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# EXAMPLES OF „CABRI GEOMETRE II" APPLICATION IN GEOMETRIC SCIENTIFIC RESEARCH

## Introduction

Computer techniques are more and more frequently used in geometric problems both theoretical and didactic as well as related to engineering constructions. Solving many problems without computer techniques, particularly their visualization, would be very difficult or quite impossible.

CABRI II is a computer program developed by French mathematicians from University of Grenoble and made by the Texas Instruments. It is mainly used in the domain of didactic, but at the same time it is a very convenient tool which may be used in scientific research of geometry.

Its application is similar to traditional drawing on a sheet of paper with compasses and triangles.

A possibility of defining additional constructions called **macroconstructions** enables extend of original collection of objects and constructions in CABRI program. It permits a later automatic replay of a wanted constructions at any moment.

One of CABRI's internal function enables creation of geometric loci executing an automatic curve plotting. This option is very convenient for research, construction and verification of geometrical relation.

As an example a few geometric relations have been showed, using macroconstructions and geometric loci.

## Macroconstructions.

In CABRI II program, inter alia has been additionally created, following simple **macroconstructions:**

**M1** - a construction of straight lines $t_{1,2}$ externally tangent to two given circles $\hat{a}_{1,2}$,

**M2** - a construction of straight lines $t_{1,2}$ internally tangent to two given circles $\hat{a}_{1,2}$,

**M3** - a construction of circle $\hat{b}$ with given centre $O_b$, perpendicular to given circle $\hat{a}$,

**M4** - a construction of circle of Apolonius $\hat{a}_p$,

**M5** - a construction of power axis $p$ for two given circles $\hat{a}_{1,2}$,

**M6** - a construction of power point $P$ for three given circles $\hat{a}_{1,2,3}$,

**M7** - a construction of inversion of straight line $m$ in relation to given circle $\hat{a}$,

**M8** - a construction of inversion of circle $\hat{m}$ in relation to given circle $\hat{a}$,

**M9** - a construction of circle $\hat{a}_\phi$. Straight lines $t_{1,2...}$ tangent to circle $\hat{a}_\phi$ including a given angle with given circle $\hat{a}$,

**M10** - a construction of circle $\hat{p}$ including given point $P$ and tangent to two given straight lines $a_{1,2}$,

**M11** - a construction of point *P( P', P")*, from which segments *P1, P2, P3, P4* tangents to two given circle $\hat{a}$ and $\hat{b}$ noncoplanar, are the same length,

**M12** - a construction of orthogonal projection *c"*, of right section of a cone of revolution. Its generatrix coinciding with given point *I* and including given angle $\phi$ with given circle $\hat{a}$ .

In the following examples

- given angles $\phi_1 = \phi_2 = \phi$
- given elements ----------
- determined elements ━━━━━

are assumed:

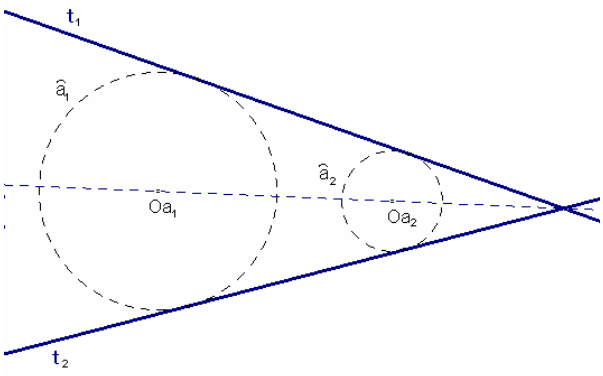


Fig.M1

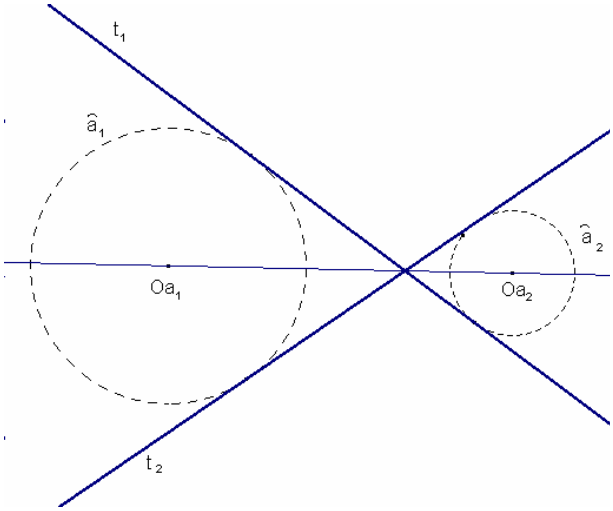


Fig.M2

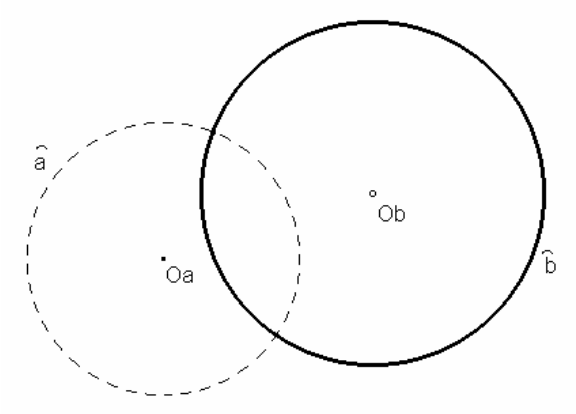


Fig.M3

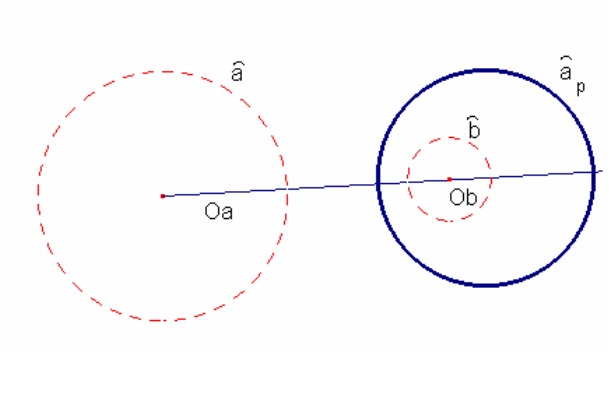


Fig.M4

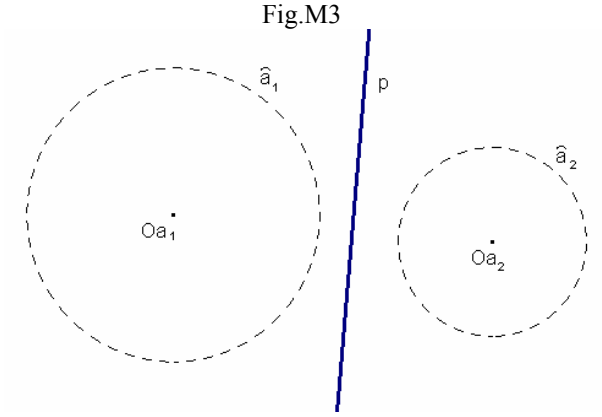


Fig.M5

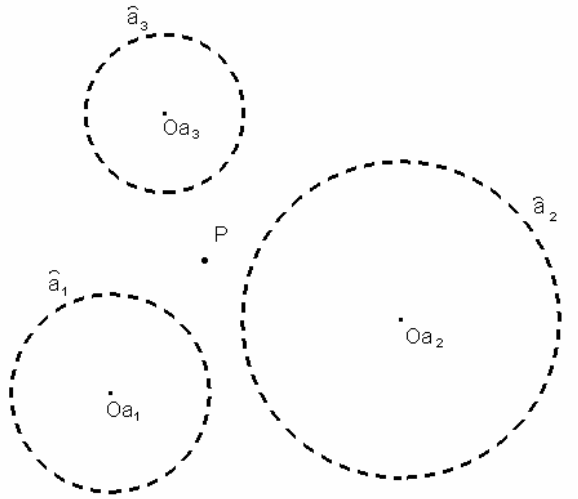


Fig.M6

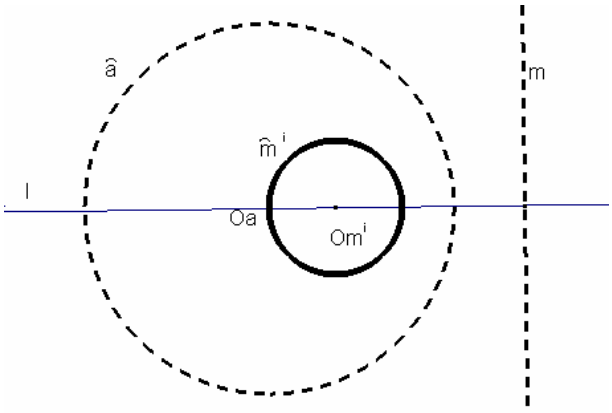


Fig.M7

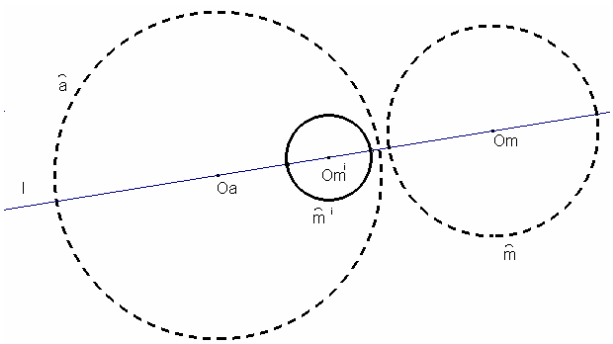


Fig.M8

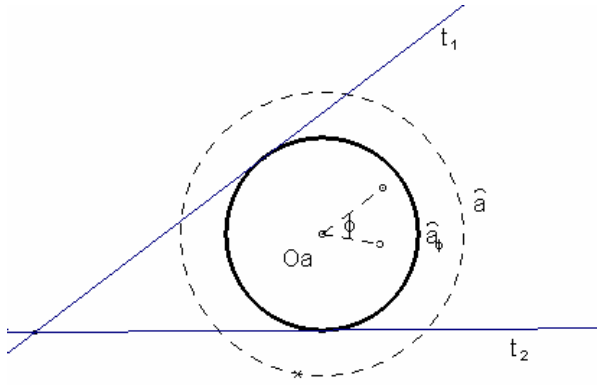


Fig.M9

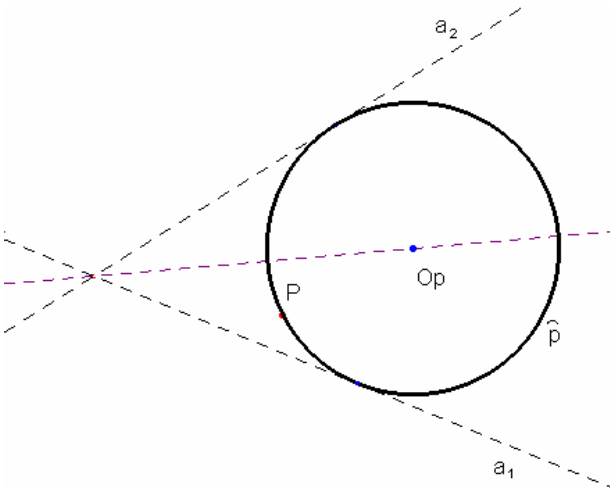


Fig.M10

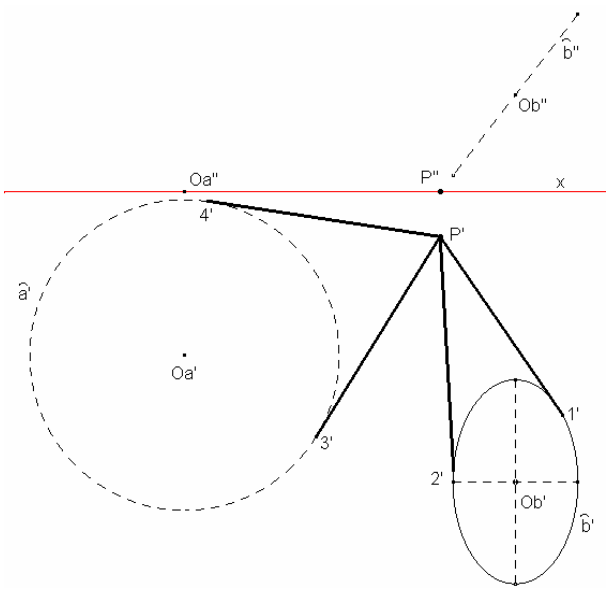


Fig.M11

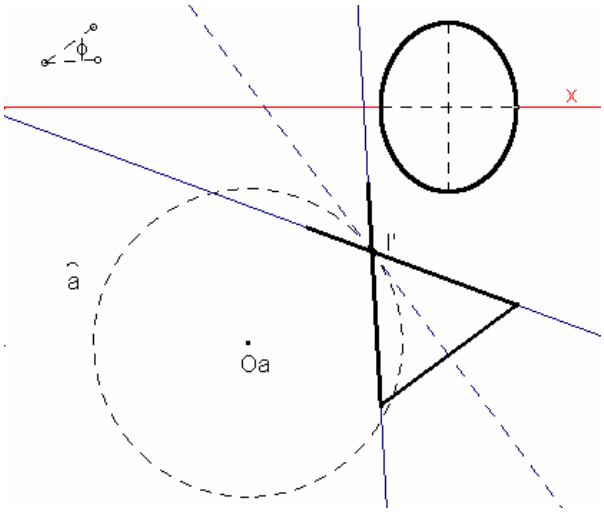


Fig.M12

Using macroconstructions „.M" bigger macroconstructions „K" have been created".
Using macroconstructions „K" a few constructions have been defined:
For example:

**$K_1$**

Given elements:

**- circle $\hat{a}$ , angle $\phi$, point *P*** (given elements are coplanar).

Determined elements:

**- circles $\hat{b}_1$ and $\hat{b}_2$** - (including given angle $\phi$ with circle $\hat{a}$ and coinciding with point *P*),

**- $O_{b1}$ and $O_{b2}$** - (centres of circles $\hat{b}_1$ and $\hat{b}_2$),

**- $c^2$** - conic section - (geometric locus of points $O_{b1}$ and $O_{b2}$).

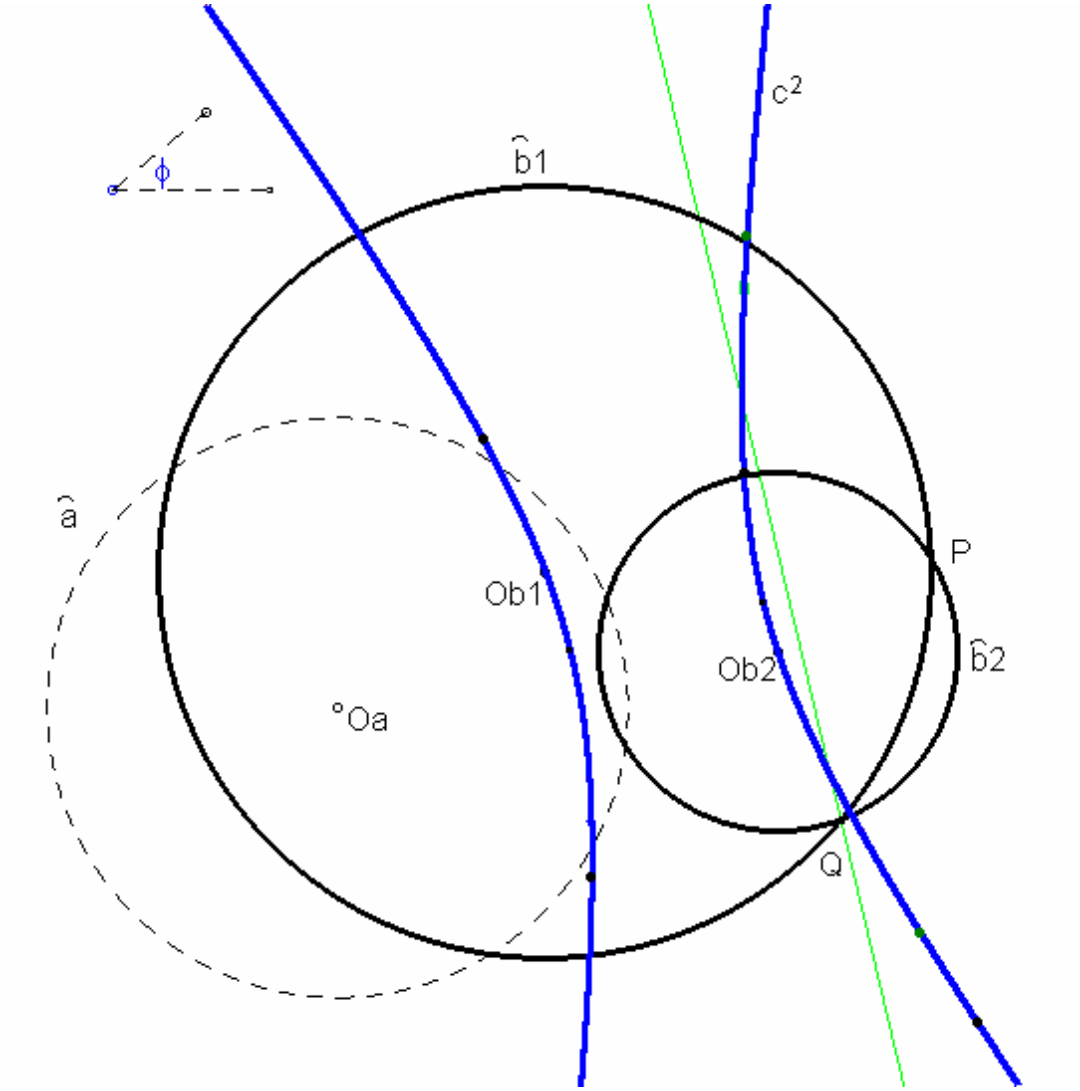


Fig.K1

**$K_2$**

Given elements**:**

- **circles** $\widehat{a}_1$ **and** $\widehat{a}_2$ **, angles** $\phi_1$ **and** $\phi_2$ - (given elements are coplanar).

Determined elements:

- **circles** $\widehat{b}_1$ **,**$\widehat{b}_2$ $\widehat{b}_3$ **and** $\widehat{b}_4$ - (including given angle $\phi_1$ and $\phi_2$ with circles $\widehat{a}_1$ and $\widehat{a}_2$ ),
- $O_{b1}$ $O_{b2}$ $O_{b3}$ **and** $O_{b4}$ - (centres of circles $\widehat{b}_1$ **,**$\widehat{b}_2$ **,**$\widehat{b}_3$ and $\widehat{b}_4$ ),
- $c_1^{\,2}$ **and** $c_2^{\,2}$ - concic sections - (geometric locus of points $O_{b1}$ $O_{b2}$ $O_{b3}$ and $O_{b4}$ ).

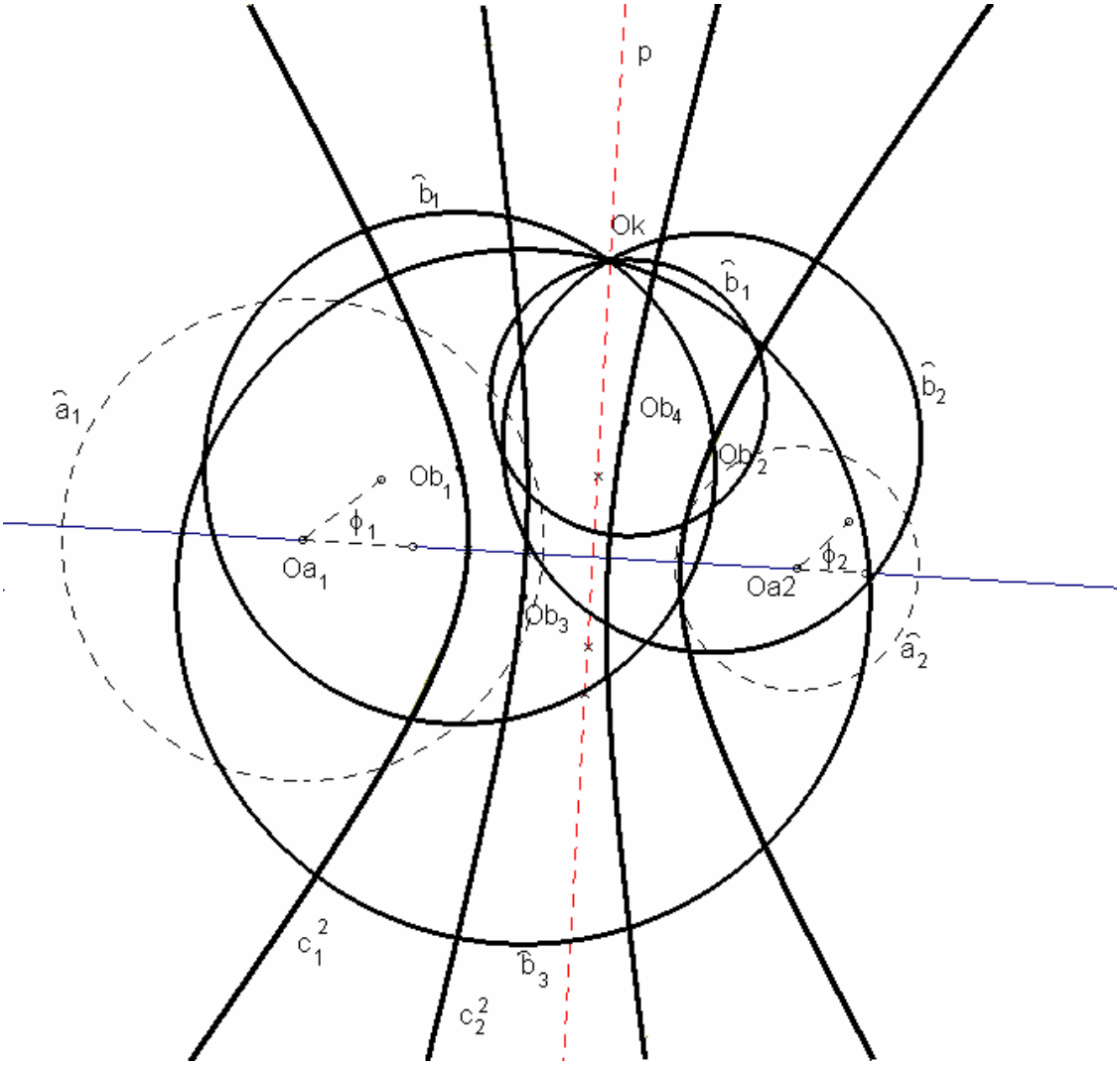


Fig.K2

**K3**

Given elements**:**

**- sphere** $\widehat{A}$**, angle** $\phi$ **and point** ***P*** *(P',P")* - ( $\bar{a}$ - an equator of sphere $\widehat{A}$, and point *P* are coplanar).

Determined elements:

**- spheres** $\widehat{B}_1$ **and** $\widehat{B}_2$ - (including given angle $\phi$ with sphere $\widehat{A}$**,** and coinciding with point *P*),

**-** $O_{B1}$ **and** $O_{B2}$ - (centres of spheres $\widehat{B}_1$ and $\widehat{B}_2$ ),

**-** $C^2$ **- a quadric** -(geometric locus of points $O_{B1}$ and $O_{B2}$),

**-** $c^2$ **- conic section** - (an intersection of quadric $C^2$ by plane of circle $\bar{a}$ and point *P*).

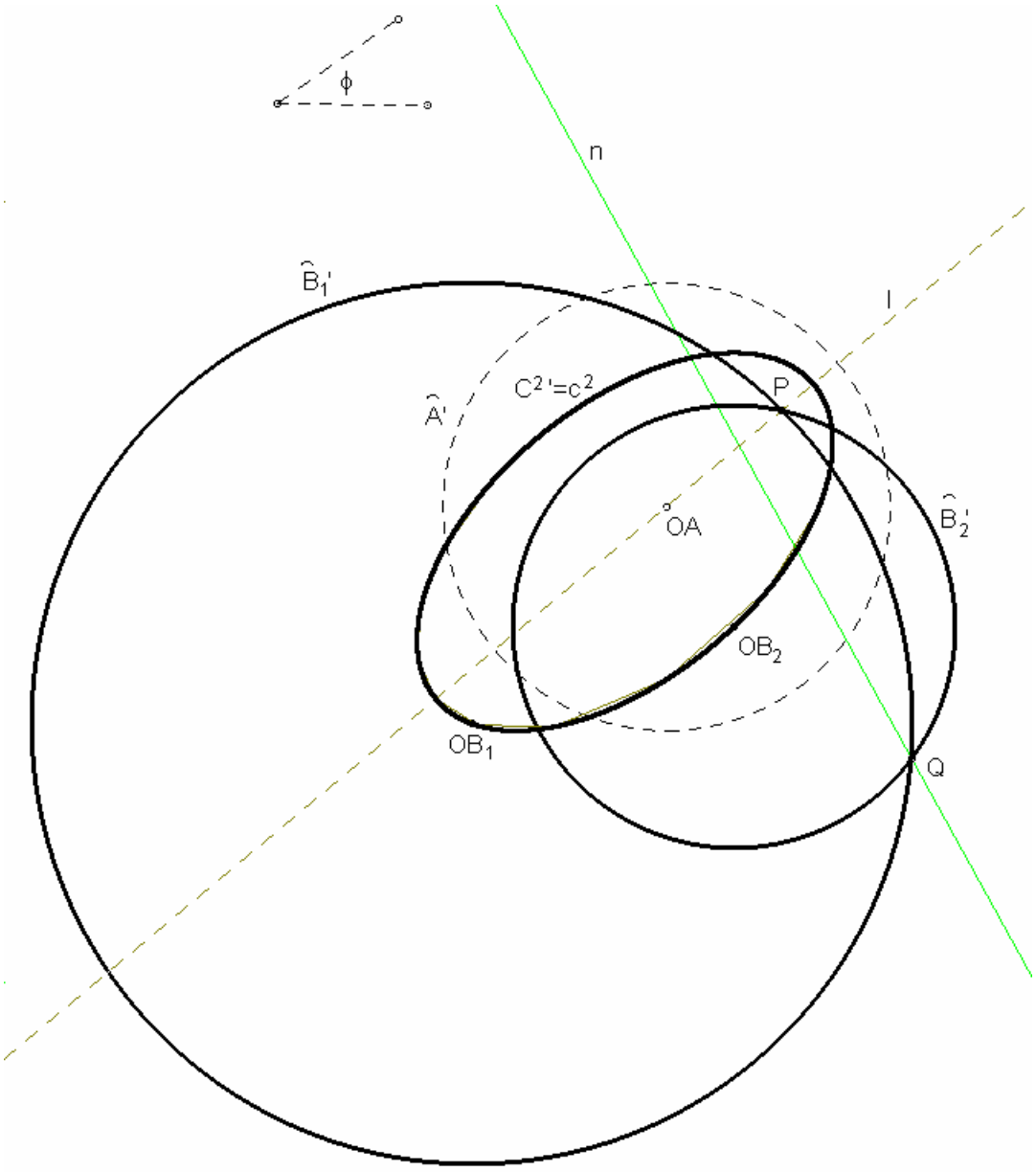


Fig.K3

**K4**

Given elements**:**

**- spheres** $\widehat{A}_1$ **and** $\widehat{A}_2$**, angles** $\phi_1$ **and** $\phi_2$,

Determined elements:

**- spheres** $\widehat{B}_1, \widehat{B}_2, \widehat{B}_3$ **and** $\widehat{B}_4$ - (including given angles $\phi_1$ and $\phi_2$ with spheres $\widehat{A}_1$ and $\widehat{A}_2$),

**-** $O_{B1}, O_{B2}, O_{b3}$ **and** $O_{b4}$ - (centres of spheres $\widehat{B}_1, \widehat{B}_2, \widehat{B}_3$ and $\widehat{B}_4$),

**-** $C_1^2$ **and** $C_2^2$ - quadrics - (geometric locus of points $O_{B1}$, $O_{B2}$, $O_{b3}$ and $O_{b4}$),

**-** $c_1^2$ **and** $c_2^2$ - conic sections - (intersections of quadrics $C_1^2$ and $C_2^2$ by a plane of circles $\widehat{a}_1$ and $\widehat{a}_2$ equators of spheres $\widehat{A}_1$, $\widehat{A}_2$).

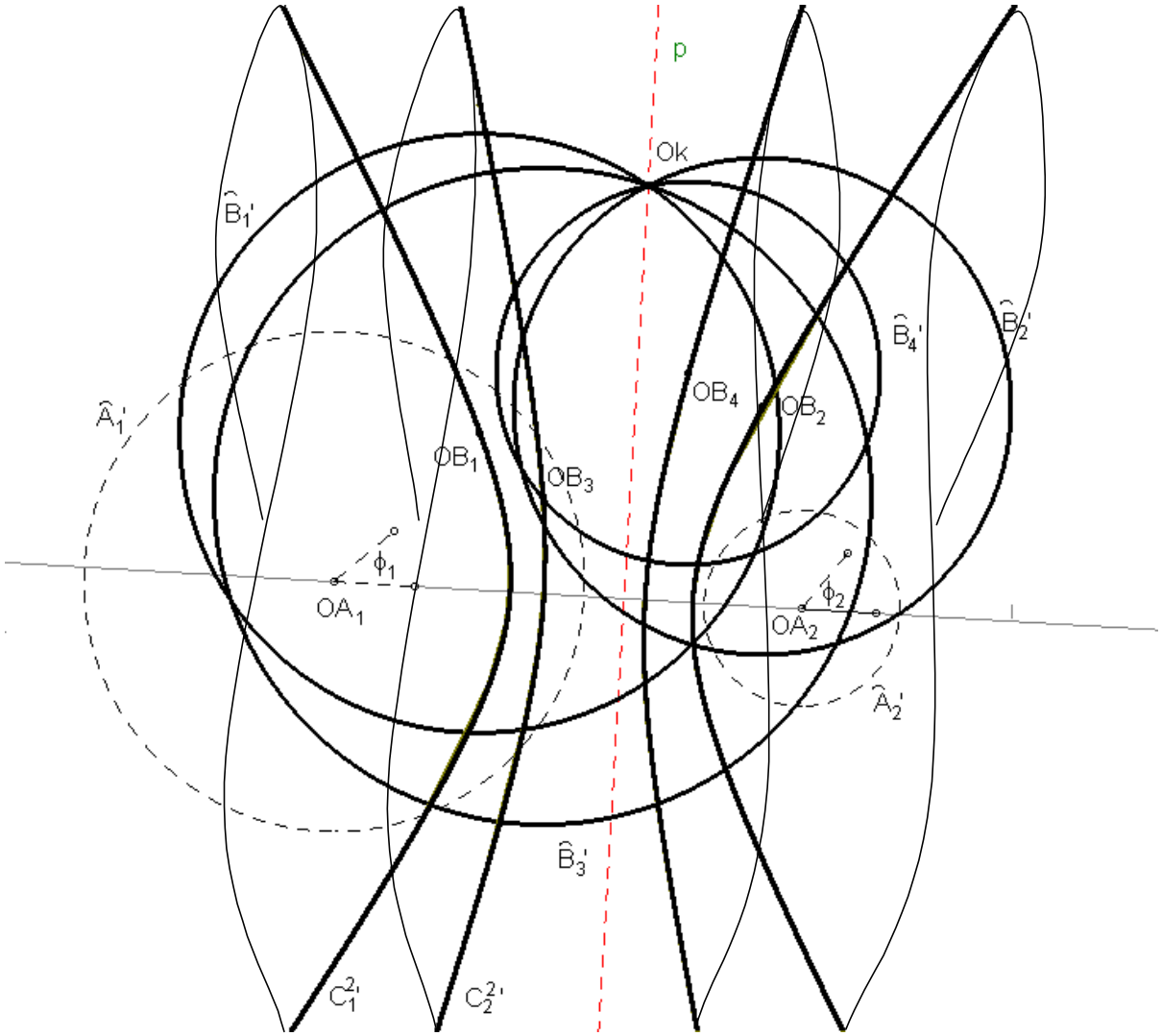


Fig.K4

**K$_5$**

Given elements**:**

**- sphere** $\widehat{A}$**, angle** $\phi$ **and point** ***P*** *(P',P*)* - ($\widehat{a}$ - an equator of sphere $\widehat{A}$, and point $P$ are not coplanar).

Determined elements:

- $\widehat{b}_1$ **and** $\widehat{b}_2$ **circles -** (including given angle $\phi$ with sphere $\widehat{A}$**,** and coinciding with point $P$),
- ***O$_{b1}$*** **and** ***O$_{b2}$*** - (centres of circles $\widehat{b}_1$ and $\widehat{b}_2$ ),
- ***C$^2$*** **- quadric of revolution with an axis** l = $O_A P$- (geometric locus of points $O_{b1}$ and $O_{b2}$),
- ***c$^2$*** **- conic section** - (an intersection of quadric $C^2$ by plane of a circle $\widehat{a}$ ) - (in this case the plane of angle $\phi$ belongs to the plane of determined circles).

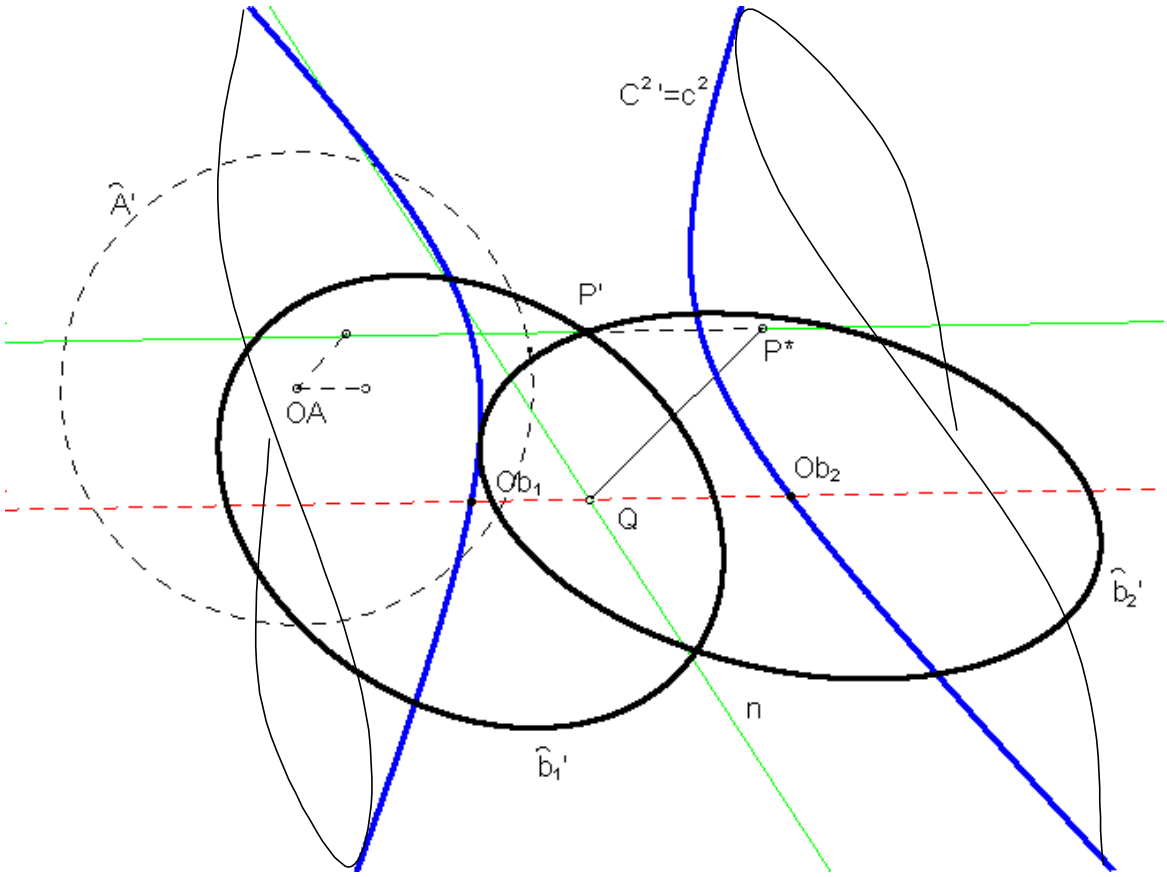


Fig.K5

**K6**

Given elements**:**

**- circle** $\hat{a}$ **, angle** $\phi$ **, point** ***P*** *(P', P*)* **and point I** - belongs to circle $\hat{a}$ - (given elements are noncoplanar).

Determined elements:

**- circles** $\hat{b}_1$ **and** $\hat{b}_2$ **- do not belong to plane of circle** $\hat{a}$ - (including given angle $\phi$ with circle $\hat{a}$ and coinciding with point *P*),

**-** $O_{b1}$ **and** $O_{b2}$ - (centres of circles $\hat{b}_1$ and $\hat{b}_2$ ),

**-** $c^4$ - curve of fourth degree (geometric locus of points $O_{b1}$ and $O_{b2}$ ).

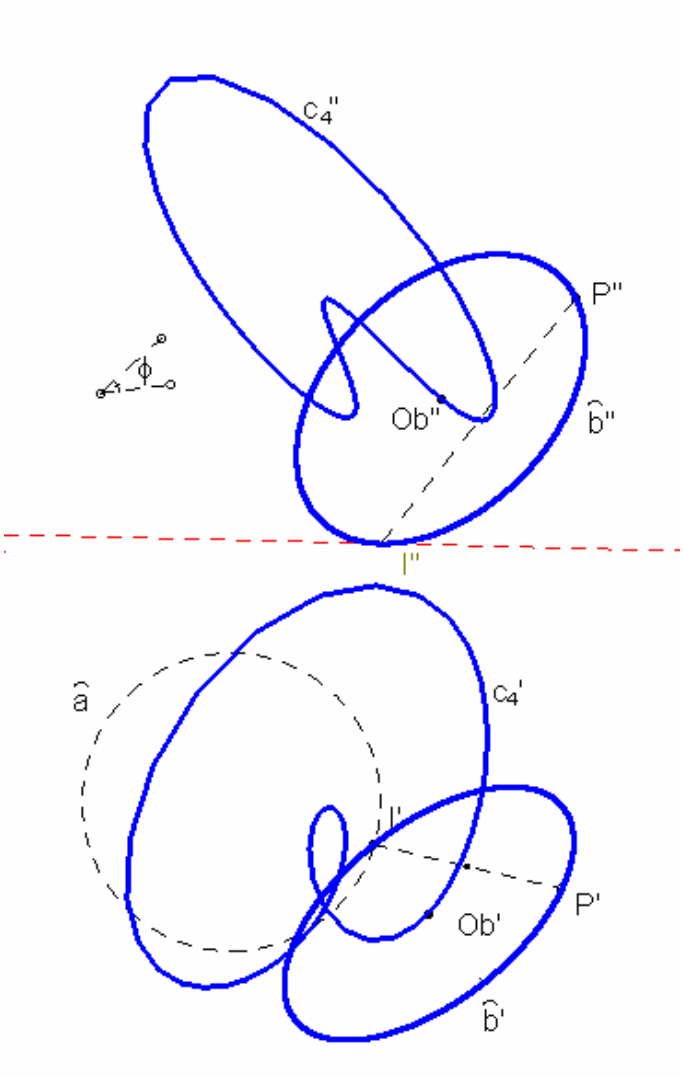


Fig.K6

# PRZYKŁADY STOSOWANIA PROGTAMU "CABRI GEOMETRE II" W ROZWIĄZYWANIU PROBLEMÓW GEOMATRYCZNYCH

Program CABRI II jest programem komputerowym opracowanym przez matematyków francuskich z Uniwersytetu w Grenoble, którego producentem jest Texas Instrument. Program ten używany jest głównie w dydaktyce, ale równocześnie jest on dobrym i przydatnym narzędziem do poszukiwań naukowych w dziedzinie geometrii.

Możliwość tworzenia własnych konstrukcji zwanych **makrokonstrukcjami** pozwala poszerzyć zbiór oryginalnych konstrukcji CABRI.

Umożliwia to późniejsze automatyczne powtarzanie, w dowolnym momencie, potrzebnych konstrukcji.

Jedna z wewnętrznych konstrukcji CABRI -"miejsce geometryczne" jest bardzo przydatna w edytowaniu skomplikowanych elementów krzywoliniowych. Również opcja ta pozwala na graficzną weryfikację przeprowadzonych analiz.

W niniejszym opracowaniu przedstawiono kilka prostych makrokonstrukcji "$M_{1,2,\ldots}$" , za pomocą których utworzono bardziej złożone makrokonstrukcje "$K_{1,2,\ldots}$" dotyczące poważniejszych zagadnień.

Translated by: mgr Barbara SKARKA

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