - \Delta Y_6 \Big], [1];$$
  

$$\Delta_0 = \frac{\Delta Y_3 + \Delta Y_7}{2K_1}, [g]; \quad \delta_{Mo(Mp)} = \frac{\Delta Y_7 - \Delta Y_3}{2K_1\delta_{o}(p)g}, [1]; \quad \delta_{K3} = \frac{\sqrt{2}}{K_1k_3g^3} \Big[ \Delta Y_6 - \Delta Y_2 \Big], [1].$$
(5)

In formulas (5) were used the following designations:  $\Delta_0$ 

- error of zero offset factor identification;  $\delta_{K1}$
- · relative error of scale factor identification;  $\delta_{K2}$ ,  $\delta_{K3}$
- · relative errors of nonlinearity factors identification;  $\delta_{Mo}$ ,
- · relative errors of additive cross sensitivity factors identification;  $\delta_{Min}$ ,
- · relative error of multiplication cross sensitivity factor identification.

In order to find  $\Delta Y_j$  its sources need to be considered, i.e., random error of NA's output signal measurement  $\Delta Y_B$  and error of NA's positioning relatively to the HP. The last one, according to Fig. 2, includes systematic (the same in every position) errors of the initial leveling ( $\beta_1$ ,  $\beta_2$ ) and a random error of testing position assignment ( $\Delta \varphi$ ).

In Fig. 2. are shown: 1) shaft of the OIH that serves as a dial of NA test positions relatively to the HP; 2) platform connected with shaft on which NA is mounted; 3) test NA;  $\varphi$  – rotation angle around the axis of shaft that is equal to the angle  $\varphi_j$  (2);  $OX_{\Gamma}Y_{\Gamma}Z_{\Gamma}$  – coordinates associated with the horizontal plane, and  $OY_{\Gamma}$  axis is in the HP co-directional to the OIH's shaft spinning axis,  $OZ_{\Gamma}$  axis is perpendicular to the HP;  $OX_{\Pi}Y_{\Pi}Z_{\Pi}$  are coordinates associated with the platform for NA mounting, and  $OY_{\Pi}$  is the spinning axis of the OIH' shaft,  $OZ_{\Pi}$  axis is perpendicular to the basic mounting surface *B* of the platform. During the calibration, NA is mounted on the platform so that

its input axes parallel to the platform axes  $OZ_{\Pi}$  and axes OA and PA are correspondingly parallel to the axes  $OY_{\Pi}$  and  $OX_{\Pi}$ .



Fig. 2. Orientation of accelerometer axis 0XYZ relatively to the HP when errors of its positioning exist [Chernyak, Baranovska, Terokhin, 2015].

According to figure 2, projections of apparent linear acceleration on the axis of the accelerometer in position *j* in first approximation (for small angles  $\Delta \varphi$ ,  $\beta_1$ ,  $\beta_2$ ) have the following form:

$$a_{ij} = -g_{ij} \approx g \Big[ \cos \varphi_j - (\beta_2 \pm \Delta \varphi) \sin \varphi_j \Big];$$
  

$$a_{pj} = -g_{pj} \approx -g \Big( (\beta_2 \pm \Delta \varphi) \cos \varphi_j + \sin \varphi_j \Big); \quad a_{oj} = -g_{oj} \approx g \beta_1.$$
(6)

In order to find the differences  $\Delta Y_j$  the difference between real and ideal output signals of NA in each test position needs to be determined. An expression for the real output signals can be found by placing the expressions (6) into MM of NA's output signal:

$$Y_{j\phi} = K_{1}(k_{0\Sigma} + (1+0,5k_{1A}signa_{i})(\cos\varphi_{j} - (\beta_{2} \pm \Delta\varphi)\sin\varphi_{j})g + k_{2}(\cos\varphi_{j} - (\beta_{2} \pm \Delta\varphi)\sin\varphi_{j})^{2}g^{2} + k_{3}(\cos\varphi_{j} - (\beta_{2} \pm \Delta\varphi)\sin\varphi_{j})^{3}g^{3} - \delta_{o}g((\beta_{2} \pm \Delta\varphi)\cos\varphi_{j} + \sin\varphi_{j}) + \delta_{p}g\beta_{1} - \delta_{ip}(\cos\varphi_{i} - (\beta_{2} \pm \Delta\varphi)\sin\varphi_{j})((\beta_{2} \pm \Delta\varphi)\cos\varphi_{i} + \sin\varphi_{i})g^{2}) \pm \Delta Y_{B}, [B].$$
(7)

The expression that describes output signals of the NA in the ideal case can be found by equating values of  $\beta_1$ ,  $\beta_2$  and  $\Delta \phi$  errors to zeros:

$$Y_{j} = K_{1}(k_{0\Sigma} + (1+0,5k_{1A}signa_{j})g\cos\varphi_{j} + k_{2}g^{2}\cos^{2}\varphi_{j} + -\delta_{jp}(\cos\varphi_{j} - (\beta_{2} \pm \Delta\varphi)\sin\varphi_{j})((\beta_{2} \pm \Delta\varphi)\cos\varphi_{j} + \sin\varphi_{j})g^{2}) \pm \Delta Y_{B}, [B]$$
(8)

The difference between (7) and (8) is the sought difference of output signals  $\Delta Y_j$  in each test position:

$$\Delta Y_{j} = K_{1}(k_{0\Sigma} - (1+0.5k_{1A}signa_{j})((\beta_{2} \pm \Delta \phi)\sin\phi_{j})g - k_{2}((\beta_{2} \pm \Delta \phi)\sin\phi_{j})g - k_{2}(\beta_{2} \pm \Delta \phi)\cos\phi_{j})g + \delta_{p}g\beta_{1} - \delta_{ip}(\beta_{2} \pm \Delta \phi)\sin\phi_{j})(\beta_{2} \pm \Delta \phi)\cos\phi_{j}g^{2}) \pm \Delta Y_{B}, [B].$$
(9)

Let us find the expressions for the MM's coefficients identification errors from the error of NA's in dependence on output signal measurement  $\Delta Y_B$ , errors of initial leveling ( $\beta_1$ ,  $\beta_2$ ) and error of testing position assignment ( $\Delta \varphi$ ). In order to achieve it, one needs to substitute (9) into (5) taking into consideration the random nature of errors  $\Delta Y_B$  and  $\Delta \varphi$ . It allows for the use of a geometric sum instead of an algebraic one. For each test position an appropriate value of angle  $\varphi_j$  needs to be chosen and calculated by the formula (2) beginning with the initial horizontal value. As a result, we receive, in the first approximation relatively to the  $K_1$  value, following the expressions for the sought identification errors calculation:

$$\begin{split} \delta_{K2} &= \sqrt{\frac{1}{32}} \Delta \varphi^2 + \frac{1}{16g^4 K_1^2 k_2^2} \Delta Y_B^2}, \ [1]; \ \delta_{K3} &= \sqrt{\frac{2}{g^4 k_3^2}} \Delta \varphi^2 + \frac{1}{g^6 K_1^2 k_3^2} \Delta Y_B^2}, \ [1]; \\ \delta_{K1} &= \sqrt{\frac{1}{18}} \Delta \varphi^2 + \frac{1}{27g^2 K_1^2} \Delta Y_B^2}, \ [1]; \ \Delta_0 &= \sqrt{\frac{g^2}{8}} \Delta \varphi^2 + \frac{1}{8K_1^2} \Delta Y_B^2}, \ [g]; \ (10) \\ \delta_{Mo(Mp)} &\approx -\beta_{2(1)} \frac{1}{\delta_{o(p)}} + \sqrt{\frac{1}{8\delta_{o(p)}^2}} \Delta \varphi^2 + \frac{1}{8g^2 K_1^2 \delta_{o(p)}^2} \Delta Y_B^2}, \ [1]; \ \delta_{Mip} &= \sqrt{\frac{1}{32}} \Delta \varphi^2 + \frac{1}{16g^4 K_1^2 \delta_{ip}^2} \Delta Y_B^2}, \ [1]; \end{split}$$

Expressions (10) are the mathematical model of instrumental errors of navigation accelerometer metrological model's coefficients identification by test-positioning method in the terrestrial gravitational field. Their analysis shows that identification errors of all MM's coefficients depend only on the error of testing position assignment ( $\Delta \varphi$ ) and the error of NA's output signal measurement  $\Delta Y_B$ . Errors of initial leveling  $\beta_1$ , influence only the tolerance of cross sensitivity factors identification.

By the means of formulas (10) instrumental errors of navigation accelerometer metrological model's (1) coefficients identification depending on the certificated calibration equipment's drifts ( $\beta_1$ ,  $\beta_2$ ,  $\Delta \phi$  and  $\Delta Y_B$ ) can be calculated.

#### 5. ENSURING THE ACCURACY OF MM'S COEFFICIENTS IDENTIFICATION

In the case when in the calibration task there are demands on allowable errors of metrological model's coefficient identification, namely specified:  $[\Delta_0]$ 

- · allowable error of zero offset factor identification;  $[\delta_{K1}]$ ,
- · allowable relative error of scale factor identification;  $[\delta_{Mo}], [\delta_{Mp}], [\delta_{M$
- · allowable relative errors of additive cross sensitivity factors identification;  $[\delta_{Mip}]$ ,
- allowable relative error of multiplication cross sensitivity factor identification. In this situation, expressions (10) help to find demands on calibration equipment tolerance that ensures specified requirements.

In order to find those demands from (10) we need to find the expressions that relate calibration equipment drifts ( $\beta_1$ ,  $\beta_2$ ,  $\Delta \varphi$  and  $\Delta Y_B$ ) to the allowed MM's coefficient identification errors, specified

in the calibration task. First, demands to the test position assignment need to be made. In order to do that, we omit the influence of errors  $\beta_1$ ,  $\beta_2$  and  $\Delta Y_B$  in formulas (10) by implementation of the following conditions:

$$A^{2}\Delta Y_{B}^{2} \leq 0, 1B^{2}\Delta \varphi^{2}; \quad C\beta_{1(2)} \leq B\Delta \varphi, \tag{11}$$

where  $A^2$ ,  $B^2$ , C – corresponding coefficients near the  $\Delta Y_B^2$ ,  $\Delta \varphi^2$  and  $\beta_{1(2)}$  in the expressions (10). Ensuring the conditions presented in (11) allows for getting the following set of inequalities that characterize demands on the error of testing position assignment  $\Delta \varphi$ :

$$\Delta \varphi_{K1} \leq 3\sqrt{2} \left[ \delta_{K1} \right]; \quad \Delta \varphi_{K0} \leq \frac{2\sqrt{2}}{g} \left[ \Delta_0 \right]; \quad \Delta \varphi_{K2} \leq \frac{8}{\sqrt{2}} \left[ \delta_{K2} \right]; \quad \Delta \varphi_{K3} \leq \frac{g^2 k_3}{\sqrt{2}} \left[ \delta_{K3} \right];$$

$$\Delta \varphi_{Mo(Mp)} \leq 2\sqrt{2} \left[ \delta_{Mo(Mp)} \right]; \quad \Delta \varphi_{Mip} \leq \frac{8}{\sqrt{2}} \left[ \delta_{Mip} \right]; \quad (12)$$

The expressions (12) and further indexes *K0*, *K1*, *K2*, *K3*, *Mip*, *Mo*, *Mp* refer to the corresponding MM's NA coefficient whose identification tolerance determines corresponding allowable calibration equipment's drifts.

In order to find demands on tolerance of NA's output signals meter and demands on leveling accuracy, it is necessary to solve inequalities (11) relative to  $\beta_1$ ,  $\beta_2$  and  $\Delta Y_B$  for every coefficient. As a result, we receive the following inequalities sets:

$$\beta_{2Mo} \le \delta_o \left[ \delta_{Mo} \right]; \ \beta_{1Mp} \le \delta_p \left[ \delta_{Mp} \right]; \tag{13}$$

$$\Delta Y_{BK1} \le 3\sqrt{3}gK_{1}[\delta_{K1}]; \ \Delta Y_{BK0} \le 2\sqrt{2}K_{1}[\Delta_{0}]; \ \Delta Y_{BK2} \le 4g^{2}K_{1}k_{2}[\delta_{K2}]; \ \Delta Y_{BK3} \le g^{3}K_{1}k_{3}[\delta_{K3}];$$
  
$$\Delta Y_{BMo(p)} \le 2\sqrt{2}gK_{1}\delta_{o(p)}[\delta_{Mo(p)}]; \ \Delta Y_{BMip} \le 4g^{2}K_{1}\delta_{ip}[\delta_{Mip}]$$
(14)

Inequalities sets (12...14) allow to determine demands on allowable calibration equipment's drifts as sources of instrumental errors of navigation accelerometer metrological model's coefficients identification in the case of specification of allowable errors of identification of those coefficients.

### 6. EXAMPLE OF OBTAINED RESULTS USAGE

As an example of the obtained results we should consider the calibration by model (1) of the navigational accelerometer with the tensoresistance angle sensor (TAS) that was studied in the article [3]. The following numerical values of its metrological model certificated coefficients were determined there:

$$K_1 = 1.5 \left[ B / g \right]; \ k_2 = 105 \left[ \mu g / g^2 \right]; \ k_3 = 87 \left[ \mu g / g^3 \right]; \ \delta_o \approx \delta_p \approx \delta_{ip} = 1.15 \, mRad.$$
(15)

According to the calibration task, it is necessary to ensure identifications of those coefficients with the following allowable errors:

$$[\Delta_0] = \pm 50 \,\text{mkg}; \left[\delta_{K1}\right] = \pm 0,01\%; \left[\delta_{K2(3)}\right] = \pm 5\%; \left[\delta_{Mo(Mp,Mip)}\right] = \pm 1\%.$$
(16)

After a substitution of allowable identification errors values (16) and numerical coefficient valued into formulas (12-14) the corresponding calibration equipment's drifts limits can be found:

From the inequalities (17) demands on identification tolerance (16) should be achieved if the calibration equipment's drifts does not exceed the following values:

$$\Delta \varphi = \Delta \varphi_{K3} \le 6"; ... \beta_2 = \beta_{2M0} \le 2,5"; \ \Delta Y_B = \Delta Y_{BK3} \le 6 \mathsf{M} \kappa B.$$
<sup>(18)</sup>

Requirements (18) are the numerical values of maximal allowable calibration equipment's drifts. They show that the error of the testing position assignment  $\Delta \varphi$  and the error of NA's output signal measurement  $\Delta Y_B$  are determined by an allowable identification error of the cube nonlinearity factor  $[\delta_{K3}]$ . Errors of initial leveling  $\beta_1$ ,  $\beta_2$  are determined by an allowable identification error of additive cross sensitivity factors  $[\delta_{Mo}]$ ,  $[\delta_{Mp}]$ .

In order to confirm the realization of the calibration task, when demands on calibration equipment's drifts (18) are provided, the experiment was made. The experiment was meant to calibrate NA with TAS whose numerical MM's coefficients values had been determined beforehand. The calibration algorithm was described in [4] and required equipment shown in Fig. 3, where: 1 was the foundation untied from the construction 2; 3 was OIH; 4 was OIH's shaft; 5 was the type of the heat chamber TWT-2; 6 was the NA's power source; 7 was the precision voltmeter; 8 was the computer; A1, A2, A3 – NA, whose MM's coefficients are determined; IA1, IA2, IA3 were input axes of the appropriate NA.



Fig. 3. Calibration equipment [Chernyak, Baranovska, Terokhin, 2015]

In the experiment, the numerical values of MM's NA coefficients were determined. After that, the errors of their identification were calculated by substraction from the founded numerical coefficients values their reference values (15).

In the first case, conditions (18) were provided by choosing appropriate calibration equipment, a precise test position alignment and a precise initial leveling. In the second case, test positions of NA relatively to the HP were not precise ( $\Delta \varphi = 50^{\circ}$ ) and the initial leveling was not precise, either

 $(\beta_{1(2)} = 20'')$ . The voltmeter with bigger drifts had been chosen. As a result, numerical values of MM's coefficients identification errors for each case that are written in table 1 were obtained.

Errors	$\Delta_0, [\mu g]$	$\delta_{K1}, [\%]$	$\delta_{K2}, [\%]$	$\delta_{K3}, [\%]$	$\delta_{Mo(p)}, [\%]$	δ <sub>Mip</sub> ,[%]
Case 1.	12,3	0,001	1,5	4,5	1	0,005
Case 2.	70	0,005	20	53	4	0,02

Tab. 1. MM's NA coefficients identification errors [Chernyak, Baranovska, Terokhin, 2015]

Comparing values of MM's NA coefficients identification errors from table 1 in each case with their allowable ones (16) we can see that the provision of conditions (18) ensures the specified accuracy of MM's NA coefficients identification. If conditions (18) are not provided, errors  $\Delta_0$ ,  $\delta_{K2}$ ,  $\delta_{K3}$  and  $\delta_{Mo(p)}$  will exceed their allowable values considerably. However, errors  $\delta_{K1}$  where  $\delta_{Mip}$  still remain within appropriate limits.

# 7. CONCLUSIONS

The mathematical model of instrumental errors of navigation accelerometer nonlinear metrological model's coefficients identification developed in this article shows that calibration equipment's errors of testing position assignment and NA's output signal measurement has an influence on the tolerance of identification of all MM's coefficients and the errors of initial leveling have an influence only on the tolerance of identification of additive cross sensitivity factors. Moreover, the influence of testing position assignment error on total error of identification of MM's NA coefficient does not depend significantly on numerical values of those coefficients and the influence of error of NA's output signal measurement inversely depends on the coefficients numerical values. This fact makes ensuring MM's NA coefficients identification tolerance much more complicated because identification of small numerical values of MM's coefficients require calibration equipment with a higher tolerance.

Choosing stand equipment that is used for calibration of a nonlinear metrological model related to the gained conditions, ensures identification with assigned accuracy of all its metrological model coefficients.

## BIBLIOGRAPHY

- Koryukin, M. S., 2003, Russian Federation Patent No. 2477864. Moscow: Federal Service for Intellectual Property.
- [2] Ustyugov, M.N. & Schipitsyna, M.A., 2006, "Calibration of accelerometer of strapdown inertial navigation system.", SUSU Bulletin, 14, pp. 140-143.
- [3] Chernyak M.G. & Hazynedarlu, E., 2009, "Navigational pendulous accelerometer calibration method of testing positions in gravitational field of the Earth", Mechanics of Gyroscopic Systems, 20, pp. 81-91.
- [4] Chernyak M.G. & Hazynedarlu, E., 2009, "Study of the metrological characteristics of the navigational compensational pendulum accelerometer with transformer angle sensor." Information Systems, Mechanics and Control, 3, 5-20.

# ZAPEWNIENIE DOKŁADNOŚCI WYZNACZANIA PARAMETRÓW FUNKCJI PRZEKSZTAŁCENIA AKCELEROMETRU NAWIGACYJNEGO W POLU GRAWITACYJNYM ZIEMI

## Streszczenie

Praca skupia się na wyznaczeniu błędów instrumentalnych wyznaczania współczynników nieliniowego modelu pomiarowego akcelerometru nawigacyjnego podczas kalibracji na jednoosiowym stoisku obrotowym, ponadto zapewnieniu zadanej dokładności powyższej kalibracji. Otrzymano wzory analityczne do wyznaczenia odpowiednich odchyłek identyfikacji oraz wymagania dla stoiska pomiarowego, spełnienie których pozwoli na utrzymanie zadanej dokładności kalibracji akcelerometru. W drodze doświadczeń potwierdzono prawidłowość wyżej opisanych wzorów i wymagań, poprzez wykonanie kalibracji nawigacyjnego akcelerometru rzeczywistego inercyjnego systemu nawigacyjnego.

<u>Słowa kluczowe:</u> akcelerometr nawigacyjny, kalibrowanie, identyfikacja, metrologiczny model, błędy instrumentalne, zadana dokładność.