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INTERPRETATION OF FIBONACCI NUMBERS IN BOTANY ON THE EXAMPLE OF SELECTED FRUITS

Abstract

Introduction and aims: The study shows the interpretation of Fibonacci numbers in botany. In particular, it is shown the interpretation of symmetry in the cross-sections of selected fruits of plants and trees. Also have been presented some definitions of Fibonacci numbers and discuss their interpretation in certain cross-sections of selected fruits. Therefore, the main aim of this work is to show the interpretation of Fibonacci numbers in the analysis of cross-sections of selected fruits.
Material and methods: Material consists some pictures of fruits and their cross-sections which were made by the Authors of this paper. The method of visual and theoretical analysis has been performed in this paper.
Results: In this paper, has been considered a series of interesting images of fruit selected plants and trees. Presented graphical interpretation of dual, triangular, pentagonal, octagonal and decagonal symmetry, which shows the occurrence of Fibonacci numbers.
Conclusions: Fibonacci numbers in botany are interpreted in fruit cross-sections of various plants. In some cross-sections of plant and tree fruits can be observed some multiples of Fibonacci numbers. The interpretation of Fibonacci numbers may be used to supplement the classification of fruit plants.

Keywords: Botany, fruits, cross-section, Fibonacci numbers, interpretation.
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INTERPREATCJA LICZB FIBONACCIEGO W BOTANICE NA PRZYKŁADZIE WYBRANYCH OWOCÓW

Streszczenie

Wstęp i cele: *W pracy pokazano interpretację liczb Fibonacciego w botanice. W szczególności pokazano interpretację symetrii występującej przekrojach poprzecznych wybranych owoców roślin i drzew. Podano definicje liczb Fibonacciego oraz omówiono ich interpretację w określonych przekrojach poprzecznych wybranych owoców. Zatem głównym celem pracy jest pokazanie interpretacji liczb Fibonacciego w analizie przekrojów poprzecznych wybranych owoców.*
Materiał i metody: *Materiałem są zdjęcia owoców i ich przekrojów poprzecznych wykonane przez autorów pracy. Zastosowano metodę analizy wizualno-teoretycznej.*
Wyniki: *W pracy otrzymano szereg interesujących zdjęć owoców wybranych roślin i drzew. Przedstawiono interpretację graficzną symetrii dualnej, trójkątnej, pięciokątnej, ośmiokątnej i dziesięciokątnej, w której pokazano występowanie liczb Fibonacciego.*
Wnioski: *Interpretację liczb Fibonacciego można znaleźć w różnych przekrojach owoców wybranych roślin. W niektórych przekrojach owoców roślin i drzew można zaobserwować krotności liczb Fibonacciego. Interpretacja liczb Fibonacciego może być użyteczna w uzupełnieniu klasyfikacji owoców roślin.*

Słowa kluczowe: *Botanika, owoce, przekroje poprzeczne, liczby Fibonacciego, interpretacja.*
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1. Fibonacci numbers

Leonardo Fibonacci
(around 1175 –1250)

Leonardo of Pisa was born around 1175 years and died in 1250. The Italian mathematician. His father, Guillermo Bonacci family, he was a diplomatic mission in North Africa and the Fibonacci there just was educated. The first lessons in mathematics charge from the Arabic teacher in *Boużia*. He has traveled extensively, first with his father and later alone, visiting and educating themselves in places like Egypt, Syria, Provence, Greece and Sicily. On his travels through Europe and the countries of the East he had a chance to see the achievement of Arab and Hindu mathematicians, especially the decimal system number. Fibonacci ended around 1200 to travel and returned to Pisa [9].

Fibonacci sequence of the general expression F(n) is determined for each natural number $n \in N$ by the following recursive definition [1], [2]:

$$F(1) = 1, \quad F(2) = 1 \quad \text{i} \quad F(n+2) = F(n+1) + F(n). \tag{1}$$

On the other hand, the general expression F(n) of Fibonacci sequence for any number of natural $n \geq 1$ is defined by Binet formula [1]-[2]:

$$F(n) \equiv \frac{1}{\sqrt{5}}\left\{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right\}. \tag{2}$$

When using any definition (1)-(2), we obtain the following sequence of numbers:

1, 1, **2**, **3**, **5**, **8**, 13, **21**, 34, ... (3)

Sometimes are used Fibonacci numbers which are multiply by any natural number. For example, doubled Fibonacci sequence of numbers is as follows:

2, 2, **4**, **6**, **10**, **16**, 26, **42**, 68, ... (4)

Moreover tripled Fibonacci sequence of numbers has the following form:

3, 3, 6, **9**, 15, 24, 39, 63, 102, ... (5)

and Fibonacci sequence of numbers multiplied by 4:

4, 4, **8**, **12**, 20, 32, 52, 84, 136, ... (5)

Hence we can construct the following multiplication 2·**2**=4, 2·**3**=6, 2·**5**=10, 2·**8**=16, 2·**21**=42, 3·**3**=9, 4·**2**=8 and 4·**3**=12 which occurs in presented in this paper cross-sections.

2. Symmetry of fruit

The interpretation of Fibonacci numbers can be found in various issues of botany. In particular, Fibonacci numbers can be read not only the cross-section of fruit and vegetables, but also in their foliage [3]-[8]. In this paper, we show the interpretation of Fibonacci numbers in the symmetry of selected fruits. We will present the dual *(strawberry)*, triangular *(banan and horned melon)*, pentagonal *(apple, pear, carambola, kumquat, lemon)*, octagonal *(lemon)* and decagonal symmetry *(lemon, green lemon)*.

Furthermore there it will be indicated the angle of rotation about the central axis of the fruit. We will also show examples of fruit, in which there is some symmetry simultaneously *(melon, grapefruit, kiwi)*.

3. Symmetry in intersectional views of fruits

3.1. Dual symmetry

This variant of radial symmetry, also called dual, with rotation angle of 180° about the central axis there is strawberries (Fig. 1a). In cross-section of the strawberry is observed Fibonacci number 2 (Fig. 1b).

Fig. 1a. Strawberry, general view
Photo: J. Śledziowski

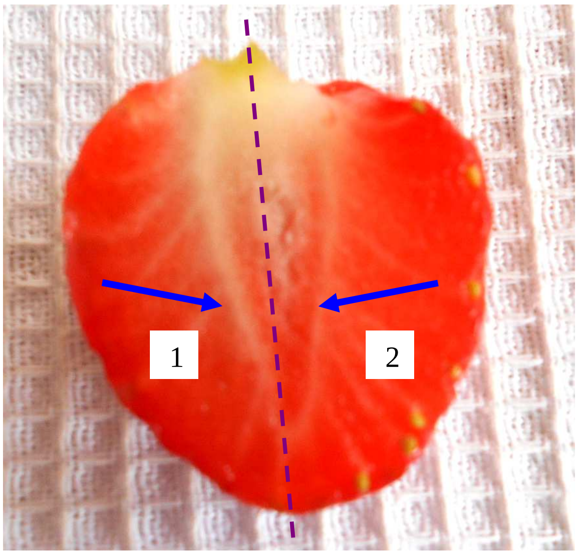

Fig. 1b. Strawberry, cross-section, number 2
Photo: J. Śledziowski

3.2. Triangle symmetry and its multiple

A triangular symmetry with rotation angle 120° also is presented in bananas (Fig. 2a). We have possibility of interpretation the twice Fibonacci number 3 for cross-sectional where the sum of baffles and chambers is 6 = 2·**3** (Fig. 2b).

Fig. 2a. Banans, general view
Photo: G.P. Skorny

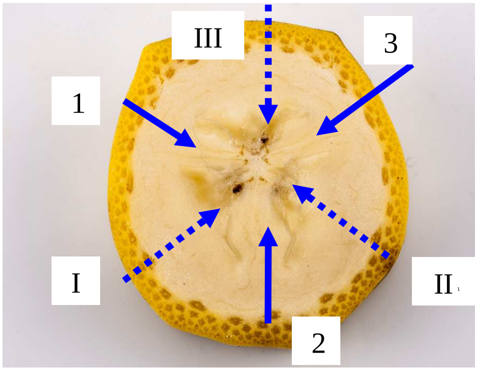

Fig. 2b. Banan, cross-section, number 3,
the sum of baffles and of chambers 6 = 2·**3**
Photo: G.P. Skorny

Very interesting sections we can found in horned melon *(also called kiwano).* In some cross-sections of the horned melon there is triangular symmetry and we can read Fibonacci number 3 (Fig. 3a,b) and its doubling 6 = 2·**3** (Fig. 3c,d).

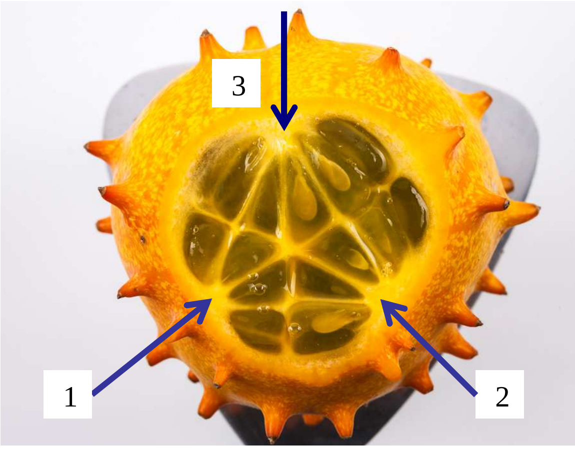

Fig. 3a. Horned melon, cross-section, number 3 *Photo: G.P. Skorny*

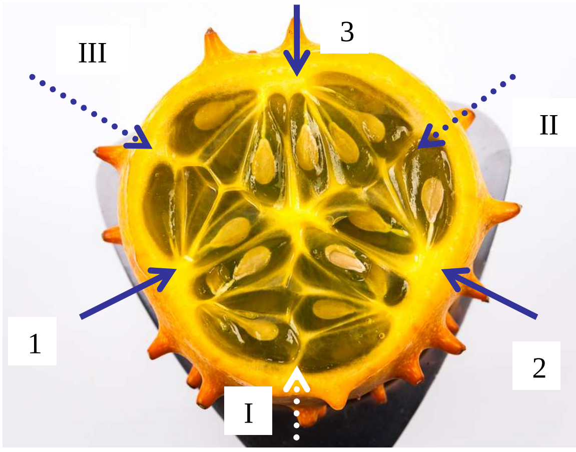

Fig. 3b. Horned melon, cross-section, number 3 *Photo: G.P. Skorny*

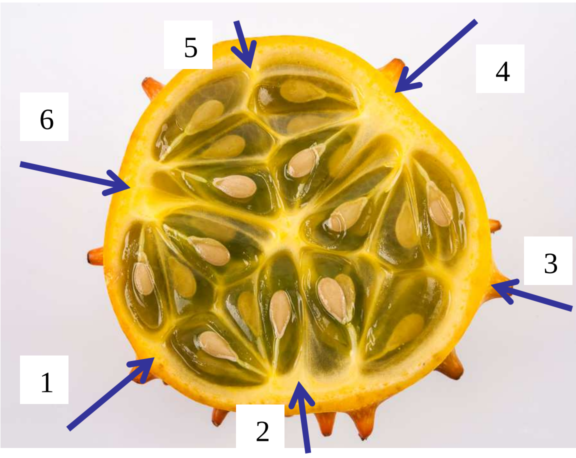

Fig. 3c. Horned melon cross-section, the number of baffles 6 = 2·**3**
Photo: G.P. Skorny

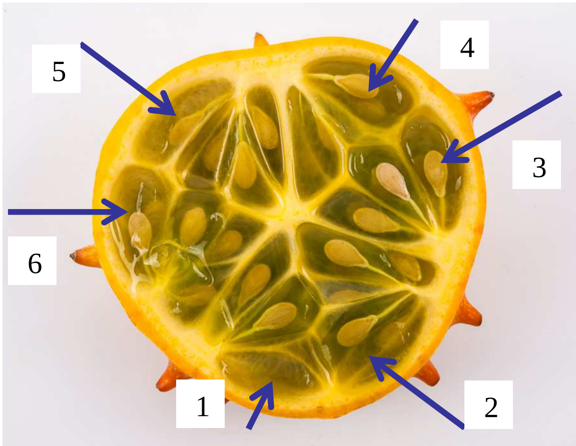

Fig. 3d. Horned melon, cross-section, number of chambers 6 = 2·**3**
Photo: G.P. Skorny

3.3. Pentagonal symmetry and its multiple

This variant of pentagonal symmetry, also called pentaradial, with rotation angle of 72° about the central axis show some crosses of *Golden Delicious* apples (Fig. 4a) and *Royal Gala* (Fig. 5a). Here there is a Fibonacci number 5 (Fig. 4b & Fig. 5b).

Fig. 4a. Apples *Golden Delicious*, general view
Photo: J. Śledziowski

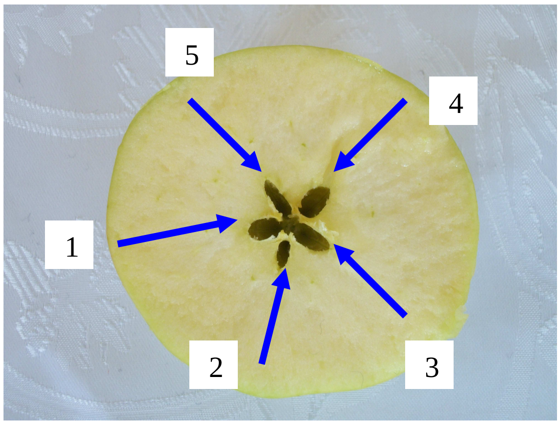

Fig. 4b. Apple, cross-section, number 5
Photo: J. Śledziowski

Fig. 5a. Apples *Royal Gala*, general view
Photo: J. Śledzioski

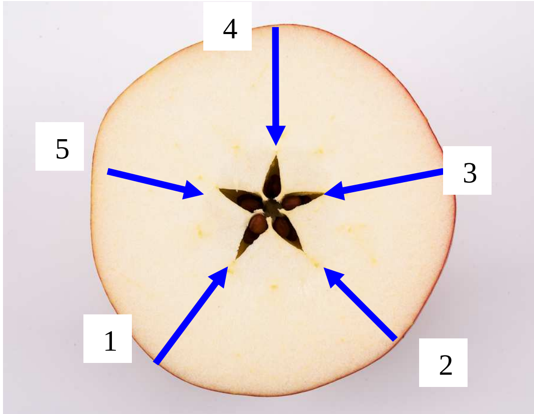

Fig. 5b. Apple, cross-section, numer 5
Photo: J. Śledzioski

A pentagonal symmetry is also a cross-sectional of pear (Fig. 6a,b)

Fig. 6a. Pears, general view
Photo: J. Śledziowski

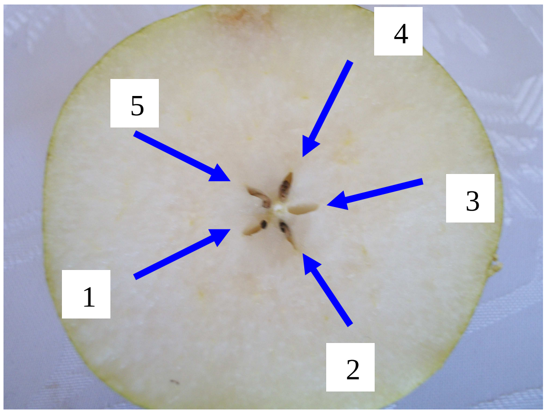

Fig. 6b Pear, cross-section, number 5
Photo: J. Śledziowski

A pentagonal symmetry is also a cross-sectional of carambola (Fig. 7a,b)

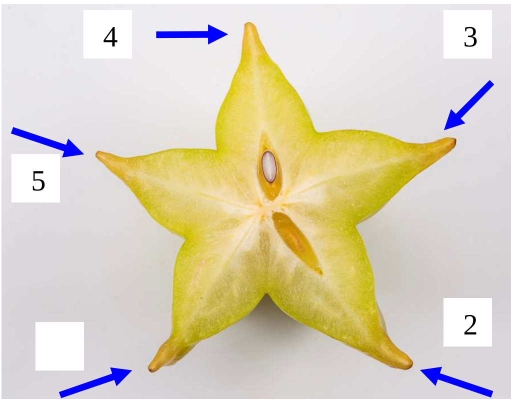

Fig. 7a. Carambola, number 5 petals
Photo: G.P. Skorny

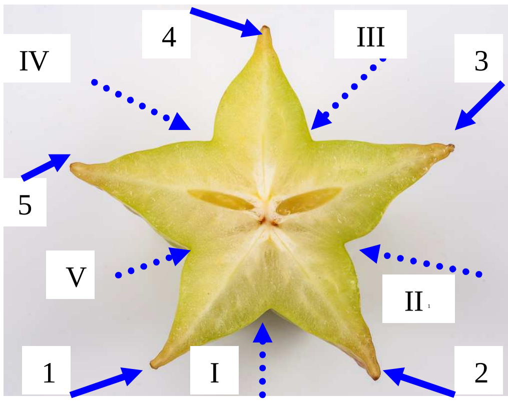

Fig. 7b. Carambola, 5 petals and 5 indentations
number 10 = 2·**5**
Photo: G.P. Skorny

A pentagonal symmetry can be observed in kumquat fruit (Fig. 8a). In the kumquat cross-section, such symmetry is both in the number of baffles (5) and number of chambers (5) (Fig. 8b). You can also see a double Fibonacci number 5, i.e. 10 = 2·**5**. Then if we take into account the sum of baffles and chambers, we have here angle of rotation 36° around a central axis (Fig. 8b).

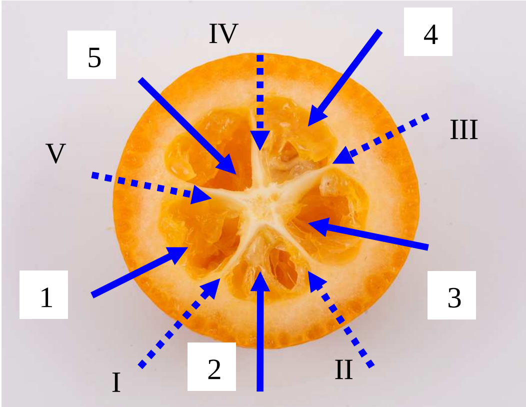

Fig. 8a. Kumquats, cross-section,
number 5 occurring 2 times
Photo: G.P. Skorny

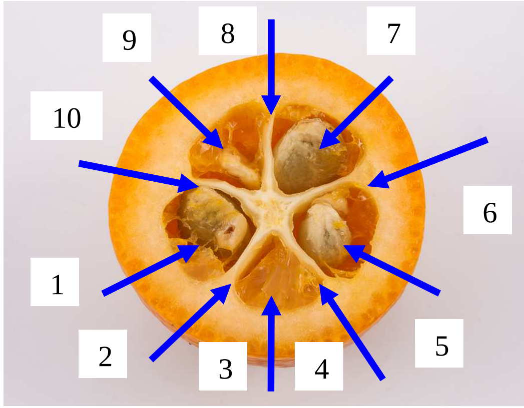

Fig. 8b. Kumquats, cross-section,
number 10 = 2·**5**
Photo: G.P. Skorny

3.4. Octangle symmetry

Type the octagonal symmetry *(i.e. octoradial)*, with rotation angle 45°, we see in the lemon (Fig. 9a). In the cross-section is a triple of Fibonacci number 3 is 9 = 3·**3** (Fig. 9b).

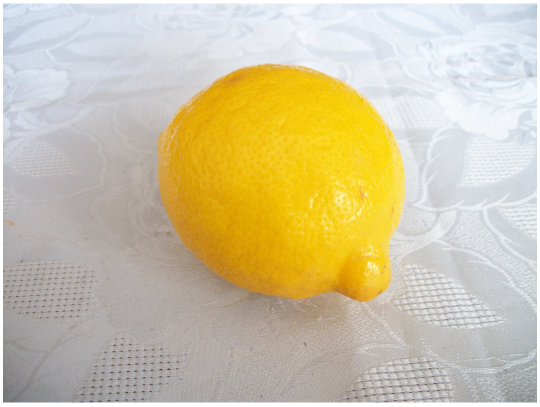

Fig. 9a. Lemon, general view
Photo: J. Śledziowski

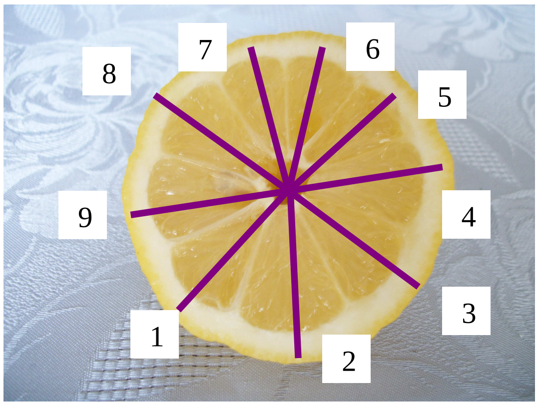

Fig. 9b. Lemon, cross-section
number of parts 9 = 3·**3**
Photo: J. Śledziowski

3.5. Decagonal symmetry

A decagonal symmetry *(i.e. decaradial)*, with rotation angle 36° about central axis occurs in lemons (Fig. 10a). There is double of Fibonacci number 5 in the certain cross-sections of lemon, i.e 10 = 2·**5**.

Fig. 10a. Lemon, general view
Photo: J. Śledziowski

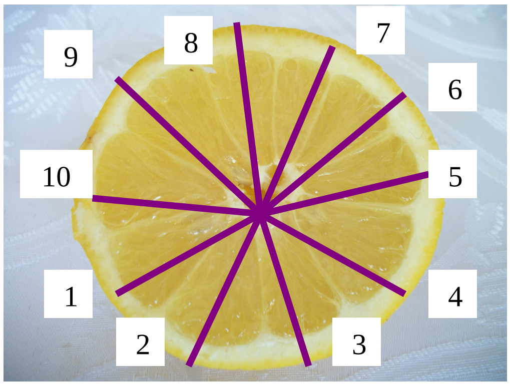

Fig. 10b. Lemon, cross-section
number of sections 10 = 2·**5**
Photo: J. Śledziowski

Fig. 11a. Green lemon, general view
Photo: J. Śledziowski

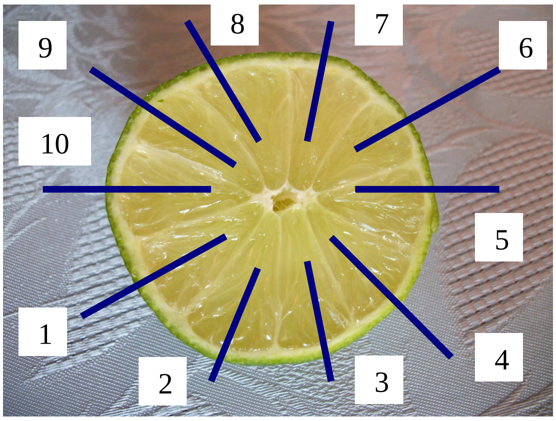

Fig. 11b. Green lemon, cross-section,
number of sections 10 = 2·**5**
Photo: J. Śledziowski

3.6. The occurrence of several symmetries

Outside the melon may be seen a decagonal symmetry in the number of parts, i.e. 10 = 2·**5**. We have here a doubling of Fibonacci number 5 (Fig. 12a).

On the other hand, in the cross-section of melon we have a quadruple symmetry in a number slits, i.e. 4 = 2·**2**. Moreover, there is some octagonal symmetry in the number of recesses with grains, i.e. 8 = 4·**2**. We have in this case some doubling of Fibonacci number 2 and the multiplication by 4 of Fibonacci number 3 (Fig. 12a,b).

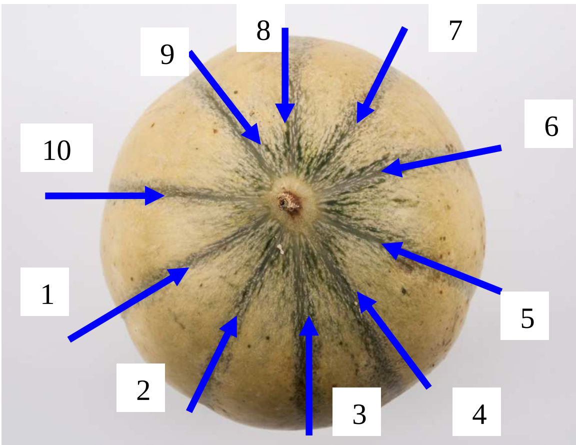

Fig. 12a. Melon, general view,
number of parts 10 = 2·**5**
Photo: G.P. Skorny

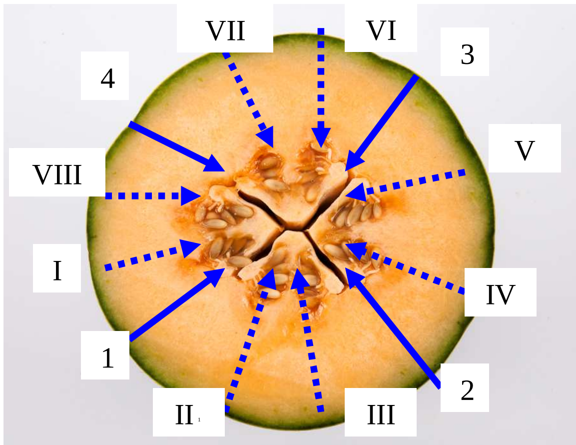

Fig. 12b. Melon, cross-section – number of parts 8
number of parts 4 = 2·**2**, number of parts 12 = 4·**3**
Photo: G.P. Skorny

The octangle symmetry *(i.e. oktoradial)* with rotation angle 45° (Fig. 13a) and hexagonal symmetry *(e.g. hexradial)* with rotation angle 22,5° is occurred in a cross-sectional of grapefruit (Fig. 13b). We have doubled Fibonacci numbers 4 and 8, i.e. 8 = 4·**2** and 16 = 2·**8**.

Wery interestig case may be observed in kiwi fruit (Fig. 14a). There are 42 baffles in kiwi fruit. There is an interpretation of Fibonacci doubled number 21, it is 42 = 2·**21** (Fig. 14b)

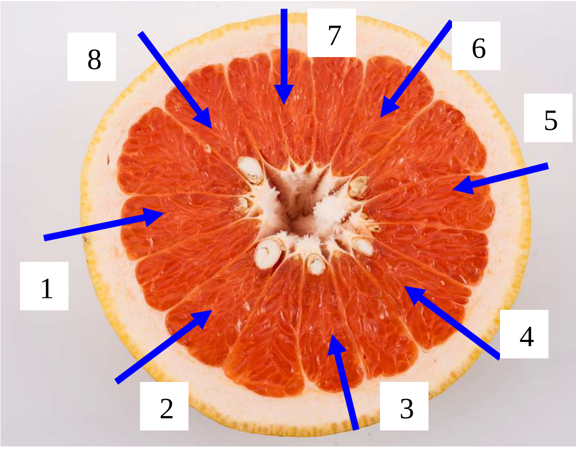

Fig. 13a. Grapefruit, cross section of the number of baffles calculated what the other 8 = 4·**2**
Photo: G.P. Skorny

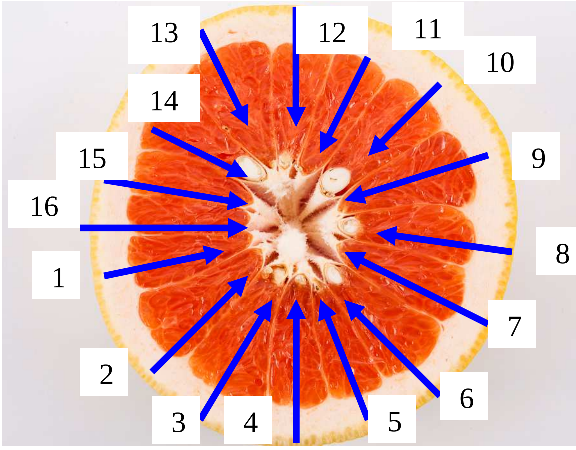

Fig. 13b. Grapefruit cross section of the total number of baffles 16 = 2·**8**
Photo: G.P. Skorny

Fig. 14a. Kiwi fruit, general view
Photo: J. Śledziowski

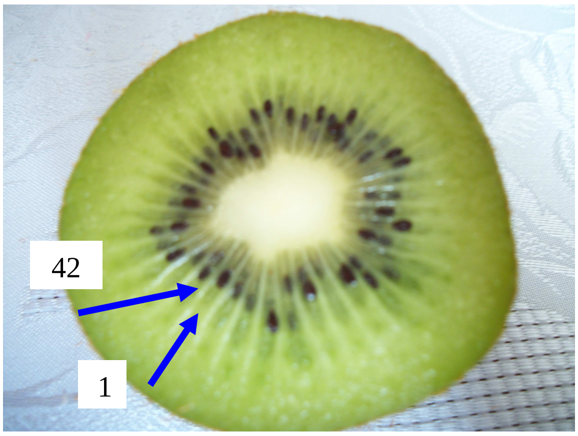

Fig. 14b. Kiwi, the cross section - number 42 the total number of baffles 42 = 2·**21**
Photo: J. Śledziowski

4. Conclusions

- Fibonacci numbers in botany are interpreted in fruit cross-sections of various fruits of plant and trees. In some cross-sections of fruit plants can be observed some multiples of Fibonacci numbers.
- The interpretation of Fibonacci numbers may be used to supplement the classification of fruit plants.

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