Effect of Fluctuation Shape Functions on Vibrations of Laminated Structures

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Abstract

In this note, the influence of the fluctuation shape functions on vibrations of the periodic laminate is analysed. The structure, used to show this impact, is the composite, consisting of the layers made of components differ in material properties like a specific heat or a thermal conductivity. The periodic laminate is microscopically heterogenous and to analyse this laminate, the tolerance averaging technique is used, therefore the influence of the thickness of the layer can be allow. One of the concepts introduced by tolerance modelling, is the fluctuation shape function, affecting on the results. The fluctuation shape function is assumed a priori and the character of vibrations is dependent on this function.

Keywords: vibrations, laminate, tolerance modelling, fluctuation shape function

1. Introduction

This paper treats of vibrations of the periodically laminated structure, shown in the Fig. 1. This structure consists of several dozen cells, called the periodicity cells, with regular thickness, denoted by λ. The volume share of the first and the second component in the periodicity cell is denoted by v_1 and v_2, respectively and is constant. L_1 is a dimension of presented structure along coordinate x_1, perpendicular to the layers and L_2 is a dimension along coordinate x_2, parallel to the layers. The analysed laminate is characterized by irregular structure, but the macroscopic properties of analysed structure are constant.

To analyse this laminate tolerance modelling is used. This technique is developed in many publications concerning vibrations in periodic plates [1-2], vibrations in periodic beams [3-7], dynamic problems in laminates [8-10], in medium thickness plates [11], in cylindrical shells [12] and in microperiodic composites [13]. The other methods, which can be used in the analysis of layered structures are the asymptotic homogenization [14], the homogenization introducing the concept of microlocal parameters [15] or Finite Element Method [16-18].

The tolerance averaging technique introduces a few new concepts, explained in Section 2 of this work. Otherwise, this way of modelling introduces new assumptions, which allow to take into account the size of the thickness of periodicity cell. These assumptions are presented in Section 3. In Section 4, the equations of the tolerance model, obtained by using the tolerance averaging technique, are shown. The conclusions are described in Section 5 and at the end of this work, the references are placed.

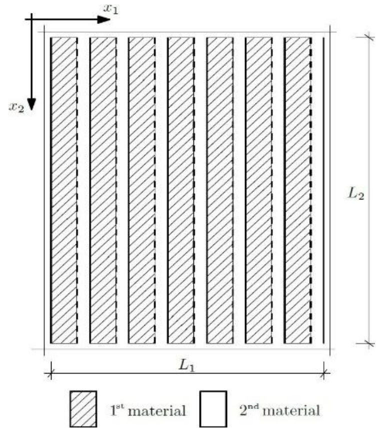

Figure 1. Periodically laminated composite

2. Basic concepts of the tolerance modelling

The first concept, underlying of the tolerance modelling is the tolerance-periodic function *TP*, expressed by following equation:

$$\left(\forall x\in\Omega\right)\left(\exists TP^{(i)}\left(x,\cdot\right)\in H^{0}\left(\Delta\right)\right)\left(\left\|\partial^{i}\,TP\big|_{\Omega_x}\left(\cdot\right)-TP^{(i)}\left(x,\cdot\right)\right\|_{H^{0}(\Omega_x)}\leq\delta\right) \tag{1}$$

and complying following term:

$$\int\limits_{\Delta(\cdot)} TP^{(i)}\left(\cdot,z\right)dz\in C^{0}\left(\overline{\Omega}\right) \tag{2}$$

where the global coordinates and the local coordinates in space **Ω** are denoted by *x* and *z*, respectively, an approximation of the tolerance-periodic function is expressed by using the symbol ~ and $H^0(\Delta)$ is a space of square integrable functions. The tolerance-periodic function *TP* is defined in relation to the cell Δ with thickness λ defined as $\Delta\equiv[-\lambda/2,-\lambda/2]$ and is connected with the tolerance parameter δ.

One of the natures of the tolerance-periodic function is the highly-oscillating function, denoted by *HO*. This function, has to fulfil the following condition, additionally:

$$\left(\forall x\in\Omega\right)\left(HO^{(i)}\left(x,\cdot\right)\Big|_{\Delta(x)}=\partial^{i}\,HO\left(x\right)\right) \tag{3}$$

where index *i* takes values 0, 1, 2.

The second nature of the tolerance-periodic function is the slowly-varying function, denoted by *SV*, which has to satisfy succeeding term:

$$\left(\forall x\in\Omega\right)\left(SV^{(i)}\left(x,\cdot\right)\Big|_{\Delta(x)}=\partial^{i}\,SV\left(x\right)\right) \tag{4}$$

where index *i* takes values 0, 1, 2.

The tolerance averaging technique, as the name suggests, based on averaging procedures, using the formula:

$$<\partial^i TP>(x)\equiv\frac{1}{|\Delta|}\int_{\Delta(x)} TP^{(i)}(x,z)\,dz \qquad (5)$$

where the local coordinate $z\epsilon\Delta(x)$ and the cell $\Delta(x)$ is assumed as $\Delta+\lambda$.

3. Main assumptions of the tolerance modelling

Tolerance modelling gives a possibility to consider the averaged part and the oscillating part of analysed unknowns by using the micro-macro decomposition assumption, according to the following formulas:

$$\begin{aligned} \theta(x_1,x_2)&=\vartheta(x_1,x_2)+g^A(x_1)\psi_A(x_1,x_2)\\ u(x_1,x_2)&=w(x_1,x_2)+h^M(x_1)V_M(x_1,x_2) \end{aligned} \qquad (6)$$

where the basic unknowns, the total temperature $\theta(x_1,x_2)$ and the total displacements $u(x_1,x_2)$ are the tolerance-periodic functions, the averaged parts of the total temperature – the macrotemperature and the total displacements – the macrodisplacements are denoted by $\vartheta(x_1,x_2)$ and $w(x_1,x_2)$, respectively. The oscillating parts are dependent on the fluctuation shape functions of the temperature $g^A(x_1)$ and the displacements $h^M(x_1)$ and the new main unknowns – the fluctuation amplitudes of the temperature $\psi_A(x_1,x_2)$ and the displacements $V_M(x_1,x_2)$.

The fluctuation shape functions $g^A(x_1)$ and $h^M(x_1)$ are assumed a priori for each considered phenomenon. It is possible to assume more than one fluctuation shape function in the analysis of specific issue (A=1,2,…,N, M=1,2,…,N). In this work, only one *g*-function and one *h*-function is presupposed, but the influence of the character of assumed fluctuation shape functions is verify, by analysing the vibrations of periodic laminate two times – with two different characters of considered fluctuation shape functions.

The first fluctuation shape function of the temperature $g^I(x_1)$ and the first fluctuation shape function of the displacements $h^I(x_1)$ is a saw-type function, consisting of linear functions, shown in the Fig. 2. The second fluctuation shape function of the temperature $g^{II}(x_1)$ and the second fluctuation shape function of the displacements $h^{II}(x_1)$ is a function shown in the Fig. 3, consisting of linear and parabolic functions.

The fluctuation shape functions of the displacements are assumed as the same as the fluctuation shape functions of the temperature.

Next to the micro-macro decomposition, the main assumption of tolerance modelling is an approximation of the derivatives of the temperature and the displacements (in general – any function), expressed by following equations:

$$\begin{aligned} \theta^{(k)}(x_1,x_2,z)&=\nabla^k\vartheta(x_1,x_2)+\partial^k g^A(x_1,z)\psi_A(x_1,x_2)+g^A(x_1,z)\overline{\nabla}^k\psi_A(x_1,x_2)\\ u^{(k)}(x_1,x_2,z)&=\nabla^k w(x_1,x_2)+\partial^k h^M(x_1,z)V_M(x_1,x_2)+h^M(x_1,z)\overline{\nabla}^k V_M(x_1,x_2) \end{aligned} \qquad (7)$$

where the periodic approximations of individual functions are denoted by symbol ~.

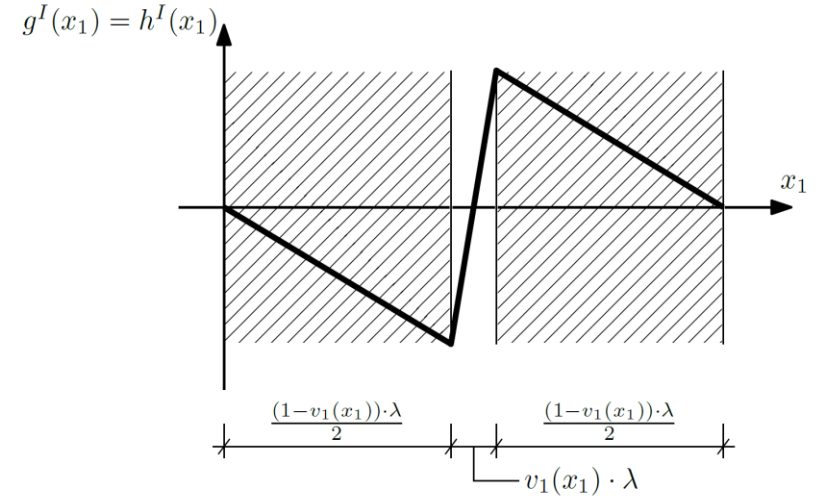

Figure 2. The character of the first fluctuation shape function

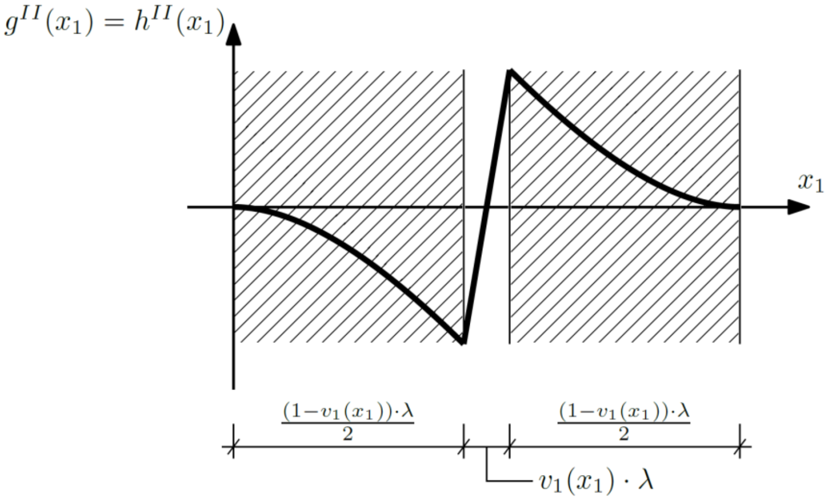

Figure 3. The character of the second fluctuation shape function

4. The tolerance model equations

The equations of the tolerance model can be obtained by using the orthogonalization method [10] or the extended stationary action principle [19], the concepts and the assumptions mentioned in Section 2 and Section 3. These equations are presented in the following form:

$$
\begin{aligned}
&< c\rho > \dot{\vartheta} - \nabla\left(<\mathbf{K}> \nabla\vartheta + <\mathbf{K}\partial g^A > \psi_A + <\mathbf{K} g^A > \overline{\nabla}\psi_A \right) = 0 \\
&< c\rho g^A g^B > \dot{\psi}_B - \overline{\nabla}\left(<\mathbf{K} g^A g^B > \overline{\nabla}\psi_B + <\mathbf{K} g^A \partial g^B > \psi_B + <\mathbf{K} g^A > \nabla\vartheta \right) \\
&+ <\mathbf{K}\partial g^A \partial g^B > \psi_B + <\mathbf{K}\partial g^A g^B > \overline{\nabla}\psi_B + <\mathbf{K}\partial g^A > \nabla\vartheta = 0 \\
&\nabla\left(<\mathbb{C}> \nabla\boldsymbol{w} + <\mathbb{C}\partial h^M > \boldsymbol{V}_M + <\mathbb{C} h^M > \overline{\nabla}\boldsymbol{V}_M \right) = <\rho> \ddot{\boldsymbol{w}} + <\mathbf{B}> \nabla\vartheta \\
&+ <\mathbf{B}\partial g^A > \psi_A + <\mathbf{B} g^A > \overline{\nabla}\psi_A + <\nabla\mathbf{B}> \vartheta + <(\nabla\mathbf{B}) g^A > \psi_A \\
&\overline{\nabla}\left(<\mathbb{C} h^M h^N > \overline{\nabla}\boldsymbol{V}_N + <\mathbb{C} h^M > \nabla\boldsymbol{w} + <\mathbb{C} h^M \partial h^N > \boldsymbol{V}_N \right) - <\partial h^M \mathbb{C}\partial h^N > \boldsymbol{V}_N \\
&- <\mathbb{C}\partial h^M > \nabla\boldsymbol{w} - <\mathbb{C} h^N \partial h^M > \overline{\nabla}\boldsymbol{v}_N = <\rho h^M h^N > \ddot{\boldsymbol{v}}_N + <\mathbf{B} h^M > \nabla\vartheta \\
&+ <\mathbf{B} h^M \partial g^A > \psi_A + <\mathbf{B} g^A h^M > \overline{\nabla}\psi_A + <(\nabla\mathbf{B}) h^M > \vartheta + <(\nabla\mathbf{B}) g^A h^M > \psi_A
\end{aligned}
\tag{8}
$$

where c is a specific heat, ρ is a mass density, **K** is a tensor of conductivity, $\mathbb{C}$ is a tensor of elasticity and **B** is a tensor of thermal extensions.

These equations can be used in the analysis of dynamic problems in relation to the periodic composites. For instance, the forced vibrations of this type of laminates can be considered. The vibrations can be caused by mechanical or thermal loads. The method, recommended for solving the above equations is a type of the Finite Difference Method – the Cranck-Nicholson Method.

5. Example of application

The forced vibrations of the laminated layer, made of two different materials, periodically distributed along direction x_1 are considered in this note. It is assumed, that analysed materials are steel and aluminium and the dimensions of this layer are equal to L_1=1[m] and L_2=1[m]. The volume share of the first material (steel) in the cell equals 0.25. The calculations were carried out for the number of the cells equals 20. The thermal (known temperature) and the mechanical (time-varying stresses and known displacements) loads on the edges of the layer are considered. The equations of the tolerance model were solved by using Finite Difference Method and the numerical results in the form of plots of the total temperature and the total displacements are obtained. The calculations were carried out two times with two different fluctuation shape functions. In the Figs 4-5, the distributions of the total temperature in one of cross-sections are presented (including the fluctuation shape functions $g^I(x_1)$ and $g^{II}(x_1)$, respectively). And in the Figs 6-7, the distributions of the total displacements in one of cross-sections are presented (including the fluctuation shape functions $h^I(x_1)$ and $h^{II}(x_1)$, respectively). The plots were made for selected time coordinate. In case of the fluctuation shape functions consisting of linear and parabolic functions, the microstructural character of the layer is more visible and the specific wedges are more noticeable and distinct, rather than in case of the fluctuation shape functions consisting of linear functions only (especially it comes to the temperature). The maximum values of the total displacements also vary, depending on the selected fluctuation shape function.

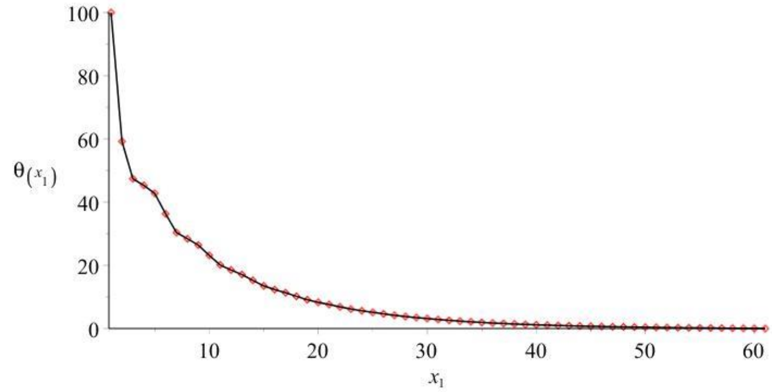

Figure 4. The total temperature including fluctuation shape function $g^I(x_1)$

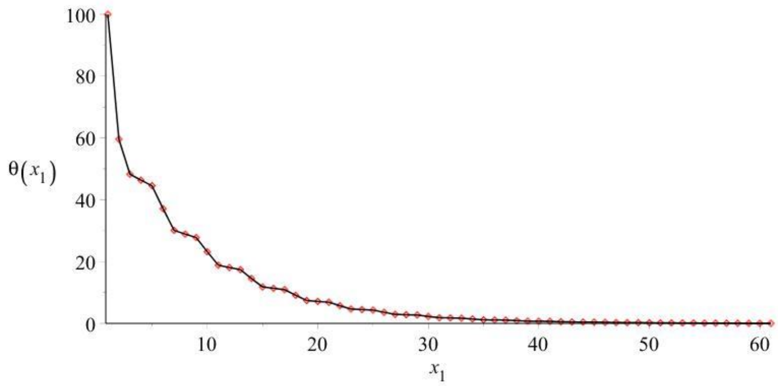

Figure 5. The total temperature including fluctuation shape function $g^{II}(x_1)$

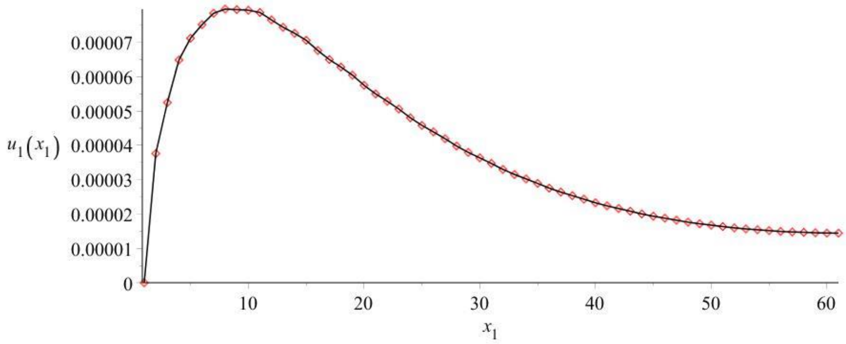

Figure 6. The total displacements including fluctuation shape function $h^I(x_1)$

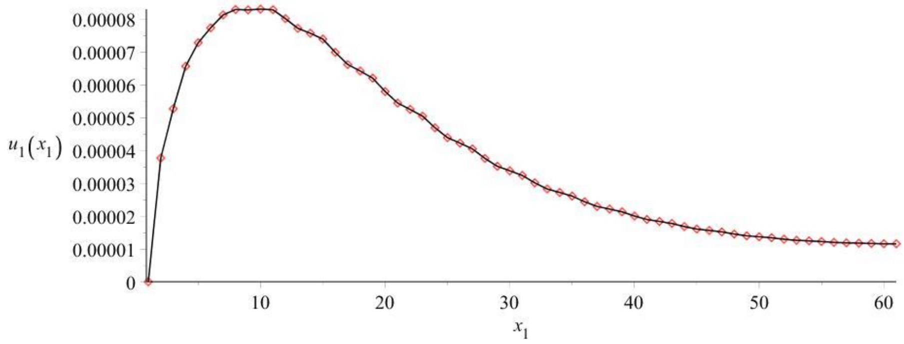

Figure 7. The total displacements including fluctuation shape function $h^{II}(x_1)$

6. Conclusions

Bases on carried out analysis, the following conclusions can be formulated:
- the equations of the tolerance model are characterized by slowly-varying or constant coefficients, in contrast to the known heat conduction equation,
- by using tolerance modelling, it is possible to take into account the impact of the thickness of the cells of considered laminates,
- tolerance modelling can be used in the analysis of dynamic problems, heat conduction issues and thermoelasticity problems in relation to the beams, plates or shells with periodic or functionally graded structures,
- the micro-macro decomposition allows to analyse the macro- and microvibrations of considered structures,
- the proposed fluctuation shape functions, assumed a priori, affect on the character of vibrations of the structures, that is why the correct fluctuation shape function is so important.

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