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RENEWABLE WARRANTY POLICY FOR MULTIPLE-FAILURE-MODE PRODUCT CONSIDERING DIFFERENT MAINTENANCE OPTIONS

POLITYKA ODNAWIANIA GWARANCJI DLA PRODUKTÓWO MNOGICH PRZYCZYNACH USZKODZEŃ UWZGLĘDNIAJĄCA RÓŻNE OPCJE OBSŁUGI

Along with the advancement of manufacturing techniques, the quality of the spares for product is likely to be improved during the warranty period. There can be two types of spares, i.e. low-quality spares and high-quality spares for replacement maintenance. And the manufacturers (customers) may have to decide whether or not to provide (buy) the warranty considering upgrading maintenance. This paper presents a renewing warranty policy considering three maintenance options for products with multiple failure modes. The cost and availability models of these maintenance options are proposed. Of these options, upgrading maintenance is taken into account with the assumption that the warrantied item will be upgraded one time during the warranty cycle. After upgrading maintenance, the high-quality spares are used to replace the failed item. By minimizing the ratio between cost and availability of the product, the optimal upgrading opportunity is obtained. In the numerical example, the results of these options are presented. Monte Carlo simulation results are compared with the analytical results to demonstrate the correctness and efficiency of the proposed models considering upgrading maintenance. The renewing warranty policy considering upgrading maintenance policy is compared with the one without considering upgrading maintenance. The results show that the former is better than the latter in some cases. The sensitivity of the cost model and availability model to different parameters is analyzed at last.

Keywords: *multiple failure modes; renewing warranty; preventive maintenance; warranty cost; product availability; upgrading maintenance.*

Wraz z postępem techniki produkcji, wzrasta prawdopodobieństwo, że jakość części zamiennych do produktu ulegnie poprawie w przeciągu okresu gwarancyjnego. Istnieją dwa rodzaje części zamiennych: części zamienne niskiej i wysokiej jakości. Producenci (klienci) mogą być zmuszeni podjąć decyzję czy objąć produkt gwarancją (wykupić gwarancję) zapewniającą konserwację modernizacyjną. W artykule przedstawiono politykę odnawiania gwarancji z uwzględnieniem trzech różnych opcji obsługi produktów narażonych na mnogie przyczyny uszkodzeń. Zaproponowano modele kosztów i gotowości dla omawianych opcji obsługi. Spośród badanych opcji, do dalszej analizy wybrano konserwację modernizacyjną zakładającą, że element podlegający gwarancji zostanie poddany jednokrotnej modernizacji podczas cyklu gwarancyjnego. Po wykonaniu konserwacji modernizacyjnej, uszkodzony element zastępuje się częściami zamiennymi wysokiej jakości. Minimalizując stosunek kosztów do gotowości produktu, uzyskuje się optymalną możliwość modernizacji Przykład numeryczny przedstawia wyniki uzyskane dla omawianych opcji. Wyniki symulacji Monte Carlo porównano z wynikami analitycznymi w celu wykazania prawidłowości i efektywności proponowanych modeli uwzględniających konserwację modernizacyjną. Politykę odnawiania gwarancji uwzględniającą konserwację modernizacyjną porównano z polityką, która takiej konserwacji nie uwzględnia. Wyniki pokazują, że pierwsza z tych opcji jest w niektórych przypadkach korzystniejsza od drugiej. Badania wieńczy analiza czułości modelu kosztów i modelu gotowości na różne parametry.

Słowa kluczowe: *mnogie przyczyny uszkodzeń; odnowienie gwarancji; obsługa profilaktyczna; koszty gwarancji; gotowość produktu; konserwacja modernizacyjna.*

1. Introduction

Warranty is an obligation attached to products that requires manufacturers to provide compensation for customers when the products fail to perform their pre-specified functions during a specified period [20]. Since product warranty can be a powerful incentive in selling a product and a useful way to protect customer from product quality defects, it plays an increasingly important role in commercial transactions and has obtained increasing attention from the manufacturers as well as the customers recently. According to Murthy and Blischke [5], warranty polices can be divided into two classes, i.e. renewing warranty and non-renewing warranty. In a renewing warranty policy, whenever an item fails under warranty, it is replaced by a new item with a new-starting warranty. In contrast, in the case of a non-renewing policy, replacement of a failed item doesn't alter the original warranty [6]. The warranty period for renewing policies begins anew with each replacement, while the replacement item for non-renewing policies will be covered for the remaining time of the item it replaced. Nowadays, most product warranties are non-renewing. But for some high-reliability-required and high-safety-required products (like expensive aviation products), the maintenance in warranty period is considered to be perfect such that the failed product is replaced by a new one. Then the customers are more preferred to choose the renewing warranty policy. However, the period of renewing warranty normally gets longer compared to the non-renewing warranty. Then reducing the warranty servicing cost and improving the product performance (like availability, mean time between failures, etc.) have become great challenges for both manufactures and customers.

From customer's perspective, product performance is a key factor when a customer is making a buying decision, although cost and availability are both taken into consideration. However, manufacturers are mainly concerned about the profitability and the warranty cost is a key factor to be considered. In order to get to a win-win situation, the warranty policy makers must take some trade-offs between the warranty cost and product performance. In reality, technology development may lead to product improvement, meaning that the spares for replacing failed product are upgraded from low-quality to high-quality. As a result, a question facing the manufacturers (customers) is: whether to provide (accept) upgrading replacement in the warranty or not? In this paper, we focus on the analysis of product cost and availability in the warranty period considering upgrading maintenance. The rest of this paper is organized as follows: Section 2 introduces the relevant work existing in the literature in regarding to product warranty policies. Section 3 outlines the model assumptions and notation, and then proposes three maintenance options including upgrading maintenance. Section 4 is dedicated to development of the mathematical models. A numerical example is given in Section 5 to illustrate our approach.

2. Literature review

Bai and Pham [2] presented discounted warranty cost model for repairable series systems assuming the impact of repair actions on components' failure time was minimal. Huang et al. [9] used a bivariate approach and taken into account periodic preventive maintenance to develop a two-dimensional warranty policy for the repairable product. Park and Pham [12] proposed warranty cost models on the quasi-renewal processes and exponential distribution assuming that a repair service was imperfect for several systems, including multi-component systems. They also [13] introduced two alternative quasi-renewal processes: altered quasi-renewal and mixed quasi-renewal processes to obtain the expected value of warranty cost, covariance of warranty cost and variance of warranty cost for the warranted product. Vahd [19] developed a renewing free replacement warranty policy for a multi-state deteriorating repairable product with *N* working states and *N* failure states. Two rectification actions should be done in case any failure has occurred: minimal repair with non-negligible needed time and replacement which was performed instantly. Banerjee and Bhattacharjee [4] analyzed the cost of a new two-dimensional warranty servicing strategy that probabilistically exercised a choice between a replacement and a minimal repair to rectify the first failure in the middle interval. Su and Shen [18] proposed two types of extended warranty policies from the manufacturer's perspective, namely one-dimensional extended warranty policy and two-dimensional non-renewing extended warranty policy. The corresponding warranty cost and profit models were presented to calculate the warranty cost considering minimal repair, imperfect repair combined with minimal repair and complete repair combined with minimal repair for failed component. Bai and Pham [3] presented full-service warranty for repairable multi-component systems under which the failed component(s) or subsystem(s) will be replaced. In addition, a (perfect) maintenance action will be performed to reduce the chance of future system failure, both free of charge to consumers. Chien [7] studied on the effects of a free-repair warranty on a periodic replacement policy with a discrete time process. Sana [16] studied on an imperfect production system with allowable shortages due to regular preventive maintenance for products sold with free minimal repair warranty. Aggrawal et al. [1] used a two dimensional innovation diffusion model to demonstrate product sales cycle, and presented a methodical approach to obtain optimal price and warranty length for a product. The model examined significance of these decision variables and estimates the overall maximum profit for the manufacturer. Exponential distribution has been used to represent the life time distribution of a product and the model has been validated using real life data set.

González-Prida et al. [8] selected the warranty period after the completion of a series of successive repairs on a product. Two stochastic failure models were used: a general renewal process (GRP) model and a non-homogeneous Poisson process (NHPP) model. Both used a Weibull distribution for the life time of the product, allowing the possibility of renewal (GRP) or not (NHPP) when successive repairs were performed. Park et al. [14] proposed a warranty cost model is proposed in consideration of both repair service and replacement service simultaneously upon the system failure to find the optimized warranty period in terms of an expected cost rate during the warranty cycle under the manufacturer's point of view. Xie et al. [21] presented an integrated model to estimate the gross profit for a new durable product to be sold in a fixed sales period at a fixed price. It was assumed that the sales over time can be characterized by a stochastic Bass model in the form of a nonhomogeneous Poisson process and the production system was a make-to-order type of system. Shafiee et al. [17] developed an optimization model to investigate the lengths of the optimal burn-in and warranty period, so that the mean of total product servicing cost was minimized. Jeon and Sohn [10] extracted association rules from warranty data of heavy duty diesel engine in order to find significant patterns of failures along with manufacturing information. They also used Weibull regression to identify influential factors that affect the variation in mean time between failures which were identified from extracted association rules. Liu et al. [11] developed a model based on renewing free-replacement warranty by considering failure interaction among components.

Obviously, lots of literatures focused on optimizing non-renewing policy. On the contrary, only a few researchers studied on renewing policy. However, renewing policy is desirable for both consumers and manufacturers since consumers receive better warranty service compared to the traditional non-renewing policy and manufacturers could attract more consumers to buy their products. In the literatures on renewing warranty policy, the optimization target is always warranty cost and the maintenance actions are only corrective maintenance. Although the warranty cost is a good measure on the overall cost of warranty, it provides little information of the product performance contained in a warranty program. Therefore, it is not sufficient enough to only use warranty cost model. The product availability can provide us a numerical measure of the product performance in the warranty period. These measures are useful for evaluating product performance. So throughout this paper, we consider both the expected cost and product availability for the renewing warranty policy together.

This paper considers three maintenance options including upgrading maintenance. The models proposed in this paper are different from the previous studies on complex system and single component models in the following perspectives:

(1) It is able to handle the multiple failure modes in calculating warranty cost and product availability. In reality, a product normally has several failure modes due to different failure reasons. So the resulting models can be more useful to the warranty policy-makers and customers to make appropriate decisions.

(2) The upgrading maintenance can be considered as a type of preventive maintenance (PM). PM action can sometimes improve the availability or decrease the warranty cost. By adjusting upgrading opportunity, the cost and availability can be optimized.

None of these situations have been considered so far, and thus this model offers warranty policy-maker and customers a useful tool to achieve a win-win situation in terms of minimal warranty cost and best product performance. We believe the models can help warranty policy makers to make optimal decisions with the objective of minimizing the ratio between warranty cost and product availability.

3. Modeling approach

This section develops a new modeling approach and provides some preliminary results. The item being considered in this paper has multiple failure modes as shown in Fig. 1. The failure modes are caused by different failure reasons, which can be chemical, physical or human errors, etc. By collecting failure data during the product life cycle, we can properly find the statistical characteristics of these failure modes. The notations of this paper are as follows:

W_1	warranty period for low-quality item
W_2	warranty period for high-quality item
k_a	failure mode number of low-quality item
k_b	failure mode number of high-quality item
$F_s(\cdot)$	cumulative distribution function of low-quality item
$G_s(\cdot)$	cumulative distribution function of high-quality item
$R_{ai}(\cdot)$	reliability function of the i^{th} failure mode for the low-quality item
$F_{ai}(\cdot)$	cumulative distribution function of the i^{th} failure mode for the low-quality item
$R_{bj}(\cdot)$	reliability function of the j^{th} failure mode for the high-quality item
$F_{bj}(\cdot)$	cumulative distribution functionof the j^{th} failure mode for the high-quality item
f_{ai}	failure density function for the i^{th} failure mode for the low-qualityitem
f_{bj}	failure density function for the j^{th} failure mode for the high-quality item
T	warranty cycle which is a time interval starting from the date of sale and ending at the warranty expiration date
T_0	preventive replacement time period
$\bar{T}_{bm}$	expected operational time between failures
$\bar{D}$	expected downtime
C_a	corrective maintenance cost of low-quality item
C_b	corrective maintenance cost of high-quality item
C_L	downtime loss of the item
T_m	replacement time for the low-quality item
C_p	preventive replacement cost for the low-quality item
T_p	preventive replacement time for the low-quality item
$\tilde{C}_p$	upgrading maintenance cost for the high-quality item
$\tilde{T}_p$	upgrading maintenance time for the high-quality item
T_r	replacement time for the high-quality item

The failure modes are assumed to be independent to each other, so we have:

$$F_s(t) = 1 - \prod_{i=1}^{k_a} R_{ai}(t) = 1 - \prod_{i=1}^{k_a} (1 - F_{ai}(t)),\ \ G_s(t) = 1 - \prod_{j=1}^{k_b} R_{bj}(t) = 1 - \prod_{j=1}^{k_b} (1 - F_{bj}(t)).$$

Due to the renewable nature of the warranty, the restored system will automatically carry a new-starting warranty. Different maintenance options will have influence on warranty cycle, so we consider the following maintenance options:

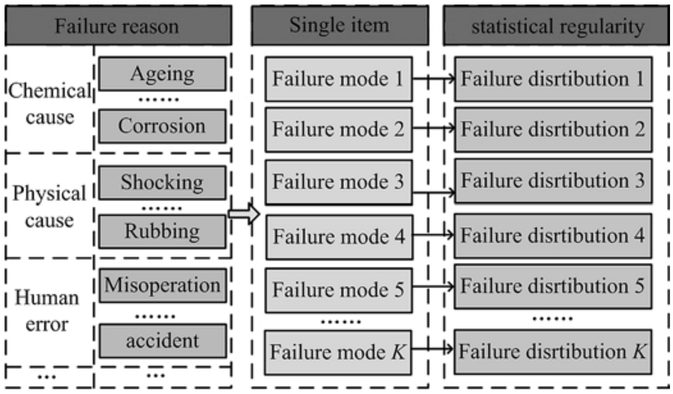

Fig. 1. An item with multiple failure modes

***Option I**:* When an item has failed, it would be replaced by a new low-quality spare in the warranty cycle. Only the low-quality spares are used in the warranty cycle.

Option II: When an item has successfully worked until T_0, a new low-quality spare would be used to replace it. Only the low-quality spares are used in the warranty cycle. After replacement at T_0, the warranty period doesn't renew. And the warranty period renews after the replacement of a failed item.

Option III: When an item has successfully worked until T_0, a new high-quality spare would be used to replace it by upgrading maintenance. Before the upgrading maintenance, the low-quality spares are applied to replace a failed item. After upgrading maintenance, the high-quality spares are used to replace a failed item. After replacement at T_0, the warranty period doesn't renew. But the warranty period renews after the replacement of a failed item.

Option II and Option III can be considered as a type of preventive maintenance. High-quality items normally have higher reliability and lower maintenance cost than low-quality items. They can increase the system availability which will catch more interest from the customers. Under the above maintenance options, manufacturers are responsible for replacing the failed components. After a replacement, the item is in good working condition. Obviously *T* is a random variable. For Option I and Option II, the value of *T* depends on W_1. For Option III, the value of *T* depends on W_1 and W_2. Warranty cycles for different maintenance options are shown in Fig. 2.

For Option I, there is one stage in the warranty cycle. *T* can be expressed as:

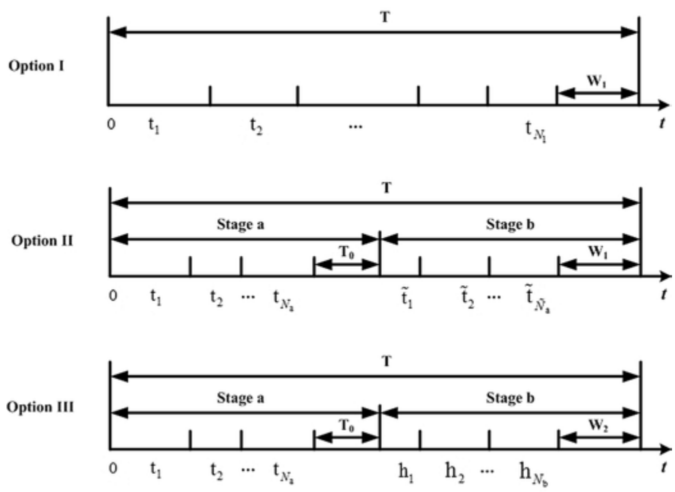

Fig. 2. System failure times for different maintenance options

$$T = t_1 + t_2 + ... + t_{N_1} + \mathrm{W}_1 , \quad (1)$$

where N_1 is the number of failures of the item and t_i (i=1,2,…, N_1) is the corresponding inter-arrival failure times. For Option II, there are two stages in the warranty cycle. T can be expressed as:

$$T = t_1 + t_2 + ... + t_{N_a} + \mathrm{T}_0 + \tilde{t}_1 + \tilde{t}_2 + ... + \tilde{t}_{\tilde{N}_a} + \mathrm{W}_1 , \quad (2)$$

where N_a is the number of failures of the item in stage a, $\tilde{N}_a$ is the number of failures of the item in stage b. t_i (i=1,2,…, N_a) is the corresponding i^{th} inter-arrival failure time in stage a and $\tilde{t}_i$ (i=1,2,…, $\tilde{N}_a$) is the corresponding i^{th} inter-arrival failure time in stage b.

For Option III, there are two stages in the warranty cycle. In stage a, the failed item is replaced by a low-quality item. In stage b, the failed item is replaced by a high-quality item.So T can be expressed as:

$$T = t_1 + t_2 + ... + t_{N_a} + \mathrm{T}_0 + h_1 + h_2 + ... + h_{N_b} + \mathrm{W}_2 , \quad (3)$$

where N_a is the number of failures of the system in stage a. N_b is the number of failures of the system in stage b. t_i (i=1,2,…, N_a) is the corresponding i^{th} inter-arrival failure time of stage a and h_i (i=1,2,…, N_b) is the corresponding i^{th}inter-arrival failure time of stage b.

All warranty claims are valid, all system failures under warranty are claimed, and any warranty service is instant. Take the steady availability of the system as one of the targets, as shown in Eq. 4.

$$\mathrm{A}=\frac{\overline{\mathrm{T}}_{\mathrm{bm}}}{\overline{\mathrm{T}}_{\mathrm{bm}} + \overline{\mathrm{D}}} . \quad (4)$$

The cost for option I can be calculated by Eq.5.

$$\mathrm{TC} = \sum_{i=1}^{k_a} (C_a + C_L T_m) N_{ai} . \quad (5)$$

where N_{ai} is the number of failures of the i^{th} failure mode for the low-quality item within warranty cycle. The cost for option II can be calculated by Eq.6.

$$\mathrm{TC} = \sum_{i=1}^{k_a} (C_a + C_L T_m)(N_{ai} + \tilde{N}_{aj}) + C_{\mathrm{p}} + C_L T_p . \quad (6)$$

where N_{ai}($\tilde{N}_{aj}$) is the number of failures of the i^{th} failure mode for the low-quality item within stage a (stage b). The cost for option III can be calculated by Eq.7.

$$\mathrm{TC} = \sum_{i=1}^{k_a} (C_a + C_L T_m) N_{ai} + \sum_{j=1}^{k_b} (C_b + C_L T_r) N_{bj} + \tilde{C}_{\mathrm{p}} + C_L \tilde{T}_p . \quad (7)$$

where N_{ai} (N_{bi}) is the number of failures of the i^{th} (j^{th}) failure mode for the low-quality (high-quality) item within stage a (stage b). Because the corrective maintenance is replacement, the corrective maintenance cost includes the spare cost and maintenance cost.

4. Analytical model

For a single component, every failure mode will cause the component to fail. We start with the availability model for one component with several failure modes, and then we develop the cost model. Before proposing the models for different options, we give some definitions.

Lemma 1. *For a low-quality item,* $p_{ai}(t)$ *is the probability that failure mode i causes the component failure before the end of time limit t, so*

$$p_{ai}(t) = P(T_{ai} \le \min(T_{aj}, \forall j, j \in \Omega_1, j \ne i), T_{ai} \le t) = \int_0^t \frac{\lambda_{ai}(u)}{\lambda_{as}(u)} f_{as}(u)\mathrm{d}u,\ \Omega_1 = (1,2,...,k_a). \quad (8)$$

$\lambda_{ai}(u)$ is the failure rate of the i^{th} failure mode and $f_{as}(u)$ is the probability density function for the low-quality item. Because the failure modes are independent to each other, $\lambda_{as}(t) = \sum_{i=1}^{k_a} \lambda_i(t)$, $R_{as}(t) = 1 - F_s(t) = \prod_{i=1}^{k_a} R_{ai}$ and $\lambda_{ai}(t) = \frac{f_{ai}(t)}{R_{ai}(t)}$.

Proof. From the definition of $p_{ai}(t)$, let $Y = \min(T_{aj}, \forall j, j \in \Omega_1, j \ne i)$ we can obtain:

$$p_{bj}(t) = \int_0^\infty P[T_{bj} \le \min(T_{bj}, \forall j, j \in \Omega_2, j \ne i), T_{bj} \le t | T_{bj} = u] \mathrm{d}F_{bj}(u)$$
$$= \int_0^t P[\min(T_{bj}, \forall j, j \in \Omega_2, j \ne i) \ge u] \mathrm{d}F_{bj}(u) = \int_0^t \frac{\lambda_{bj}(u)}{\lambda_{bs}(u)} f_{bs}(u)\mathrm{d}t .$$

Lemma 2. Let $p_{bj}(t)$ *be the probability that failure mode j of the high-quality product causes a component failure in t,* $\Omega_2 = (1,2,...,k_b)$ *then:*

$$p_{bj}(t) = \int_0^\infty P[T_{bj} \le \min(T_{bj}, \forall j, j \in \Omega_2, j \ne i), T_{bj} \le t | T_{bj} = u] dF_{bj}(u)$$
$$= \int_0^t P[\min(T_{bj}, \forall j, j \in \Omega_2, j \ne i) \ge u] dF_{bj}(u) = \int_0^t \frac{\lambda_{bj}(u)}{\lambda_{bs}(u)} f_{bs}(u)dt \quad .(9)$$

Proof. The proof process is similar to Lemma 1.

4.1. Availability and cost model for Option I

In order to derive the statistical properties of the product availability and cost for Option I, it is necessary to obtain the distribution of N_1. The following lemma gives the probability mass function (pmf) of N_1. Obviously, $N_1 = \sum_{i=1}^{k_a} N_{ai}$. Under the perfect maintenance assumption, the pmf of N_1 is:

$$P(N_1 = n_1) = [F_s(\mathrm{W}_1)]^{n_1} (1 - F_s(\mathrm{W}_1))\ \forall n_1, n_1 = 0,1,2,... \quad (10)$$

The proof process is similar to [3].We can formulate the product availability and expected warranty cost for Option I as:

$$A=\frac{\sum_{i=1}^{k_a}E(N_{ai})\int_0^{W_1}tf_{ai}(t)dt+W_1}{\sum_{i=1}^{k_a}E(N_{ai})(\int_0^{W_1}tf_{ai}(t)dt+T_m)+W_1},\ E(C)=(C_a+C_LT_m)\sum_{i=1}^{k_a}E(N_{ai}),\quad(11)$$

where $E(N_{ai})=\dfrac{p_{ai}(W_1)}{R_{as}(W_1)}$.

Proof. From Eq.11, obviously the product availability and warranty cost can be determined as long as the joint distribution of $N_{ai}(i=1,2,\ldots k_a)$ is known. From Lemma 1, we can obtain the probability that failure mode i causes an item failure before the end of warranty period W_1. Then when the item failed within W_1, the probability that it is caused by failure mode i is $\dfrac{p_{ai}(W_1)}{F_s(W_1)}$. So the conditional joint distribution of $N_{a1},N_{a2},...,N_{ak_a}$ given $N_1=n_1$ is multinomial as shown by Equation 12:

$$P(N_{a1}=n_{a1},N_{a2}=n_{a2},...,N_{ak_a}=n_{ak_a}\mid N_1=n_1)=\frac{n_1!}{n_{a1}!n_{a2}!...n_{ak_a}!}\prod_{i=1}^{k_a}\left(\frac{p_{ai}(W_1)}{F_s(W)}\right)^{n_{ai}}.\quad(12)$$

According to the properties of multinomial distribution, we can get:

$$P(N_{ai}=n_{ai}\mid N_1=n_1)=\frac{n_1!}{n_{ai}!(n_1-n_{ai})!}\left(\frac{p_{ai}(W_1)}{F_s(W_1)}\right)^{n_{ai}}\left(1-\frac{p_{ai}(W_1)}{F_s(W_1)}\right)^{n_1-n_{ai}}.\quad(13)$$

So the moment generating function of N_{ai} is:

$$E(e^{tN_{ai}})=E[E[e^{tN_{ai}}\mid N_1]]=E[(1-\frac{p_{ai}(W_1)}{F_s(W_1)}+\frac{p_{ai}(W_1)}{F_s(W_1)}e^t)^{N_1}]=\frac{\dfrac{R_{as}(W_1)}{R_{as}(W_1)+p_{ai}(W_1)}}{1-\dfrac{p_{ai}(W_1)}{R_s(W_1)+p_{ai}(W_1)}e^t}.\quad(14)$$

Eq.14 shows that N_{ai} follows a geometric distribution:

$$P(N_{ai}=n_{ai})=\left(\frac{p_{ai}(W_1)}{R_{as}(W_1)+p_{ai}(W_1)}\right)^{n_{ai}}\left(1-\frac{p_{ai}(W_1)}{R_{as}(T_0)+p_{ai}(W_1)}\right).\quad(15)$$

So $E(N_{ai})=\dfrac{\dfrac{p_{ai}(W_1)}{R_{as}(W_1)+p_{ai}(W_1)}}{1-\dfrac{p_{ai}(W_1)}{R_{as}(W_1)+p_{ai}(W_1)}}=\dfrac{p_{ai}(W_1)}{R_{as}(W_1)}$.

4.2. Availability and cost model for Option II

In order to derive the statistical properties of the product availability and cost for Option II, it is necessary to obtain the distributions of N_{ai} and $\tilde{N}_{aj}$. The following lemma gives the probability mass functions (pmf) for Na and $\tilde{N}_a$. Obviously, $N_a=\sum_{i=1}^{k_a}N_{ai}$ and $\tilde{N}_a=\sum_{j=1}^{k_a}\tilde{N}_{aj}$.

For Stage a, under the perfect maintenance assumption, the pmf of N_a is:

$$P(N_a=n_a)=[F_s(T_0)]^{n_a}(1-F_s(T_0))\ \forall n_a, n_a=0,1,2,\ldots.\quad(16)$$

Proof. Let t_1, t_2, …,be the subsequent inter-arrival failure times within T of stage a which follow a distribution F_s. Obviously,

$$N_a=\min\{i:t_i>T_0\}-1.$$

Therefore:

$$P(N_a=n_a)=P(N_a\le n_a)-P(N_a\le n_a+1)=[F_s(T_0)]^{n_a}-[F_s(T_0)]^{n_a+1}.\quad(17)$$

Lemma 3. *For Stage b, under the perfect maintenance assumption, the pmf of* $\tilde{N}_a$ *for Option II can be expressed as:*

$$P(\tilde{N}_a=\tilde{n}_a)=\begin{cases}1-F_s(W_1\text{-}T_0) & \tilde{n}_a=0\\ F_s(W_1\text{-}T_0)[F_s(W_1)]^{\tilde{n}_a-1}(1-F_s(W_1)) & \tilde{n}_a=1,2,...\end{cases}.\quad(18)$$

Proof. Let $\tilde{t}_1$, $\tilde{t}_2$,... be the subsequent inter-arrival failure times within T of stage b which follow a distribution F_s. Obviously,

$$\tilde{N}_a=\begin{cases}0 & ,\tilde{t}_i\ge W_1-T_0\\ \min\{i:\tilde{t}_i>W_1\}-1, & \tilde{t}_i<W_1-T_0\end{cases}.$$

When $\tilde{t}_1\ge W_1-T_0$, then $\tilde{n}_a=0$ and:

$$P(\tilde{N}_a=\tilde{n}_a)=1-F_s(W_1\text{-}T_0).$$

The PM is perfect, so when $\tilde{t}_1<W_1-T_0$, $\tilde{n}_a>0$ and:

$$P(\tilde{N}_a=\tilde{n}_a)=F_s(W_1\text{-}T_0)[F_s(W_1)]^{\tilde{n}_a-1}(1-F_s(W_1)).\quad(19)$$

Lemma 4. We can formulate the product availability and warranty cost for Option II as:

$$\mathrm{A}=\frac{\sum_{i=1}^{\mathrm{k_a}}E(N_{\mathrm{a}i})\int_0^{\mathrm{T_0}}tf_{ai}(t)\mathrm{d}t+\mathrm{W_1}}{\sum_{i=1}^{\mathrm{k_a}}E(N_{\mathrm{a}i})(\int_0^{\mathrm{T_0}}tf_{ai}(t)\mathrm{d}t+\mathrm{T_m})+\mathrm{T_p}+\mathrm{W_1}}(1-F_s(\mathrm{W_1}-\mathrm{T_0}))$$
$$+\frac{\sum_{i=1}^{\mathrm{k_a}}E(N_{\mathrm{a}i})\int_0^{\mathrm{T_0}}tf_{ai}(t)\mathrm{d}t+\mathrm{T_0}+\sum_{i=1}^{\mathrm{k_a}}(\frac{p_{\mathrm{a}i}(\mathrm{W_1}\text{-}\mathrm{T_0})}{F_s(\mathrm{W_1}\text{-}\mathrm{T_0})}\int_0^{\mathrm{W_1}\text{-}\mathrm{T_0}}tf_{\mathrm{a}i}(t)\mathrm{d}t)+\sum_{i=1}^{\mathrm{k_a}}E(\tilde{N}'_{\mathrm{a}i})\int_0^{\mathrm{W_1}}tf_{\mathrm{a}i}(t)\mathrm{d}t+\mathrm{W_1}}{\sum_{i=1}^{\mathrm{k_a}}E(N_{\mathrm{a}i})(\int_0^{\mathrm{T_0}}tf_{ai}(t)\mathrm{d}t+\mathrm{T_m})+\mathrm{T_0}+\mathrm{T_p}+(\sum_{i=1}^{\mathrm{k_a}}\frac{p_{\mathrm{a}i}(\mathrm{W_1}\text{-}\mathrm{T_0})}{F_s(\mathrm{W_1}\text{-}\mathrm{T_0})}\int_0^{\mathrm{W_1}\text{-}\mathrm{T_0}}tf_{\mathrm{a}i}(t)\mathrm{d}t+\mathrm{T_m})+\sum_{i=1}^{\mathrm{k_a}}E(\tilde{N}'_{\mathrm{a}i})(\int_0^{\mathrm{W_1}}tf_{\mathrm{a}i}(t)\mathrm{d}t+\mathrm{T_m})+\mathrm{W_1}}F_s(\mathrm{W_1}-\mathrm{T_0}), \quad (20)$$

$$\mathrm{E(C)}=(C_a+C_LT_m)\sum_{i=1}^{k_a}\mathrm{E}(N_{\mathrm{a}i})+F_s(\mathrm{W_1}-\mathrm{T_0})(C_a+C_LT_m)\sum_{i=1}^{k_a}\mathrm{E}(N'_{\mathrm{a}i})+C_p+C_LT_\mathrm{p}, \quad (21)$$

where $E(N_{ai})=\frac{p_{\mathrm{a}i}(\mathrm{T_0})}{R_{\mathrm{as}}(\mathrm{T_0})}$ and $E(\tilde{N}'_{\mathrm{a}i})=\frac{p_{ai}(\mathrm{W_1})}{R_{\mathrm{as}}(\mathrm{W_1})}$.

Proof. Obviously, $E(N_{ai})=\frac{p_{\mathrm{a}i}(\mathrm{T_0})}{R_{\mathrm{a}s}(\mathrm{T_0})}$, which can be easily proved. For stage b, there are two situations: $\tilde{N}_{ai}=0$ and $\tilde{N}_{ai}>0$. When $\tilde{N}_{ai}=0$, the availability is:

$$\mathrm{A}=\frac{\sum_{i=1}^{\mathrm{k_a}}E(N_{\mathrm{a}i})\int_0^{\mathrm{T_0}}tf_{ai}(t)\mathrm{d}t+\mathrm{T_0}+\mathrm{W_1}-\mathrm{T_0}}{\sum_{i=1}^{\mathrm{k_a}}E(N_{\mathrm{a}i})(\int_0^{\mathrm{T_0}}tf_{ai}(t)\mathrm{d}t+\mathrm{T_m})+\mathrm{T_p}+\mathrm{T_0}+\mathrm{W_1}-\mathrm{T_0}}=\frac{\sum_{i=1}^{\mathrm{k_a}}E(N_{\mathrm{a}i})\int_0^{\mathrm{T_0}}tf_{ai}(t)\mathrm{d}t+\mathrm{W_1}}{\sum_{i=1}^{\mathrm{k_a}}E(N_{\mathrm{a}i})(\int_0^{\mathrm{T_0}}tf_{ai}(t)\mathrm{d}t+\mathrm{T_m})+\mathrm{T_p}+\mathrm{W_1}}.$$

When $\tilde{N}_{ai}>0$, the probability that failure mode *i* causes the item to fail in $\mathrm{W_1}$-$\mathrm{T_0}$ is $p_{ai}(\mathrm{W_1}\text{-}\mathrm{T_0})=\int_0^{\mathrm{W_1}\text{-}\mathrm{T_0}}\frac{\lambda_{\mathrm{a}i}(t)}{\lambda_{\mathrm{as}}(t)}f_{\mathrm{bs}}(t)\mathrm{d}t$. So $\frac{p_{ai}(\mathrm{W_1}\text{-}\mathrm{T_0})}{F_s(\mathrm{W_1}\text{-}\mathrm{T_0})}$ is the conditional probability that failure mode *i* causes the item to fail given that the item has failed within $\mathrm{W_1}$-$\mathrm{T_0}$. Then the availability is:

$$\mathrm{A}=\frac{\sum_{i=1}^{\mathrm{k_a}}E(N_{\mathrm{a}i})\int_0^{\mathrm{T_0}}tf_{ai}(t)\mathrm{d}t+\mathrm{T_0}+\sum_{i=1}^{\mathrm{k_a}}(\frac{p_{\mathrm{a}i}(\mathrm{W_1}\text{-}\mathrm{T_0})}{F_s(\mathrm{W_1}\text{-}\mathrm{T_0})}\int_0^{\mathrm{W_1}\text{-}\mathrm{T_0}}tf_{\mathrm{a}i}(t)\mathrm{d}t)+\sum_{i=1}^{\mathrm{k_a}}E(\tilde{N}'_{\mathrm{a}i})\int_0^{\mathrm{W_1}}tf_{\mathrm{a}i}(t)\mathrm{d}t+\mathrm{W_1}}{\sum_{i=1}^{\mathrm{k_a}}E(N_{\mathrm{a}i})(\int_0^{\mathrm{T_0}}tf_{ai}(t)\mathrm{d}t+\mathrm{T_m})+\mathrm{T_0}+\mathrm{T_p}+(\sum_{i=1}^{\mathrm{k_a}}\frac{p_{\mathrm{a}i}(\mathrm{W_1}\text{-}\mathrm{T_0})}{F_s(\mathrm{W_1}\text{-}\mathrm{T_0})}\int_0^{\mathrm{W_1}\text{-}\mathrm{T_0}}tf_{\mathrm{a}i}(t)\mathrm{d}t+\mathrm{T_m})+\sum_{i=1}^{\mathrm{k_a}}E(\tilde{N}'_{\mathrm{a}i})(\int_0^{\mathrm{W_1}}tf_{\mathrm{a}i}(t)\mathrm{d}t+\mathrm{T_m})+\mathrm{W_1}}.$$

Because $P(A)=P(A\mid B)P(B)+P(A\mid\overline{B})P(\overline{B})$, the availability model is proved. Obviously, $\tilde{N}'_a=\sum_{j=1}^{k_\mathrm{b}}N'_{ai}=\tilde{N}_\mathrm{a}-1$. $\tilde{N}'_a$ is the number of failures after the first failure within stage b. So

$$P(\tilde{N}'_{a1}=\tilde{n}'_{a1},\tilde{N}'_{a2}=\tilde{n}'_{a2},...,\tilde{N}'_{ak_a}=\tilde{n}'_{ak_a}\mid\tilde{N}'_\mathrm{a}=\tilde{n}'_\mathrm{a})=\frac{\tilde{n}'_\mathrm{a}!}{\tilde{n}'_{\mathrm{a}1}!\tilde{n}'_{\mathrm{a}2}!...\tilde{n}'_{\mathrm{a}k_\mathrm{a}}!}\prod_{j=1}^{k_a}\left(\frac{p_{\mathrm{a}i}(\mathrm{W_1})}{F_s(\mathrm{W_1})}\right)^{\tilde{n}'_{\mathrm{a}i}}, \quad (22)$$

$$P(\tilde{N}'_{\mathrm{a}i}=\tilde{n}'_{\mathrm{a}i})=\left(\frac{p_{ai}(\mathrm{W_1})}{R_{\mathrm{as}}(\mathrm{W_1})+p_{ai}(\mathrm{W_1})}\right)^{\tilde{n}'_{\mathrm{a}i}}\left(1-\frac{p_{ai}(\mathrm{W_1})}{R_{\mathrm{as}}(\mathrm{W_1})+p_{ai}(\mathrm{W_1})}\right). \quad (23)$$

As a result, $E(\tilde{N}'_{ai})=\frac{p_{ai}(\mathrm{W_1})}{R_{\mathrm{as}}(\mathrm{W_1})}$. According to the property of conditional expectation [15], we can obtain:

$$\begin{aligned}E(\tilde{N}_{ai}) &= 0\times P(\tilde{t}_1 > \mathrm{W}_1-\mathrm{T}_0) + P(\tilde{t}_1 \le \mathrm{W}_1-\mathrm{T}_0)\left[1\times\frac{p_{ai}(\mathrm{W}_1\text{-}\mathrm{T}_0)}{F_s(\mathrm{W}_1\text{-}\mathrm{T}_0)} + E(\tilde{N}_{ai}^{'})\right] \\ &= \left[\frac{p_{ai}(\mathrm{W}_1\text{-}\mathrm{T}_0)}{F_s(\mathrm{W}_1\text{-}\mathrm{T}_0)} + \frac{p_{ai}(\mathrm{W}_1)}{R_{as}(\mathrm{W}_1)}\right]F_s(\mathrm{W}_1-\mathrm{T}_0)\end{aligned} \tag{24}$$

And the warranty cost for Option II is

$$\begin{aligned}\mathrm{E(C)} &= (C_a + C_L T_m)\sum_{i=1}^{k_a}\mathrm{E}(N_{ai}) + (1-F_s(\mathrm{W}_1-\mathrm{T}_0))(C_p + C_L T_\mathrm{p}) + F_s(\mathrm{W}_1-\mathrm{T}_0)[C_p + C_L T_\mathrm{p} + (C_a + C_L T_m)\sum_{i=1}^{k_a}\mathrm{E}(N_{ai}^{'})] \\ &= (C_a + C_L T_m)\sum_{i=1}^{k_a}\mathrm{E}(N_{ai}) + F_s(\mathrm{W}_1-\mathrm{T}_0)(C_a + C_L T_m)\sum_{i=1}^{k_a}\mathrm{E}(N_{ai}^{'}) + C_p + C_L T_\mathrm{p}\end{aligned}$$

4.3. Availability and cost model for Option III

In order to derive the statistical properties of the product availability and cost for Option III, it is necessary to obtain the distribution of N_a and N_b. The following lemma gives the probability mass functions (pmf) for N_a and N_b. Obviously, $N_\mathrm{a} = \sum_{i=1}^{\mathrm{k_a}} N_{\mathrm{a}i}$ and $N_\mathrm{b} = \sum_{j=1}^{\mathrm{k_b}} N_{\mathrm{b}j}$. For stage a, under the perfect maintenance assumption, the pmf of N_a is:

$$P(N_\mathrm{a} = n_\mathrm{a}) = [F_s(\mathrm{T}_0)]^{n_\mathrm{a}}(1-F_s(\mathrm{T}_0)) \;\; \forall n_\mathrm{a}, n_\mathrm{a} = 0,1,2,\dots \tag{25}$$

For stage b, under the perfect maintenance assumption, the pmf of N_b for Option III is:

$$P(N_\mathrm{b} = n_\mathrm{b}) = \begin{cases} 1-\mathrm{G}_s(\mathrm{W}_2\text{-}\mathrm{T}_0) & n_\mathrm{b} = 0 \\ \mathrm{G}_s(\mathrm{W}_2\text{-}\mathrm{T}_0)[\mathrm{G}_s(\mathrm{W}_2)]^{n_\mathrm{b}-1}(1-\mathrm{G}_s(\mathrm{W}_2)) & n_\mathrm{b} = 1,2,\dots \end{cases} \tag{26}$$

The proof process of Eq. 25 is similar to Eq. 10. And the proof process of Eq. 26 is similar to Eq. 18.

Lemma 5. We can formulate the product availability and warranty cost for Option III as:

$$\begin{aligned}\mathrm{A} &= \frac{E(\sum_{i=1}^{\mathrm{k_a}} N_{\mathrm{a}i}\int_0^{\mathrm{T}_0} tf_{ai}(t)\mathrm{d}t + \mathrm{W}_2)}{E(\sum_{i=1}^{\mathrm{k_a}} N_{\mathrm{a}i}(\int_0^{\mathrm{T}_0} tf_{ai}(t)\mathrm{d}t + \mathrm{T_m}) + \tilde{\mathrm{T}}_\mathrm{p} + \mathrm{W}_2)}(1-G_s(\mathrm{W}_2-\mathrm{T}_0)) \\ &+ \frac{\sum_{i=1}^{\mathrm{k_a}} E(N_{\mathrm{a}i})\int_0^{\mathrm{T}_0} tf_{ai}(t)\mathrm{d}t + \mathrm{T}_0 + \sum_{j=1}^{\mathrm{k_b}}(\frac{p_{\mathrm{b}j}(\mathrm{W}_2\text{-}\mathrm{T}_0)}{G_s(\mathrm{W}_2\text{-}\mathrm{T}_0)}\int_0^{\mathrm{W}_2\text{-}\mathrm{T}_0} tf_{\mathrm{b}j}(t)\mathrm{d}t) + \sum_{j=1}^{\mathrm{k_b}} E(N_{\mathrm{b}j}^{'})\int_0^{\mathrm{W}_2} tf_{\mathrm{b}j}(t)\mathrm{d}t + \mathrm{W}_2}{\sum_{i=1}^{\mathrm{k_a}} E(N_{\mathrm{a}i})(\int_0^{\mathrm{T}_0} tf_{ai}(t)\mathrm{d}t + \mathrm{T_m}) + \mathrm{T}_0 + \tilde{\mathrm{T}}_\mathrm{p} + (\sum_{j=1}^{\mathrm{k_b}}\frac{p_{\mathrm{b}j}(\mathrm{W}_2\text{-}\mathrm{T}_0)}{G_s(\mathrm{W}_2\text{-}\mathrm{T}_0)}\int_0^{\mathrm{W}_2\text{-}\mathrm{T}_0} tf_{\mathrm{b}j}(t)\mathrm{d}t + \mathrm{T_r}) + \sum_{j=1}^{\mathrm{k_b}} E(N_{\mathrm{b}j}^{'})(\int_0^{\mathrm{W}_2} tf_{\mathrm{b}j}(t)\mathrm{d}t + \mathrm{T_r}) + \mathrm{W}_2} G_s(\mathrm{W}_2-\mathrm{T}_0)\end{aligned}$$

$$\mathrm{E(C)} = (C_a + C_L T_m)\sum_{i=1}^{k_a}\mathrm{E}(N_{ai}) + G_s(\mathrm{W}_2-\mathrm{T}_0)(C_b + C_L T_r)\sum_{j=1}^{k_b}\mathrm{E}(N_{bj}^{'}) + \tilde{C}_p + C_L\tilde{T}_\mathrm{p}. \tag{27}$$

where $E(N_{\mathrm{a}i}) = \frac{p_{\mathrm{a}i}(\mathrm{T}_0)}{R_{as}(\mathrm{T}_0)}$ and $E(\tilde{N}_{\mathrm{b}j}^{'}) = \frac{p_{\mathrm{b}j}(\mathrm{W}_2)}{R_{\mathrm{bs}}(\mathrm{W}_2)}$.

The proof process of Eq. 27 is similar to Eq. 21.

5. Numerical example

In this section, we consider a particular item with two types of spares (high-quality and low-quality) in the warranty renewing cycle. The failure distributions of these spares and the parameters needed for the warranty availability and cost analysis are provided in Table 1. The low-quality spare has three failure modes. However, the high-quality spare has only two failure modes since its quality has been improved by some engineering

Table 1. distributions and parameters

	Failure modes	Failure density function	α	β
Low-quality spare	F11	$f_{a1}(t)=\frac{\alpha_{a1}}{\beta_{a1}}\left(\frac{t}{\beta_{a1}}\right)^{\alpha_{a1}-1}\mathrm{e}^{-\left(\frac{t}{\beta_{a1}}\right)^{\alpha_{a1}}}$	2.42	12.24
	F12	$f_{a2}(t)=\frac{\alpha_{a2}}{\beta_{a2}}\left(\frac{t}{\beta_{a2}}\right)^{\alpha_{a2}-1}\mathrm{e}^{-\left(\frac{t}{\beta_{a2}}\right)^{\alpha_{a2}}}$	2.31	11.37
	F13	$f_{a3}(t)=\frac{\alpha_{a3}}{\beta_{a3}}\left(\frac{t}{\beta_{a3}}\right)^{\alpha_{a3}-1}\mathrm{e}^{-\left(\frac{t}{\beta_{a3}}\right)^{\alpha_{a3}}}$	6.16	18.92
High-quality spare	F21	$g_{b1}(t)=\frac{\alpha_{b1}}{\beta_{b1}}\left(\frac{t}{\beta_{b1}}\right)^{\alpha_{b1}-1}\mathrm{e}^{-\left(\frac{t}{\beta_{b1}}\right)^{\alpha_{b1}}}$	5.14	6.54
	F22	$g_{b2}(t)=\frac{\alpha_{b2}}{\beta_{b2}}\left(\frac{t}{\beta_{b2}}\right)^{\alpha_{b2}-1}\mathrm{e}^{-\left(\frac{t}{\beta_{b2}}\right)^{\alpha_{b2}}}$	2.11	9.21

Table 2. E(C), A and E(C)/A with different T_0

T_0	E(C)	A	E(C)/A
0.5	192.3316	0.9913	194.0241
1.0	164.6387	0.9914	166.0631
1.5	145.5575	0.9910	146.8825
2.0	134.7430	0.9898	136.1375
2.5	131.5604	0.9877	133.2052
3.0	135.4621	0.9846	137.5794
3.5	146.1847	0.9806	149.0747
4.0	163.8199	0.9757	167.8969
4.5	188.8039	0.9701	194.6238

techniques.In this paper, Weibull distribution is applied to model the failure modes. However, any other life time distributions can be used because all integrals and differentiations are manipulated numerically. C_L=100 and W_1=W_2=5.

Fig. 3 shows the warranty cost, product availability and E(C)/A with different W_1 of Option I. T_m=0.29 and C_a=200. When W_1=5, E(C)=138.8441, A=0.9692, and E(C)/A =143.2600.

For Option II, $C_p = 95$ and $T_p = 0.2$. The warranty cost, product availability and E(C)/A with different T_0 of Option II are shown in Fig. 4.

For Option III, $\tilde{C}_p = 80$, $\tilde{T}_p = 0.03$, C_b=250 and T_r=0.05. Fig. 5 shows the warranty cost, product availability and E(C)/A with different T_0. The optimal T_0 is different when the warranty cost is minimal and the product availability is maximal. So we consider E(C)/A to find the optimal T_0. The partial results of E(C)/A with different T_0 is shown in Table 2. We can conclude that when T_0=2.4 years, the renewing warranty policy is optimal and E(C)/A=133.1947.

In order to validate our previous modeling procedure and results, Monte Carlo method is used to obtain simulated results for the purpose of comparing simulated results to analytical results for Option III. Fig. 6 shows the flow chart for the simulation algorithms of Option III.

T_1 (T_3) is the simulated time of stage a (stage b). And T_2 (T_4) is the simulated working time of stage a (stage b). C_1 (C_2) is the simulated warranty cost of stage a (stage b).Fig.7 shows the comparison between the simulated results and the analytical results.

In order to identify the difference between simulated results and analytical results, we apply root-mean-square error (RMSE) method, mean-absolute error (MAE) method, variance-absolute error (VAE) method, mean-average-relative error (MARE) method, and variance-relative error (VRE) method. RMSE, MAE, and VAW belong to absolute error. And MARE and VRE belong to relative error. The relevant equations are:

$$\mathrm{RMSE}=\frac{1}{n}\sqrt{\sum_{\mathrm{w}=1}^{n}(\hat{x}_{\mathrm{w}}-x_{\mathrm{w}})^2},\ \mathrm{MAE}=\frac{1}{n}\sum_{\mathrm{w}=1}^{n}\left|\hat{x}_{\mathrm{w}}-x_{\mathrm{w}}\right|,\ \mathrm{VAE}=\frac{1}{n}\sum_{\mathrm{w}=1}^{n}(\left|\hat{x}_{\mathrm{w}}-x_{\mathrm{w}}\right|-\mathrm{MAE})^2;$$

$$\mathrm{MARE}=\frac{1}{n}\sum_{\mathrm{w}=1}^{n}\left|\frac{\hat{x}_{\mathrm{w}}-x_{\mathrm{w}}}{x_{\mathrm{w}}}\right|,\ \mathrm{VRE}=\frac{1}{n}\sum_{\mathrm{w}=1}^{n}(\left|\frac{\hat{x}_{\mathrm{w}}-x_{\mathrm{w}}}{x_{\mathrm{w}}}\right|\text{-MARE})^2.$$

where x_{w} is the simulation result and $\hat{x}_{\mathrm{w}}$ is the analytical result. The results of these errors are shown in Table 3. The error is insignificant, so we can prove that our models are valid.

We change the parameters to analyze the sensitivity of the cost model and availability model of Option III asshown in Fig. 8-9. As can be seen, when α_{a1} and β_{a1} are increasing, the product availability is increasing and the warranty cost is decreasing. When α_{b1} and β_{b1} are increasing, the product availability is increasing and the warranty cost is decreasing. The sensitivity analysis can prove the stability of the models.

Table 3 error analysis between simulated results and analytical results

	RMSE	MAE	VAE	MARE	VRE
Warranty cost	4.4340	26.1987	316.2951	5.6441×10^{-4}	1.5616×10^{-5}
Product availability	9.8000×10^{-4}	4.9771×10^{-3}	2.3249×10^{-5}	1.0031×10^{-4}	4.8317×10^{-7}

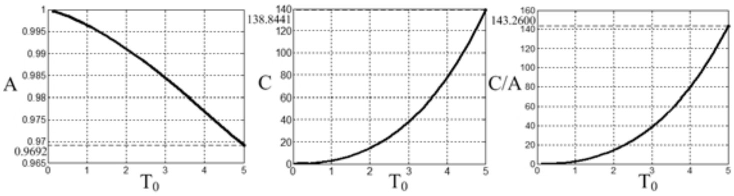

Fig. 3. Results with different W_1 for option I

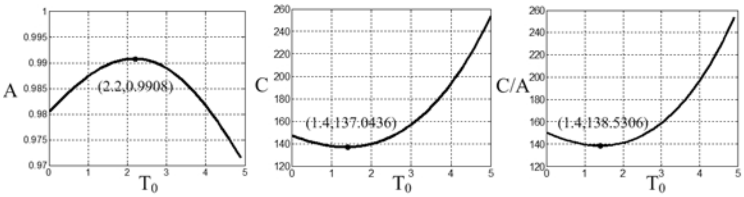

Fig. 4. Results with different T_0 for option II

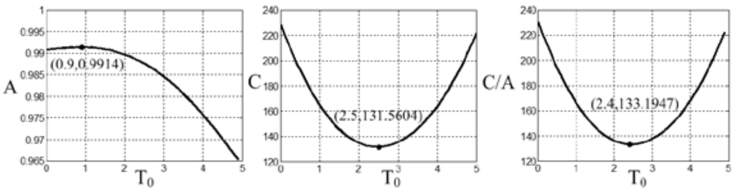

Fig. 5. Results with different T_0 for option III

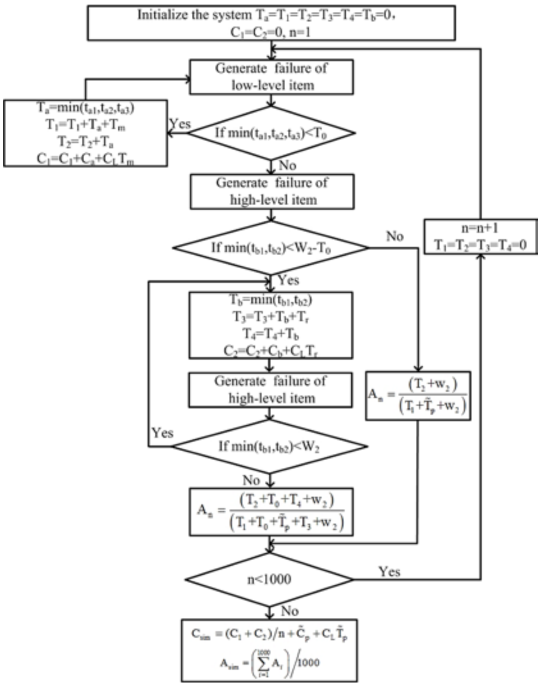

Fig. 6. Simulation flow chart of option III

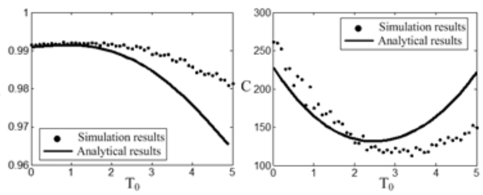

Fig. 7. Simulated results and analytical results for option III

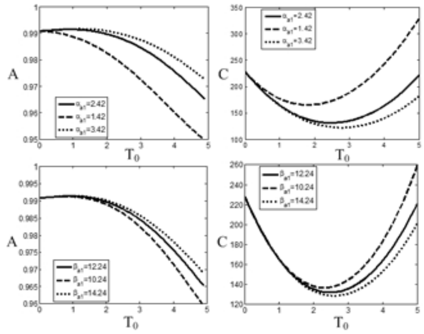

Fig. 8. Product availability and warranty cost analysis with different parameters for low-quality item

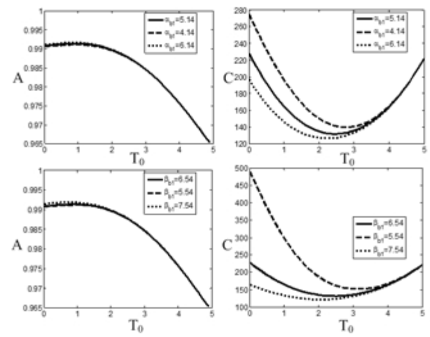

Fig. 9. Product availability and warranty cost analysis with different parameters for high-quality item

6. Conclusions

In this paper, we analyzed three maintenance options in warranty renewing period which includes a new renewing warranty policy considering upgrading maintenance for products with multiple failure modes. The cost and availability models of these maintenance options have been deveoped. For Option II and Option III, the optimal upgrading time T_0 for both warranty cost and product availability can be obtained. The proposed models are illustrated by numerical examples. Monte Carlo simulation method is applied to validate the analytical models of Option III by analyzing the errors between simulation results. Finally, sensitivity analysis for Option III is conducted. We find Option III is better than Option II in some cases. And Option II is better than Option I. As a result, in warranty renewing period, upgrading maintenance can improve the performance and decrease warranty cost in some cases.

There are several potential extensions to the study of the renewing policy considering upgrading maintenance. Firstly, the perfect maintenance assumption can be relaxed. Although this assumption is used widely in practice as well as in warranty and maintenance literature, we believe that more research shall be conducted for the policies considering imperfect maintenance. Secondly, it is needed to consider the case when upgrading maintenance is conducted during the corrective maintenance time. In this paper, we considered upgrading maintenance as a preventive maintenance. Only the low-quality product is working successfully till T_0, then the high-quality spare is used to replace the low-quality product. However, when the corrective maintenance is upgrading maintenance, the models would be different.

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