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Mathematical modelling of the process of pipeline deformation through which gas-liquid mixtures with aggressive components are transported

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ABSTRACT

Purpose: Mathematical modelling of the process of deformation of pipeline, transporting gas-liquid mixtures with aggressive components and a comparative analysis of the value of the specified velocity depending on the dynamic viscosity of the multicomponent gas mixture is conducted.

Design/methodology/approach: A mathematical model of the process of leakage of the transported product due to the loss of tightness of the pipe based on the system of Navier-Stokes equations with boundary conditions with considering the geometry of the leakage zones and the value of the leakage rate is implemented.

Findings: Models of the process of deformation of the pipeline due to displacements of a certain set of surface points by specifying different types of functions, describing the geometry of deformed sections are constructed. The method of calculating the tensely deformed state based on the data on the movement of surface points by comparing different ways of setting functions, taking into account the actual configuration of sections and axes is improved. The change of flow characteristics in the pipeline when changing the structure of the mix, transported by studying of influence of change of dynamic viscosity is investigated; The method of calculating the rate of leakage of the mixture in case of loss of tightness due to the occurrence of critical stresses in the pipe material is improved.

Research limitations/implications: Building a model of the deformation process, information about the nature, duration of forces and loads affecting the pipeline is not used. The law of the pipeline movement was constructed having taken into account the deformation of the sections in three directions. The necessity to take wind loads into account, estimating the real tensely deformed state was displayed.

Practical implications: Using the method of calculating the tensely deformed state based on the data on the movement of surface points by comparing different ways of setting functions, taking into account the actual configuration of sections and axes.

Originality/value: According to the computational algorithms created on the basis of the specified models, the calculations of the tense state of the pipelines and the flow rate of the mixture depending on its composition were performed. An analysis of the results of calculations - tense intensity and flow rate depending on the dynamic viscosity of the mixture is performed. The influence on the flow parameters - the flow rate of the mixture and the force of hydraulic resistance - changes in the dynamic viscosity of the mixture is analyzed.

Keywords: Tense state, Pipeline, Viscous fluid flow, Dynamic viscosity, Navier-Stokes equation, Gas mixture

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ANALYSIS AND MODELLING

1. Introduction

The study of the technical condition of pipelines through which gas-liquid mixtures with aggressive components are transported is an important scientific and technical problem, the solution of which allows the identification of potentially dangerous areas. In particular, the study of the tensely deformed state allows to identify those areas where it is possible to lose the integrity of the pipeline material, and, as a consequence, emissions of chemically aggressive substances transported. Harmful impurities in the inner cavity of the main gas pipelines are divided into two groups: - liquid low-viscosity accumulations - gas condensate and water; - high-viscosity solidifying resinous deposits [1]. So it is necessary to consider the full range of factors of force affecting the pipeline.

Such problem has been studied by different authors [2-7], but in these works one or another model of the process of deformation and tense is taken without taking into account the characteristics of the transported product.

The data obtained in the work [8] on the patterns of corrosion processes and the impact of hydrate formation on them will allow to identify potentially dangerous areas of flow lines and prevent emergencies.

Dependences for determining stresses and displacements under arbitrary load law and type of the support are described [9].

The considered research [10] finds localization of erosion defects, allows to estimate durability and to find residual resource of tees with erosion wear of a wall for maintenance of their reliability, to divide these defects on degree of danger, to define which of them. are critical and in need of immediate repair.

In [10], the dynamic reaction of the pipeline to external influences, such as the impact of the impact force on the pipe wall and the soil near the pipe and gas pressure jumps caused by the operation of the valve shut-off system are considered.

When studying the technical condition of pipelines through which gas-liquid mixtures with aggressive components are transported, it is also important to take into account the real spatial configuration of real pipeline systems both in the above-ground [4] and in the underground position [5], condition, and to study certain types of force effects in more detail (for example, wind loads).

2. Mathematical models of processes

Determination of the parameters of the tensely deformed state of the studied objects on the basis of known information about the change of their spatial configuration - as a rule, such information is data on the displacement of surface points of the studied body, on the basis of which it is possible to construct a mathematical representation [4] of the radius-vector of any point of this body at control moments of time.For objects of cylindrical shape (pipelines for various purposes, wells with gas-liquid flows, etc.), the specified representation can be written in the form:

$$\begin{aligned}&\vec{r}(s,\phi,r,t)=\vec{r}_l(S,\phi,r,t)+\\&+\rho(S,\phi,r,t)(\cos\omega\,(S,\phi,r,t)\vec{b}_l+\\&+\sin\omega\,(S,\phi,r,t)\vec{n}_l)+\psi(S,\phi,r,t)\vec{\tau}_l-\tfrac{D}{2}\vec{n}_l\end{aligned} \quad (1)$$

where s,ϕ,r,t – the coordinates associated with the curvilinear cylindrical body, respectively, along the axis of the body $0<s<L$, the polar angle $0\le\phi\le 2\pi$ the radius of the object $R_{vnut}<r<R_{zovn}$, L – the length of the object under study; $\vec{r_l}$ – radius vector of a point on the generating object; D – is its diameter; $\rho(s,\phi,r,t)$;$\omega(s,\phi,r,t)$; $\psi(s,\phi,r,t)$ – functions that characterize the movement of points of the studied body, respectively, in the radial, polar and longitudinal directions, $\vec{\tau_l},\vec{b_l},\vec{n_l}$ – vectors of tangent, binormal and normal to the generating object. Setting the function $\rho(s,\phi,r,t)$; $\omega(s,\phi,r,t)$ and $\psi(s,\phi,r,t)$ on the

basis of data on the movement of surface points allows at the control time to perform calculations of the components of deformation tensors ε_{ij} [5]:

$$\varepsilon_{ij}(s,\phi,r,t_k)=\frac{1}{2}(g_{ij}(s,\phi,r,t_k)-g_{ij}(s,\phi,r,t_0)), \quad (2)$$

where g_{ij} – the components of the matrix tensor, built on (1) [12], as well as the components of the tensetensor – in the case where the deformations are considered elastic, there calculation of these components is carried out according to Hooke'slaw:

$$\sigma_{ij}=\lambda I_1(\varepsilon)g_{ij}(s,\phi,r,t_0)+2\mu\varepsilon_{ij}(s,\phi,r,t)\,, \quad (3)$$

- for anisotropic body (from use (2)):
  where $I_1(\varepsilon)=\sum_{i=1}^{3}\sum_{j=1}^{3}\varepsilon_{ij}(s,\phi,r,t)g_{ij}(s,\phi,r,t_0)$ – the first invariant of the tense tensor; λ,μ – Lamé parameters of the material [13], which are related to the Young's modulus E and the Poisson's ratio σ of the material in proportion:

$$\mu=\frac{E}{2(\sigma+1)};\lambda=\frac{E\sigma}{(1+\sigma)(1-2\sigma)}; \quad (4)$$

- for the anisotropic model, the use of proportion (3) and (4) is replaced by the formula

$$\sigma_{ij}(s,\phi,r,t)=\sum_{k,l=1}^{3}C_{ijkl}\varepsilon_{kl}(s,\phi,r,t), \quad (5)$$

where C_{ijkl} – are the components of the tensors of elastic modules of the material.

This technique is substantiated in detail in [4], its feature is that the conclusion about the tensely deformed state of the object is made on the basis of certain integral indicators - moving points on the surface of the body without detailing the causes of these displacements. Based on the data on the change of the tensely deformed state of the object it is possible to find out its potentially dangerous areas where the tenses take a critical level, or the change in stenses is such that can lead to depressurization of the object, as well as potentially dangerous environmental influences. Studies conducted in recent years [5, 14-16] allow to summarize the results [4] in the case of underground pipelines, which allows to predict dangerous from the point of view of depressurization of the underground pipelines zone based on the information about the spatial configuration of the main pipeline axis. Studying the tensely deformed state of open, and ,in particular, mountain areas, the effect of aerodynamic loads is taken into account, and they are determined by known methods [17,18]. It is established that the introduction of additional loads leads to a significant change in the tensely deformed state of the investigated section of the pipeline [19,20], which is considered to be especially important when transporting gas-liquid mixtures with aggressive components (Fig. 1).

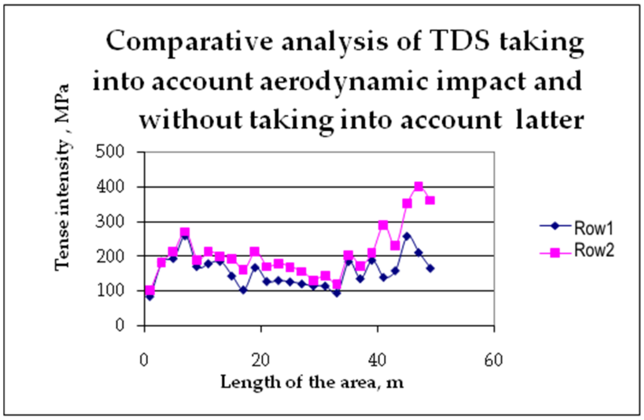

Fig. 1. Comparative analysis of TDS taking into account aerodynamic impact (row 2) and without taking into account latter (row 1)

Aerodynamic calculation of loads was carried out according to the methods [18,21], which are based on the use of Fredholm equations of the second kind. The diameters of the pipelines are 520-1420 mm with standard wall thicknesses.

For the model section of the pipeline, the tense state based on the information about the movement of a certain set of surface points was calculated (Fig. 2), which allowed to identify potentially dangerous zone , taking into account possible loss of continuity of the pipeline material.

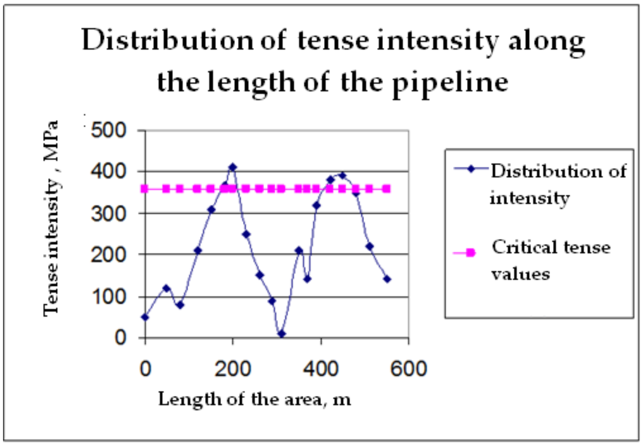

Fig. 2. Distribution of tense intensity along the length of the underground section of the pipeline

It is possible to establish the spatial configuration of these zones, but the analysis is performed for tense intensity [13]:

$$\sigma_i = \sqrt{\sum_{i,j=1}^{3} \sigma_{ij}^2}. \tag{6}$$

The calculation of the parameters of the tensely deformed state is carried out according to the linear algorithm (1)-(6) according to the known values of displacements of a certain set of surface points.

3. Estimation of intensity of leakage of substance at infringement of tightness of object

The problem of estimating the flow parameters in pipelines and downhole flows is reduced to the need to solve the system of Navier-Stokes equations [13,14] in a two-dimensional formulation [22,23]:

$$\begin{cases} u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) + y \\ u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + v(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}) + y, \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \end{cases} \tag{7}$$

with the condition: $p=p0-kx$; k – is the coefficient of pressure drop;

$$u|_{x=0} = -\frac{ky^2}{4\mu} + \frac{kRy}{2\mu},$$
$$u|_{y=0} = u|_{y=2R} = 0,$$
$$v|_{x=0} = v|_{y=0} = 0,$$
$$v|_{y=2R} = \begin{cases} 0, x \leq x_1; x \geq x_2 \\ \pm v_{vutr}, x \in [x_1 x_2] \end{cases}. \tag{8}$$

A numerical way for solving the system (7) has been developed; (8), its convergence and stability are proved, the calculation algorithm is constructed and implemented, the optimal parameters of the calculation grid are revealed. It allows to model the velocity fields at a given value v_{vutr}, which is not always possible to determine in practice. Therefore, to solve the system (7), the method [13,24] is used, which allows to reduce the system (7) to the Poisson equation:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 2\rho(\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial x}). \tag{9}$$

There are methods for solving system (6), however, using the results [19], we can make conclusion that,

$$\left|2\rho(\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial x})\right| << 1, \tag{10}$$

and, taking into account (10), we can propose the following solution algorithm:

1. In the first step, equation (8) is solved with boundary conditions (Fig. 3).
2. The system (7) is solved with conditions (8), andv_{vutr} is determined by the found *P(x,y,t)* by Darcy's law:

$$\begin{cases} u = -\frac{k}{\mu}\frac{\partial p}{\partial x} \\ v = -\frac{k}{\mu}\frac{\partial P}{\partial y} + \frac{kg\rho'}{\mu} \end{cases} \tag{11}$$

k – is the permeability of the medium; μ – dynamic viscosity of the liquid.

3. According to the found distribution of velocities, equation (9) is solved with the newly listed right-hand side. The procedure is repeated until the solution coincides. The peculiarity of the obtained solution is that the boundary conditions (8) are changed at each step of the iterative procedure, thus, in case of convergence of the iterative process, it is possible to determine the flow rate of liquids from the object being studied.

The study of the technical condition of pipelines through which gas-liquid mixtures with aggressive components are transported involves the study of the properties of these mixtures. When studying the flow in a pipe of cylindrical

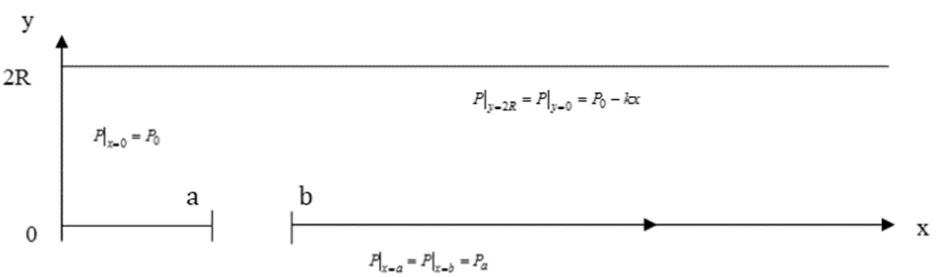

Fig. 3. Boundary conditions for the Laplace equation (Dirichlet problem); It uses the method of surface relaxation [19]

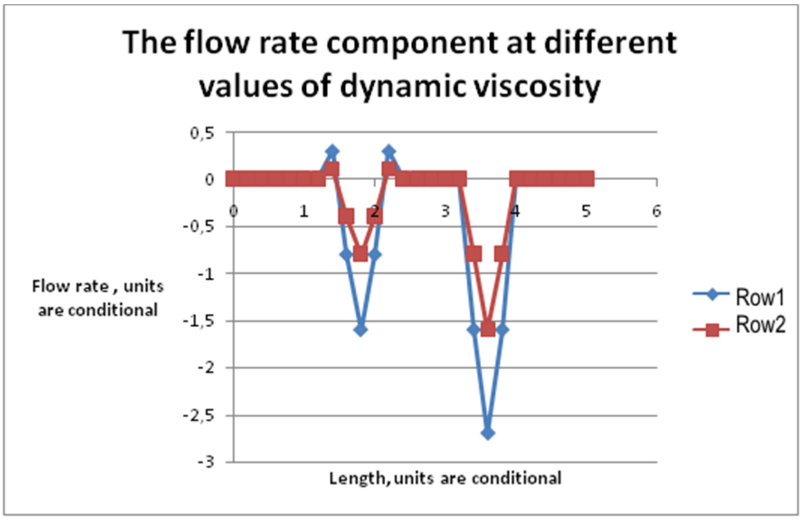

Fig. 4. Distribution of the flow rate component at different values of dynamic viscosity (row 1 – μ_1, row 2 – μ_2, $\mu_1 < \mu_2$)

cross section of a viscous liquid (gas) (pipe radius a, viscosity μ, pressure drop i) the known results [24,25] are the Poiseuille flow, according to which the velocity is determined by the formula:

$$\omega = \frac{i}{4\mu}(a^2 - r^2). \tag{12}$$

The velocity profile (12) in the cross section of the round tube is a paraboloid of rotation, the maximum speed:

$$\omega \frac{ia^2}{4\mu}_{max} \tag{13}$$

Volume consumption is:

$$Q = \int_0^a \omega \cdot 2\pi r dr = \frac{i\pi a^4}{8\mu}. \tag{14}$$

When transporting gas mixtures with aggressive components, it is necessary to determine the real value of dynamic viscosity:

$$\mu = \sum_{i=1}^{n} \sigma_i \mu_i, \tag{15}$$

where σ_i – mass fraction i – components, μ_i – its dynamic viscosity. The coefficient of friction C_f – is the ratio of the force of resistance

$$R = 2\pi a l \cdot \frac{ia}{2} = \pi a^2 l i, \tag{16}$$

to the speed of pressure and to some characteristic area S:

$$C_f = \frac{R}{\frac{\rho \omega_{ser}^2}{2} S} = \frac{16}{Re} \tag{17}$$

where Re – is the Reynolds number: $Re = \frac{d\omega_{ser}}{\frac{\mu}{\rho}}$, where d – is the diameter of the pipe; $d = 2a$; μ and ρ – respectively, the dynamic viscosity and density of the mixture. For a two-component mixture:

$$\mu = (1 - c_1)\mu_1 + c_1\mu_2, \tag{18}$$

where μ_1– is the viscosity of the main component of the mixture, μ_2– is the viscosity of the additional component, $0 \le c_1 \le 1$. The change in speed will be:

$$\omega_1 - \omega_2 = \frac{i}{4\mu_1}(a^2 - r^2) - \frac{i}{c_1\mu_2 + (1-c_1)\mu_1}(a^2 - r^2) = \\ = \frac{i(a^2 - r^2)}{4}\left[\frac{1}{\mu_1} - \frac{1}{c_1\mu_2 + (1-c_1)\mu_1}\right], \tag{19}$$

cost change

$$Q_1 - Q_2 = \frac{i\pi a^4}{8}\left[\frac{1}{\mu_1} - \frac{1}{c_1\mu_2 + (1-c_1)\mu_1}\right]. \tag{20}$$

Knowing these values in absolute units, you can assess how to change the values of pressure drops to ensure a constant, constant level of flow Q. It is obvious that when implementing the model that provides for the solution of (7), (8), (11) it is necessary to take into account the composition of these mixtures by using the dependence (18). The developed model allows to study the technical condition of pipelines through which gas-liquid mixtures with aggressive components are transported, and, in particular, a wide class of two-component mixtures – for example, hydrogen-methane mixtures. Figure 4 shows a graph of the distribution of the component of the flow rate of the mixture from the pipeline lossing tightness at different values of dynamic viscosity (row 1 μ_1 – row 2 – μ_2, $\mu_1 < \mu_2$).

4. Conclusions

According to the study, the following conclusions can be made:

1. The developed model that allows to study the technical condition of pipelines through which gas-liquid mixtures with aggressive components are transported, and, in particular, a wide class of two-component mixtures;
2. A comparative analysis of the parameters of the tensely deformed state, taking into account wind loads and without taking into account latter is done;
3. The method of calculating the rate of leakage of the mixture in case of loss of tightness due to the occurrence of critical stresses in the pipe material is improved;
4. The change of flow characteristics in the pipeline when changing the structure of the mix, transported by studying of influence of change of dynamic viscosity is investigated;
5. The tense state based on the information about the movement of a certain set of surface points was calculated, which allowed to identify potentially dangerous zone, taking into account possible loss of continuity of the pipeline material.

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