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TRAJECTORY PLANNING FOR TRACTOR TURNING USING THE TRIGONOMETRIC TRANSITION CURVE

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ABSTRACT

This paper presents a method of utilizing a transition curve in planning the movement trajectory of agricultural machinery during the headland turns. The approach involves using the transition curve, whose curvature and tangent angle are described by the trigonometric function. For the designed course, the kinematic quantities, wheel turning angles, and their rates of change were determined for two models of agricultural tractors. The algorithm proposed in this study ensures continuity and smooth changes in the kinematic quantities and can be applied to the trajectory planning of agricultural implements and machines, autonomous vehicles, mobile robots, manipulators, and CNC machines.

Introduction

Trajectory planning for tractors and agricultural implements, including the headland maneuvers, is a subject of research carried out by many scientists. Oksanen and Visala (2004) as well as Oksanen (2007), modelled the implement in the form of a tractor with a trailer as a dynamic system moving in the planar motion. The length of the paths generated using the Bézier curves was minimized, employing the proposed cost function that took into account the mechanical constraints of the vehicle and the geometrical limitations of the field. Bochtis and Vougioukas (2008) presented the algorithm for calculating the sequence of passes for the parallel field tracks, which improve machine efficiency by minimizing the total non-working distance. Their results show that by applying optimal sequences, the total non-working distance can be reduced by up to 50%. Cariou et al. (2010) described the method for generating trajectories and controlling movement for the autonomous agricultural implement during the headland turn. They utilized clothoids and circular arcs, as well as considered the control and speed regulation algorithms. Backman et al. (2012) proposed the 'Spiral Connection' method that minimizes passage time on the headlands by modifying the principle of the shortest path, such as the Dubins' spiral. Sabelhaus et al. (2013) presented the method for shaping a path using the clothoid, allowing for the connection of paths with different curvatures. The authors stated that forward and backward motions can be planned using the Dubins' curves as well as the modified Reed and Shepp's curves. Koc (2014) presented the analytical modelling of the curvature of the transport road based on differential equations. He analyzed transition curves with curvature described by the linear function, polynomials as well as the trigonometric function. Backman et al. (2015) presented the algorithm for generating a smooth path during turns, taking into account the maximum turning speed and the vehicle acceleration. Sabelhaus et al. (2015) discussed planning a turning maneuver using a path with the continuous curvature, taking into account the driving speed, the vehicle minimum turning radius and the permissible wheel turning speed. Cariou et al. (2016) used the clothoid for the trajectory planning of a mobile robot, taking into account the additional dynamic constraints. Boryga et al. (2017) presented the method of using the clothoid as the transition curve during the trajectory planning of the movement of a tomato picking manipulator in the greenhouse. They proposed three variants of the gripper movement path, determined and compared the courses of kinematic quantities. Wang and Noguchi (2018) proposed the use of the adaptive algorithm for controlling the implement movement during a turn with the real-time optimization, depending on the local conditions such as soil properties or vehicle speed. Bulgakov et al. (2019) conducted an analysis of the maneuverability of an asymmetric tool-tractor unit during the turn. Considering the design parameters, they obtained analytical relations allowing the determination of the minimum turning radius, the angle between the tractor longitudinal axis and the tool hitch beam, as well as the width necessary to perform the turn. They used these relationships in the numerical simulation for calculation of the characteristics related to the unit maneuverability. Tu and Tang (2019) developed the kinematic models of the tractor and the tractor-tool attached system as well as formulated the optimization problem for turning taking into account the operating constraints. For the proposed models, they developed a range of scenarios for different paths. The optimization problem was solved using the TOMLAB/SNOPT software. Boryga et al. (2019, 2020) presented the method of using the polynomial transition curves in planning the trajectory of agricultural machines during a turn. They planned the use of transition curves whose curvature is described by the linear function and the third, fifth, seventh, and ninth degree polynomials. They determined and compared the courses of motion parameters for the analyzed trajectory variants.

Purpose and scope of work

In this study, the method of using the transition curve for planning the trajectory of agricultural machines during the headland turns is presented. The use of the transition curve, whose curvature and the angle of the tangent to the curve are described by a trigonometric function, has been planned. For the planned driving path, the kinematic quantities, wheel steering angles, and rates of change have been determined for two models of agricultural tractors.

Objective, scope, and method of work

The curvature of the planar curve is defined as:

$$\kappa(l) = \lim_{\Delta l \to 0} \frac{\Delta \theta}{\Delta l} = \frac{d\theta}{dl} \tag{1}$$

where:

$\Delta\theta$ – the angle between the tangents to the curve on the arc ends, (rad)

Δl – the arc length, (m)

The $\theta(l)$ function is defined as

$$\theta(l)=\int \kappa(l)dl \tag{2}$$

whereas the coordinates of any point of the transition curve are calculated from:

$$x(l)=\int \cos\theta(l)dl \tag{3}$$

$$y(l)=\int \sin\theta(l)dl \tag{4}$$

Fig. 1 presents the curve with the marked geometrical elements.

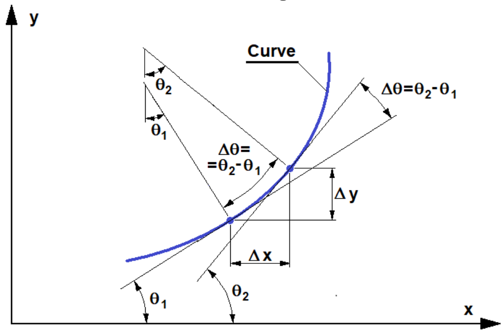

Figure 1. The geometrical elements of the planar curve

The paper will analyze the 'half of chi' trajectory presented in Fig. 2, which consists of two transition curves *BT* and *TE*, forming the so-called transition bi-curve (Boryga et al., 2020). The following designations are used in the figure: *B* – the starting point, *T* – the point of tangency, which is half the total length of the path during the turn, *E* – the end point, θ – the angle of the tangent to the curve at any point of the trajectory, θ_T – the angle of the tangent to the curve at point *T*.

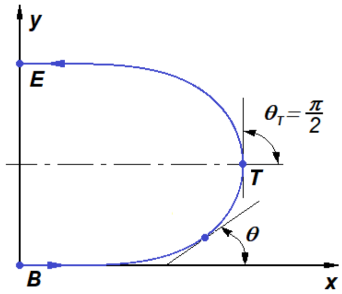

Figure 2. The analyzed 'half of chi' motion path

The total length of the transition bi-curve is $2L$. The curvature at the initial point *B* is equal to $\kappa_B=0$, and the angle of tangent to the curve $\theta_B=0$. At the tangency point *T*, the curvature is $\kappa_T=\kappa$ ($\theta_T=\pi/2$), whereas at the terminal point *E*, similar to the initial point, it is $\kappa_E=0$ ($\theta_E=\pi$). Taking into account the above conditions as well as the formulae for the curvature and the angle of tangent in the trigonometric form (Koc, 2014), the curvature $\kappa(l)$ and the angle of tangent $\theta(l)$ for the *BT* curve are given by:

$$\kappa(l) = \frac{\kappa}{2}\left(1 - cos\frac{\pi l}{L}\right) \tag{5}$$

$$\theta(l) = \frac{\kappa}{2}\left(l - \frac{L}{\pi}sin\frac{\pi l}{L}\right) \tag{6}$$

while for the *TE* curve

$$\kappa(l) = \frac{\kappa}{2}\left(1 + cos\frac{\pi l}{L}\right) \tag{7}$$

$$\theta(l) = \frac{\pi}{2} + \frac{\kappa}{2}\left(l + \frac{L}{\pi}sin\frac{\pi l}{L}\right) \tag{8}$$

where:

κ – the curvature at the midpoint of the transition bi-curve, (m^{-1})
l – the displacement along the transition bi-curve, (m)
L – the length of the single transition curve, (m)

For the given working width W_{set} and the minimum turning radius R_{min} of the tractor, trajectory planning could be presented in the following steps:

– Calculate the minimum length of the single transition curve

$$L = 2\theta_T R_{min} \tag{9}$$

where for the 'half of chi' path $\theta_T = \pi/2$.

– Calculate the minimum distance between the points *B* and *E*

$$W_{min} = 2\int_0^L sin\,\theta(l)dl \tag{10}$$

– Depending on the values of W_{min} and W_{set}, there should be distinguished three cases:

Case 1:

If $W_{set} < W_{min}$ - the 'half of chi' path cannot be realized.

Case 2:

If $W_{set} = W_{min}$ - at point *T*, the minimum turning radius of the tractor R_{min} should be applied.

Case 3:

If $W_{set} > W_{min}$ - at point *T*, the turning radius $R > R_{min}$ should be applied.

– When the realization of the 'half of chi' path is possible (case 2 or 3), for the assumed constant velocity along the transition curve *v*, and by introducing the substitutions $l = v\tau$ and $dl = vd\tau$ into equations (3) and (4), one should determine the displacements of the transition bi-curve *x(t)* and *y(t)*, and then calculate velocities, accelerations, and jerk as the first, second, and third derivatives of displacement with respect to time.

Figure 3 presents the exemplary pseudocode of the program calculating the required turn radius *R* if case 3 occurs. The algorithm works iteratively, increasing the radius by the step ΔR, then calculates the length of the curve *L* and the value of *W*. If the value of *W* exceeds the given width W_{set}, it reverts back to the previous radius, returns its calculated value and ends the operation.

```
function Radius_R(Rmin, θ, ΔR, W)
        R←Rmin
                repeat
                        R←R +ΔR
                        L←2* θ*R
                        W←2*integrate[sin(θ(l))] from l = 0 to l = L
                until (W>=Wset)
        R=R-ΔR
        return R
endfunction
```

Figure 3. The exemplary pseudocode for calculating the required turn radius R with the step of ΔR

In Fig. 4 two models of a wheeled tractor moving along the curvilinear track are presented. In the case of the model in Fig. 4a (model 1), it is assumed that only the front axle can turn while the slip angle for all the tractor wheels is neglected. In the case of the model presented in Fig. 4b (model 2), the possibility of changing the steering angle individually for each tractor wheel was assumed. There were introduced the following notations: O – the instantaneous center of the tractor rotation, O_T – the center of the mass of the tractor or another reference point, R – the tractor turn radius, δ_F – the steering angle of the front axle, δ_{FL} – the steering angle of the front-left wheel, δ_{FR} – the steering angle of the front-right wheel, δ_{RL} – the steering angle of the rear-left wheel, δ_{RR} – the steering angle of the rear-right wheel, B_{TF} – the front wheel track, B_{TR} – the rear wheel track, L_{TF} – the position of the front wheel axle, and L_{TR} – the position of the rear wheel axle.

a) *b)*

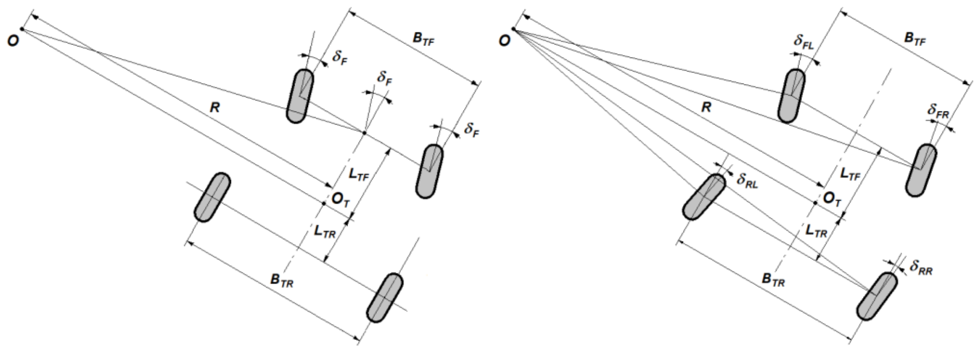

Figure 4. The model of a wheeled tractor moving along the curvilinear track: a) with the possibility of turning the front axle wheels (model 1), b) with the possibility of turning all wheels (model 2)

In the case of model 1, the steering angle of the front axle wheels can be calculated from the relationship:

$$\delta_F = arctan(\frac{L_{TF}}{R}) \tag{11}$$

however, for model 2, the steering angles of individual wheels are as follows:

$$\delta_{FL} = arctan(\frac{L_{TF}}{R - \frac{B_{TF}}{2}}) \tag{12}$$

$$\delta_{FR} = arctan(\frac{L_{TF}}{R + \frac{B_{TF}}{2}}) \tag{13}$$

$$\delta_{RL} = arctan(\frac{L_{TR}}{R - \frac{B_{TR}}{2}}) \tag{14}$$

$$\delta_{RR} = arctan(\frac{L_{TR}}{R + \frac{B_{TR}}{2}}) \tag{15}$$

Figure 5 presents the scheme of the tractor turn with a tool hanging on the headland in the form of a belt of equal width D lying between the working area and the 'prohibited area'. The tractor is to move from the initial position B to the final E, which are on the boundary line between the working area and the headland. During the turning process itself, the trajectory is confined to the headland area, which means that the tractor with the tool cannot enter the prohibited area. It was assumed that the working width is W.

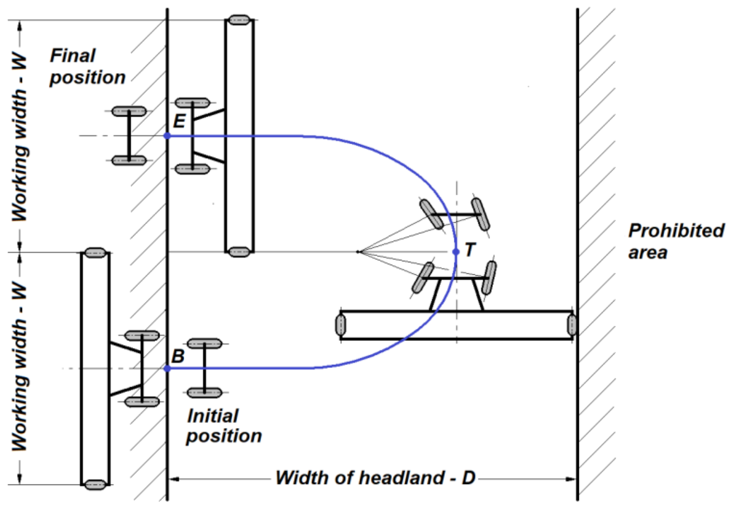

Figure 5. The scheme of the tractor turning on the headland

Research results

Firstly, using the algorithm the pseudocode of which is shown in Fig. 3 for the assumed minimum turn radius R_{min}=3 m and the radius increment ΔR=0.05 m, the turn radius to be applied at point *T* was determined. It is *R*=3.25 m. The simulation was conducted using the Matlab program for the following data: $v=2\pi/3$ m/s, L_{TF}=0.65 m, L_{TR}=0.8 m, $B_{TF}=B_{TR}$=1.65 m. The above data was used for both tractor model 1 and model 2.

In Figure 6 the courses of the kinematic quantities for the planned trajectory of the tractor motion are presented. Fig. 6a shows the displacement along the *x* and *y* axes. The maximum displacement along the *x*-axis is 8.31 m while that along the *y*-axis - 8.06 m. The course of velocity along the *x*, *y* axes and the resultant velocity is shown in Fig. 6b. The course of the acceleration along the *x*, *y* axes and the resultant acceleration is presented in Fig. 6c. Due to the constant velocity, the observed acceleration is normal acceleration and results from the motion along the curvilinear track. The maximum value of the resultant acceleration appears in the middle of the total motion time (point *T*) and is 1.349 m/s^2. Owing to the use of the transition curve, the acceleration course is smooth without the abrupt jump (from zero to the maximum value), which would occur if the track consisted of a straight line and a circle arc.

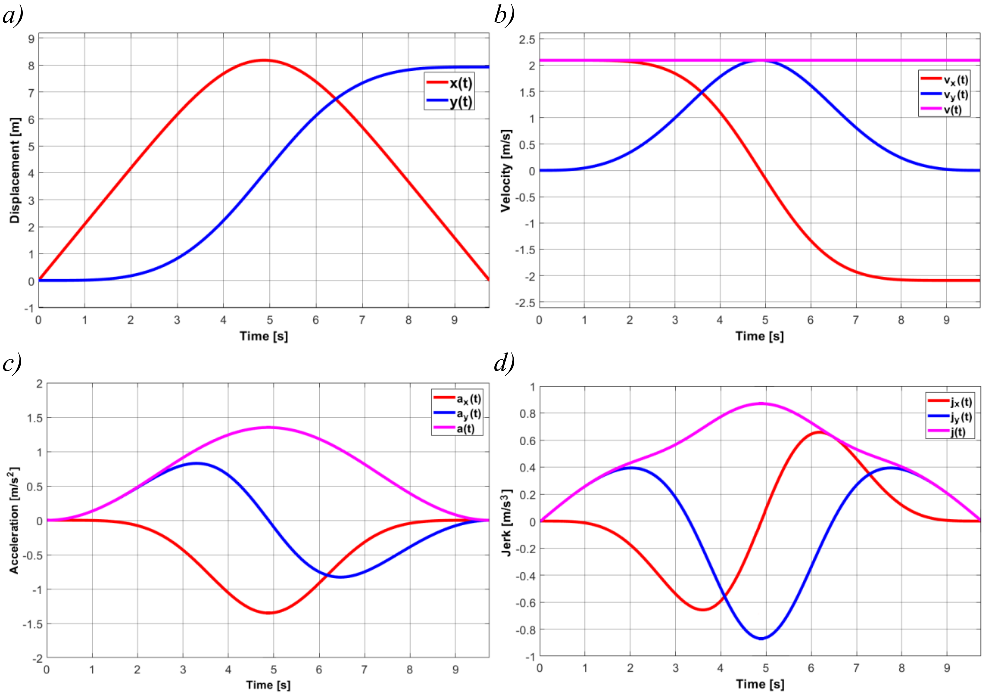

Figure 6. The courses of kinematic quantities: a) displacement, b) velocity, c) acceleration, d) jerk

Owing to the smooth changes in acceleration, the jerk courses (derivative of acceleration with respect to time) are also smooth as shown in Fig. 6d. In the case where the track would consist of a straight line segment and a circle arc, the jerk would reach infinite values during the abrupt increase in acceleration.

Figure 7 presents the courses of both changes in the steering angle of the front axle wheels for model 1 of the tractor (Fig. 7a) and the steering speed of these wheels (Fig. 7b). For the assumed trajectory, the maximum value of the steering angle of the front axle wheels is δ_F=0.2 rad (11.46°) while the maximum steering speed is ω_F=0.064 rad/s.

a) *b)*

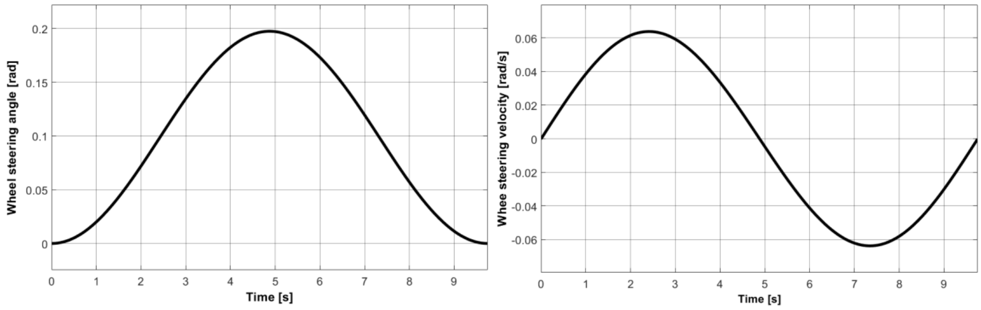

Figure 7. The courses: a) steering angle of the front axle wheels for tractor model 1, b) steering speed of the front axle wheels for tractor model 1

Fig. 8 shows the course of changes in the steering angles of all the wheels of tractor model 2 - δ_{FL}, δ_{FR}, δ_{RL}, δ_{RR} (Fig. 7a) and the steering speed of these wheels - ω_{FL}, ω_{FR}, ω_{RL}, ω_{RR} (Fig. 7b). For the assumed trajectory, the maximum absolute value appears for the rear left wheel being δ_{RL}=0.32 rad (18.26°) while the maximum steering speed of this wheel is ω_{RL}=0.105 rad/s.

Comparing the maximum values of the wheels steering angles, it should be stated that a larger value occurs for tractor model 2. However, it should be kept in mind that the steering angles are dependent on a number of factors, such as the wheelbase, their position, wheel track as well as the geometry of the path along which the tractor moves. Therefore, these results should be treated as exemplary.

a) *b)*

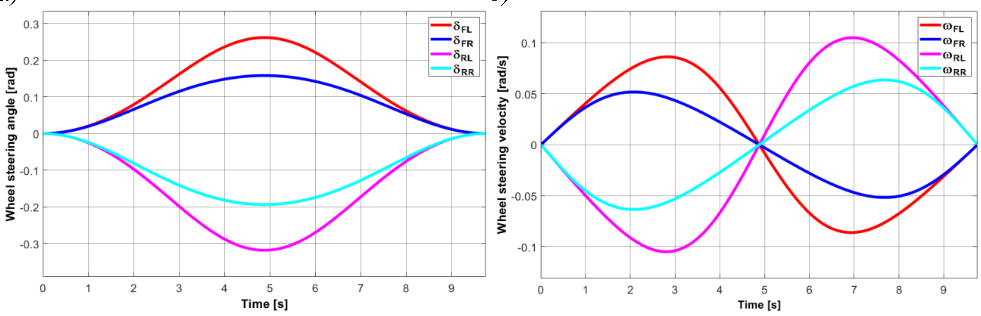

Figure 8. The courses: a) steering angles of the tractor wheels for model 2, b) steering velocity of the tractor wheels for model 2

Conclusions

At the time of transition from the straight-line motion to the circular arc motion, a sudden increase in the normal acceleration is observed. This phenomenon can lead to the generation of additional dynamic loads which as a result can lead to the loss of motion stability. To avoid this problem, it is proposed to use the transition curve, whose curvature and the angle of the tangent to the curve are described using the trigonometric function. The simulation studies proved that the displacement, speed, acceleration, and jerk courses are continuous and change gently over the range of motion. Similar characteristics also describe the changes in the steering angles and their speeds.

The presented approach can be used to:

– identify the optimal speed during turnarounds,
– develop safety strategies, including recommendations on speed and maneuvering,
– create effective control algorithms for the autonomous tractors and those equipped with the automatic driving system.

In addition to the above applications the algorithm proposed in this paper can be also used to plan the trajectory of mobile robots, manipulators and CNC machines.

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PLANOWANIE TRAJEKTORII NAWROTU CIĄGNIKA Z WYKORZYSTANIEM TRYGONOMETRYCZNEJ KRZYWEJ PRZEJŚCIOWEJ

Streszczenie. W pracy przedstawiono sposób wykorzystania krzywej przejściowej w planowaniu trajektorii ruchu maszyn rolniczych podczas jazdy na uwrociu. Zaplanowano zastosowanie krzywej przejściowej, której krzywizna i kąt stycznej do krzywej opisane są funkcją trygonometryczną. Dla zaplanowanego toru jazdy wyznaczono przebiegi wielkości kinematycznych oraz kąty skręcenia kół i prędkości ich zmian dla dwóch modeli ciągnika rolniczego. Zaproponowany w pracy algorytm, zapewnia ciągłość oraz łagodne zmiany wielkości kinematycznych i może być stosowany do planowania trajektorii ruchu agregatów i maszyn rolniczych, pojazdów autonomicznych, robotów mobilnych, manipulatorów i maszyn CNC.

Słowa kluczowe: planowanie trajektorii, manewry na uwrociach, trygonometryczna krzywa przejściowa, gładka trajektoria