DEVELOPMENTS IN MECHANICAL ENGINEERING
ISSN: 2720-0639 DOI: 10.37660/dme.2020.16.8.3 **16(8)2020**

Determination of the thickness of anodized layer on the basis mathematical models

Anna Piochacz[1], Jarosław Robert Mikołajczyk[2]

[1] Zakład Obróbki Aluminium ZOBAL, Trzcianka k. Piły, Poland; e-mail: ania.piochacz@gmail.com
[2] Stanisław Staszic University of Applied Sciences in Piła, Poland; e-mail: jmikolajczyk@puss.pila.pl

Summary: The process of aluminium surface anodizing carries many variable factors that can be influenced to optimize this process. Efforts to obtain surfaces resistant to corrosion are desirable and always valid. The paper presents preliminary studies on the influence to selected parameters of the anodizing process on the thickness of the anodic coating obtained. On the basis of obtained test results, implemented into R software [1–20], appropriate models were built from which one, the most adequate model, was selected.

Key words: electroplating, anodizing, statistic, software R

1. Introduction

Anodizing is one of the conversion coating and it consists in creating aluminium oxide on the aluminium surface [6, 7, 15, 16]. The coating created during this process is harder and more resistant to corrosion than the layer formed in a natural way.

The aluminium alloy EN AW-6060 was selected as the sample material which is characterized by a big to corrosion, average tensile strength and average fatigue strength [4]. Tested samples measured 30x10x100 [mm]. The goal of the tests was to determine the impact of anodizing time on the thickness of obtained layers. TOP CHECK test apparatus was used for examining the thickness of anodized layers.

Four numbered samples (for their further identification) numbered 1, 4, 5, 6, were prepared for testing which were subjected to the following processes:

- degreasing;
- etching;
- washing.

Above-mentioned samples were anodized in identical conditions as regards bath temperature and current density values. Only the time of anodized was a variable parameter for the listed samples. The measured thickness of obtained coating is presented in Table 2. The bath temperature for these samples was +18°C, and the current density 1.2 [A/dm^2]. The time of anodizing process for tested samples is presented in Table 1 [1, 2, 3].

In order not to hinder or disrupt the factory production process it was decided that the duration of the anodizing process of tested samples should correspond to the currently fulfilled production orders. Also, the shapes of samples corresponded to the orders realized so that no material could be wasted.

The purpose of this work is to determine the thickness of the anodized layer depending on the duration of the anodizing process at the assumed technological parameters. And on the basis of this relationship an appropriate mathematical model/models shall be developed.

Carrying out tests in a production facility is usually very difficult because production is governed by its own laws. Then, the most important factor is the economic factor. And this is understandable. Therefore, it was decided to analyze the anodizing process in real production conditions.

Table 1. Duration of the anodizing process [min] for individual samples [own study]

Sample No.	Time [min]
1	40
4	10
5	30
6	54

2. Test results

Below presented are results obtained during the performed tests. And Table 2 presented the measured thickness of obtained coating for tested samples.

Table 2. Measured thickness of obtained anodized coating [μm] for samples No. 1, 4, 5 and 6 [own study]

Sample No.	Meas. 1	Meas. 2	Meas. 3	Meas. 4	Meas. 5	Meas. 6	Meas. 7	Meas. 8	Meas. 9	Meas. 10
1	16.01	15.98	15.94	16.02	16.00	16.15	15.98	15.97	16.03	15.99
4	3.02	2.90	2.95	3.05	3.10	2.94	2.92	2.90	3.10	3.10
5	9.98	10.20	10.10	9.95	10.00	9.85	9.80	10.05	10.10	10.00
6	25.88	26.20	26.10	26.15	25.85	25.80	26.00	26.30	25.90	25.95

For measured thickness of anodized layers calculated were average values for individual samples which are presented in Table 3. Next, a diagram was drawn up showing the dependence of average thickness of obtained coating on the duration of anodizing process, which is presented in Fig. 1.

Table 3. Average thickness values of the anodized coating for the tested samples [own study]

Sample No.	Average thickness values of the anodized coating [μm]
4	3.00
5	10.00
1	16.01
6	26.01

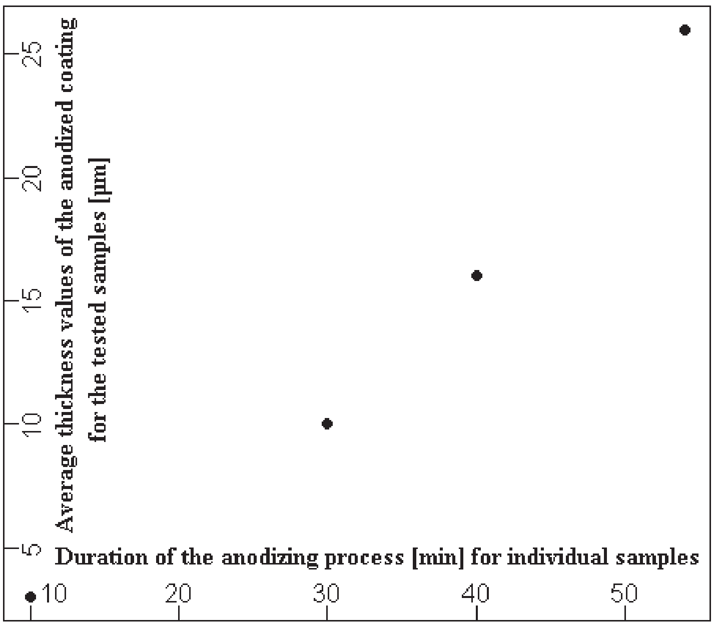

Fig. 1. Dependence of the average thickness of the anodized coating on the duration of the anodizing process for a fixed bath temperature +18°C and a fixed current density 1.2 A/dm² [own study]

Then, the average values of the anodized coating thickness were implemented into the R software to generate the models.

- model No. 1 (anodized time – anodized layer thickness; simple regression)

$$y = 0.5178 \cdot x - 3.591$$

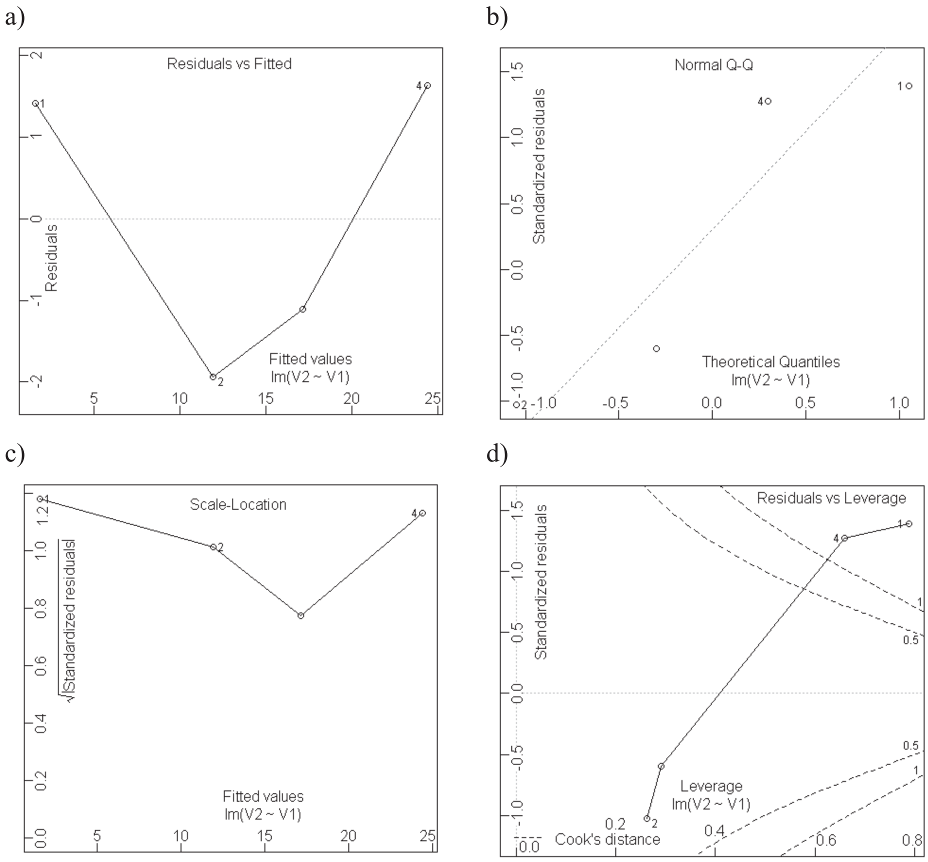

Fig. 2. Diagnostic graphs for model No. 1 (anodizing time – anodized layer thickness, simple regression) [own study]

In the graph headed "Residuals vs Fitted", y_i values, fitted by the model, are represented on the axis of abscissae and ε_i residual values are shown on the axis of ordinates. For an adequate model, the residuals should not functionally depend on the dependent variable; they should have a conditional mean equal to zero regardless of the value of y_i. On this diagnostic graph we can assess whether the mean value of residuals depends on y_i (this is bad) or not (this is good).

In the graph with the heading "Normal Q-Q" (quantile graph for the normal distribution) on the axis of abscissae presented are values of the quantiles of normal distribution corresponding to the residuals, and the empirical (experimental) quantiles for the standardized residuals are presented on the vertical axis. For an adequate model, the residuals have a normal distribution, so the points on the graph should be arranged along a straight line (marked with a dashed line). Deviations from this line indicate an abnormality.

In the graph headed "Scale Location", the axis of abscissae shows the values fitted by the y_i model, and the axis of ordinates shows the roots of standardized residuals. For an adequate model, the variance of residuals should be homogeneous and, in particular, it should not be functionally dependent on the values fitted by the model. The presence of any trend line suggests a deviation from the assumption of homogeneous variance. Heterogeneous variance can be reduced by using an appropriate data transformation which stabilizes the variance.

The graph headed "Residuals vs Leverage" allows to identify atypical values (outliers). The standardized residuals are presented on the axis of ordinates, and the so-called h_i leverages (measures of the influence of this observation on the assessment of model coefficients, called leverage) are shown on the axis of abscissae. The leverage determines the influence of the observation of y_i on the valuation of the explained variable. In an adequate model, a single observation should not have a significantly greater impact on the values of coefficient ratings than other observations. This graph is based on the rule of thumb.

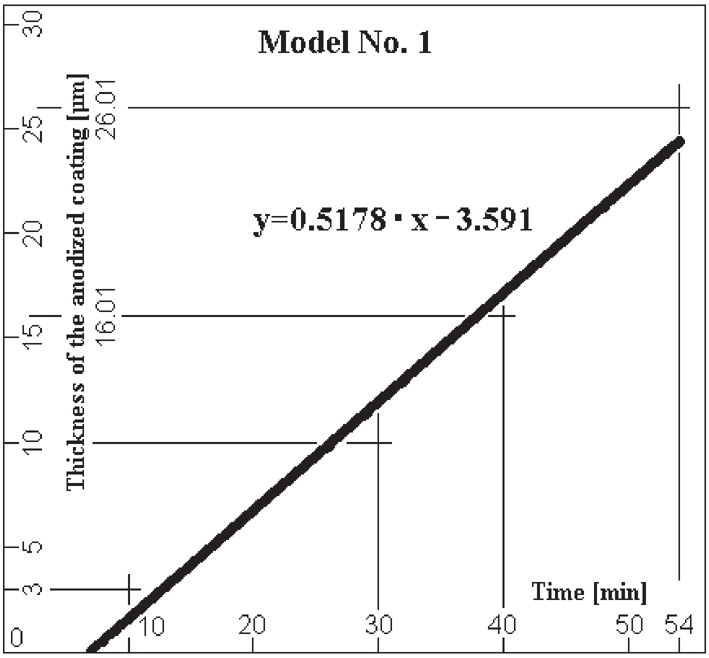

Fig. 3. Dependence of the thickness of the anodized coating on the duration of the anodizing process for model No. 1 [own study]

- model No. 2 (anodizing time – anodized layer thickness, second degree curvilinear regression)

$$y=3.103 \cdot x^2 + 16.594 \cdot x + 13.755$$

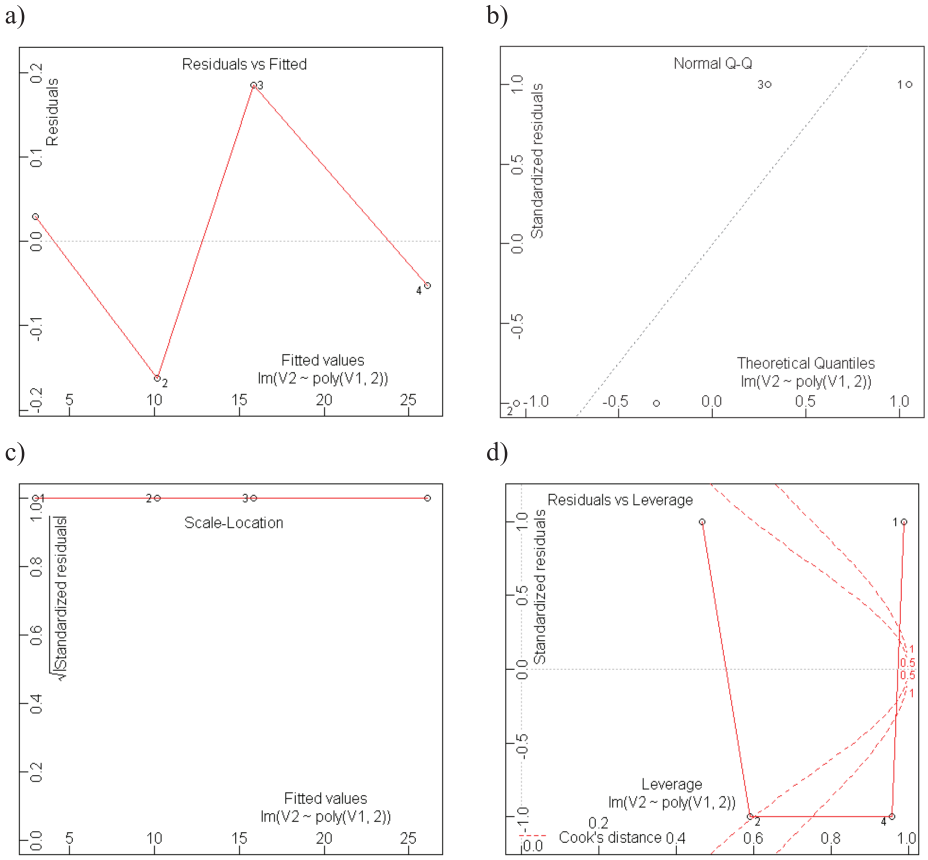

Fig. 4. Diagnostic graphs for model No. 2 (anodizing time – anodized layer thickness, second degree curvilinear regression) [own study]

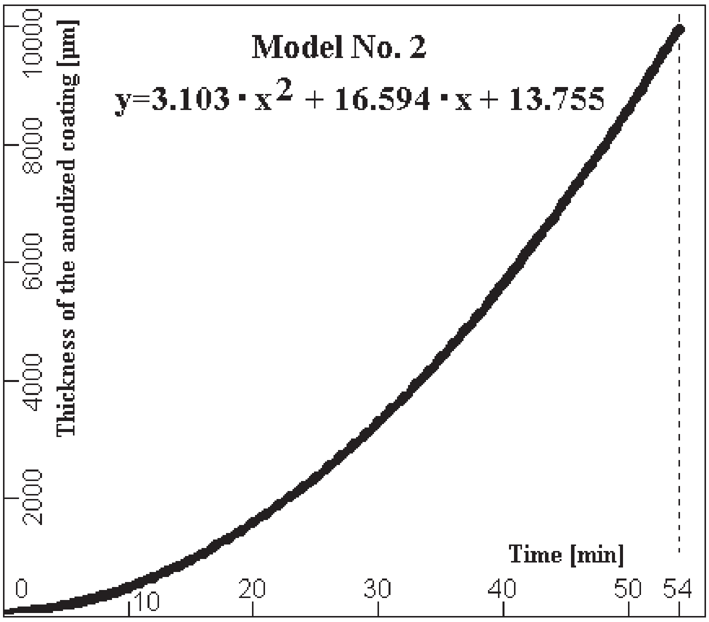

Fig. 5. Dependence of the thickness of the anodized coating on the duration of the anodizing process for model No. 2 [own study]

- model No. 3 (anodizing time – anodized layer thickness; third degree curvilinear regression)

$$y = -0.2544 \cdot x^3 + 3.1033 \cdot x^2 + 16.5935 \cdot x + 13.755$$

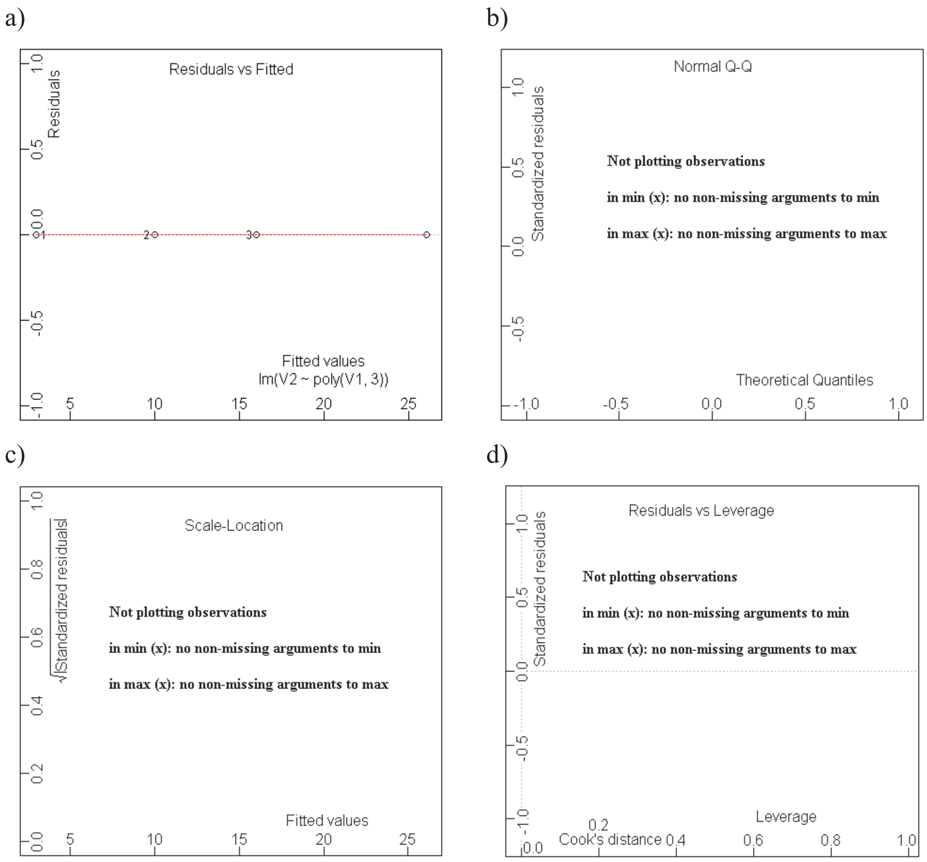

Fig. 6. Diagnostic graphs for model No. 3 (anodizing time – anodized layer thickness, third degree curvilinear regression) [own study]

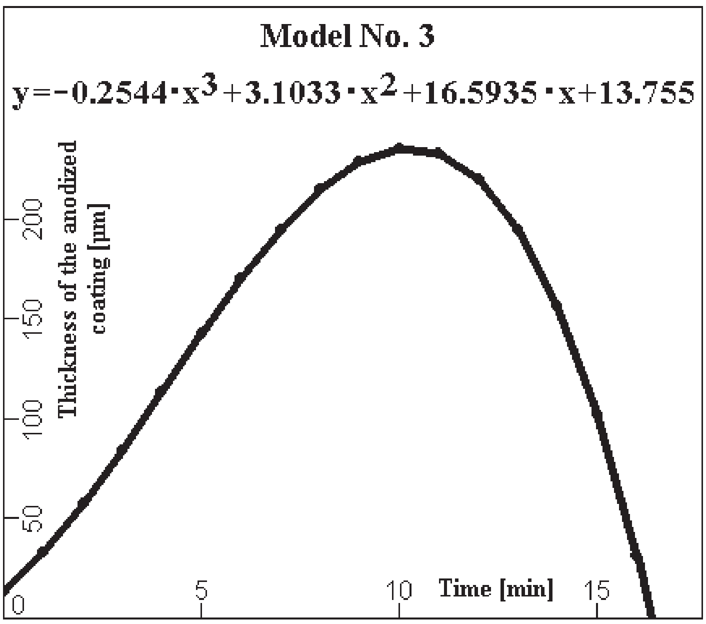

Fig. 7. Dependence of the thickness of the anodized coating on the duration of the anodizing process for model No. 3 [own study]

Next, the value of the correlation between the duration of the anodizing process and the obtained anodized layer thickness was examined (data from Tables 1 and 3 were implemented into R software). The following values were obtained:

- correlation calculated by Pearson's method: 0.983;
- correlation calculated by Spearman's method: 1.000.

3. Summary

All models mentioned above have some disadvantages, and a small number of tests seems to be the greatest drawback (only four values of the average thickness of the anodized coating obtained from 40 measurements). In principle, mathematical models work well for large data sets. Therefore, building adequate mathematical models was rendered very difficult for this small set of results. Generating higher-order models, also for this particular case, does not bring anything beneficial. It just becomes irrational. The most sensible way out of this situation is to complete the results so as to obtain more data, e.g. one reading of anodized layer thickness every minute (average thickness e.g. from ten measurements). Then, for a one hour, we obtain sixty results of the average thickness of the anodized layer for 600 measurements. Technically, it is certainly difficult and very labour-consuming. Then, models based on these results could be compared with the models obtained and presented in this paper.

It is also a well-known fact that the thickness of the anodized layer increases only up to a certain time. Then, this process dies out. Mathematical models should then also take into account this specific feature of the anodizing process.

The thickness of the anodized layer is related to the speed of its growth (formation). Moreover, the anodized layer consists of two layers with different densities and different properties and their monitoring is difficult. The above features significantly complicate the possibility of building a model (models) which would faithfully reproduce the ongoing processes.

It is also worth paying attention to the fact that the thickness of the anodized layer is greatest on the edges of a given surface. The sharper the edges, the greater the thickness of the anodized layer. Building mathematical models dedicated only to the edge of a surface depending on its acuity (radius r) is also very difficult, and it was not analyzed in this work. In this paper, the thicknesses of the anodized layer were measured as far from the edge as possible in order to avoid the influence of edge acuity on results.

On the basis of organoleptic tests and diagnostic graphs from the anodized polynomial models, model No. 1 is the most adequate model for the data obtained from measurements. There is an evident trend showing that the thickness of the anodized layer is increasing along with the increase of the time anodizing process. The values of correlation between the duration of the anodizing process and the thickness of the anodized layer, calculated both by Pearson's and Spearman's method, prove the functional relationships between these values. Unfortunately, the values of these correlations are calculated for a small set (there are only four time values and four values of the anodized layer thickness). It would seem beneficial to conduct a further analysis of other model families, e.g. logarithmic or trigonometric models, and to increase the size of data set taken for the study.

Literature

[1] Arnold, S.F., "The theory of linear models and multivariate analysis", Wiley, New York 1981.
[2] Baldin, D.J., "A tutorial on statistical methods for population association studie", https://cge.mdanderson.org/~instructor/Public/Balding.pdf, 2006.
[3] Biecek, P., "Analiza danych z programem R. Modele liniowe z efektami stałymi, losowymi i mieszanymi". Wydawnictwo Naukowe PWN, Warszawa 2013.
[4] Gągolewski, M., "Programowanie w języku R. Analiza danych, obliczenia, symulacje", Wydawnictwo Naukowe PWN, Warszawa 2016.
[5] Krzyśko, M., „Wielowymiarowa analiza statystyczna", Wydawnictwo Naukowe UAM, Poznań 2000.
[6] Piochacz, A., Mikołajczyk, J.R., "Influence of aluminum type EN AW-6060 anodizing process duration on the thickness and hardness of the obtained layer". *Developmens in Mechanical Engineering* 12(6), (2018): 49-56.
[7] Rekść, W., "Galwanotechnika", Wydawnictwo Politechniki Poznańskiej, Poznań 1992.
[8] Scheffe, H., "The analysis of variance". Wiley, New York 1959.
[9] Searle, S.R., Casella, G., McCulloch, C.E., "Variance components", Wiley, New York 1992.
[10] Seber, G.A.F., Lee, A.J., "Linear regression analysis", Wiley, Hoboken, New Jersey 2003.
[11] Sheather, S.J., "A Modern approach to regression with R", Springer, New York 2010.
[12] Sneynin, O.B., "C.F. Gauss and the Theory of Errors", *History of Exact Sciences* 20(1), (1979): 21-72. http://www.springerlink.com/index/U07K6T656NVL068.pdf.
[13] Stigler, S.M., "Gauss and the invention of least squares", *The Annals of Statistics* 9(3), (1981): 465-474. http://www.jstor.org/stable/2240811.
[14] Tibshirani, R., "Regression shrinkage and selection via the lasso", *Journal of the Royal Statistical Society*, Series B 58(1), (1996): 267-288.
[15] Wernik, S., Pinner, R., "Surface Treatment and Finishing of Aluminum and its Alloys", R. Draper, Teddington 1956.
[16] Wirbilis, S., "Galwanotechnika dla rzemieślników", Wydawnictwo Naukowo-Techniczne, Warszawa 1986.
[17] Wheeler, B., "Permutation tests for linear models in R", http://cran.r-project.org/web/packages/lmPerm.pdf, 2010.
[18] Wolffinger, R., Tobias, R., Sall, J., "Computing Gaussian Likelihoods and their Derivatives for General Linear Mixed Models", *SIAM Journal on Scientific Computing* 15(6), (1994): 1294-1310.
[19] Zeileis, A., Hothorn, T., "Diagnostic checking in regression relationships", http://cran.r-project.org/web/packages/lmtest/vignettes/lmtest-intro.pdf, 2010.
[20] Zou, H., Hastie, T., "Regularization and variable selection via the elastic net", *Journal of the Royal Statistical Society*, Series B. 67 (2005): 301-320.