Scientific Journals
of the Maritime University of Szczecin

Zeszyty Naukowe
Akademii Morskiej w Szczecinie

2016, 45 (117), 150–156
ISSN 1733-8670 (Printed)
ISSN 2392-0378 (Online)
DOI: 10.17402/099

Received: *31.08.2015*
Accepted: *15.02.2016*
Published: *25.03.2016*

Presentation of Satellite Based Augmentation System integrity data in an Electronic Chart System display

Paweł Zalewski

Maritime University of Szczecin, Faculty of Navigation, Centre of Marine Traffic Engineering
1–2 Wały Chrobrego St., 70-500 Szczecin, Poland, e-mail: p.zalewski@am.szczecin.pl

Key words: ECS, GNSS, SBAS, Integrity Data, MVPA, HPL

Abstract

We propose a Marine Vessel Protection Area (MVPA) model as the means of satellite-based augmentation system (SBAS) integrity data presentation in an electronic chart system (ECS) display. The model takes into account several factors that influence the shape and dimensions of the MVPA. These factors include GNSS signal aspects, where measurement errors depend on the geometry of visible satellites and the signal propagation, the ship's size, the ship's heading and its estimated accuracy, position of the GNSS/SBAS antenna relative to the ship's hull, and SBAS integrity data. The resultant safety contour or domain can be displayed within the ECS or ECDIS as a graphical representation of the marine ship position and the protection level of the electronic position/course fixing equipment, equivalent to the horizontal protection level (HPL) used in aviation.

Introduction

Satellite-based augmentation systems (SBAS), such as the European Geostationary Navigation Overlay Service (EGNOS) system, provide ranging signals transmitted by GEO satellites. Wide area differential corrections and additional parameters aim to guarantee the integrity of the GNSS. Integrity monitoring according to IMO (IMO, 2001) is the process of determining whether the system performance, or individual observations, can be used for navigational purposes. Overall GNSS system integrity is described by three parameters: the threshold value or alert limit (AL), the time to alarm (TAL) and the integrity risk (IR). The output of integrity monitoring is a decision on whether individual error observations, or the overall GNSS system, can or cannot be used for navigation. In other words, the output is an alert for the user to inform them if they experience a position error larger than the fixed AL value. To enable the user to estimate the position error, or its equivalent protection level (PL), the SBAS integrity data consist of estimations of each satellite ranging error. These data have been successfully used for calculations of instantaneous point positioning PL which is the standard in aviation (ICAO, 2006; RTCA, Inc. SC-159, 2013).

For example in the EGNOS system this concept is based on the broadcast of differential GPS/GLONASS corrections in message types MT 1-5, 7, 9, 17-18, 24-26 and corresponding integrity data in MT 2-6, 10, 24, 26-28 (Pisonero Berges, 2006; ESA, CNES, 2009; RTCA, Inc. SC-159, 2013). The input quantities derived from GNSS and SBAS messages for the integrity algorithm on the user side are:

1) The geometry between GNSS satellites and user derived position from observations of the GNSS satellites is described by the geometry matrix **G** of size $n \times 4$:

$$\mathbf{G} = \begin{bmatrix} -\cos el_1 \sin Az_1 & -\cos el_1 \cos Az_1 & -\sin el_1 & 1 \\ -\cos el_2 \sin Az_2 & -\cos el_2 \cos Az_2 & -\sin el_2 & 1 \\ \cdots & \cdots & \cdots & \cdots \\ -\cos el_n \sin Az_n & -\cos el_n \cos Az_n & -\sin el_n & 1 \end{bmatrix} \tag{1}$$

Where arguments el_i and Az_i are the elevation and azimuth angles, respectively, between the receiver antenna and the i^{th} satellite ($i = 1,2,...,n$), where n is the number of visible satellites.

2) The weight matrix **W** is constructed under the assumption of uncorrelated, SBAS-corrected measurements characterized by the inverse variances of the distances to the observed satellites.

$$\mathbf{W} = \begin{bmatrix} \frac{1}{\sigma_2^2} & 0 & \cdots & 0 \\ 0 & \frac{1}{\sigma_2^2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{\sigma_n^2} \end{bmatrix} \tag{2}$$

$$\sigma_i^2 = \sigma_{i,flt}^2 + \sigma_{i,UIRE}^2 + \sigma_{i,tropo}^2 + \sigma_{i,mr}^2 \tag{3}$$

where:

$\sigma^2_{i,flt}$ – is the estimated variance for the residual error associated with user differential range error $\sigma_{i,\mathrm{UDRE}}$, which can be calculated in a similar way to the model of (RTCA, Inc. SC-159, 2013) [m^2];

$\sigma^2_{i,UIRE}$ – is the estimated variance for the slant range ionospheric error associated with the grid ionospheric vertical error $\sigma_{i,\mathrm{GIVE}}$, which can again be calculated in reference to (RTCA, Inc. SC-159, 2013) [m^2];

$\sigma^2_{i,tropo}$ – is the estimated variance for the residual tropospheric error (RTCA, Inc. SC-159, 2013) [m^2];

$\sigma^2_{i,mr}$ – is the estimated variance of a marine or shipborne receiver error, which depends on the receiver properties and site-specific GNSS signal propagation effects, like the multipath effect, which must be locally evaluated. This variance cannot be derived from the SBAS message [m^2].

Based on (1) and (2) the covariance matrix can be found:

$$\begin{bmatrix} s_E^2 & s_{EN} & s_{EU} & s_{ET} \\ s_{EN} & s_N^2 & s_{NU} & s_{NT} \\ s_{EU} & s_{NU} & s_U^2 & s_{UT} \\ s_{ET} & s_{NT} & s_{UT} & s_T^2 \end{bmatrix} = \left(\mathbf{G}^{\mathrm{T}}\mathbf{W}\mathbf{G}\right)^{-1} \tag{4}$$

where:

s^2_E – is the variance of the receiver antenna Easting measurements in the local reference frame centered on the GNSS antenna (East, North, Up, ENU) [m^2];

s^2_N – is the variance of the receiver antenna Northing measurements in the local reference frame (ENU) [m^2];

s^2_U – is the variance of the receiver antenna vertical measurements [m^2];

s^2_T – is the variance of the receiver time-correction measurements, multiplied by the speed of light [m^2]. The mixed terms, for example s_{EN}, are the covariances of the respective measurements [m^2].

Finally the circular assessment of the point positioning integrity on the user side can be given as a length of the protection circle radius, named the horizontal protection level (HPL) – see Figure 1:

$$\mathrm{HPL} = k\sqrt{\frac{s_E^2 + s_N^2}{2} + \sqrt{\left(\frac{s_E^2 - s_N^2}{2}\right)^2 + s_{EN}^2}} \tag{5}$$

where k is a coverage factor calculated using normally distributed uncertainties in both the North and East directions of position coordinates.

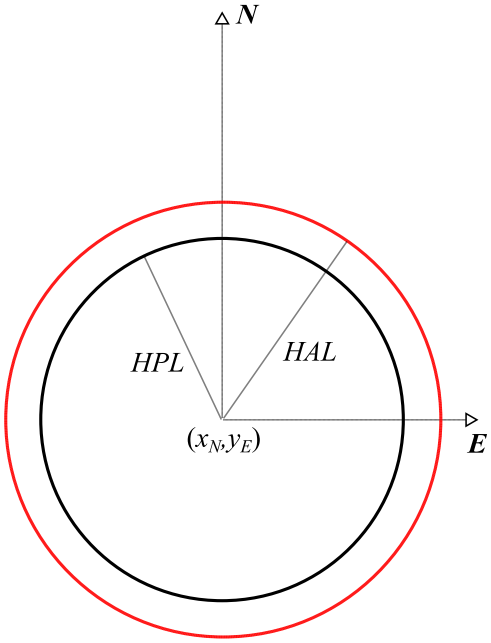

Figure 1. SBAS circular HPL and HAL

The elaborated concept treats the moving vessel as a point. This is why the new concept of a Marine Vessel Protection Area (MVPA) has been developed for the marine ECS or ECDIS, where a vessel is shown as a two-dimensional spatial object that is a contour model of a ship.

SBAS uncertainty Ellipse Determination

The "elliptical" assessment of the SBAS point positioning user integrity can be given as a protection ellipse (*PE*), which is specified by 4 parameters: (1) the semi-major axis of the estimated position error ellipse, d_a [m]; (2) the semi-minor axis of the error ellipse, d_b [m]; (3) the orientation of the error ellipse, Φ; and (4) the coverage factor, k, based on the confidence intervals. The integrity risk – or probability of Misleading Information (MI) – is the probability that the user will experience a true position outside the protection ellipse, PE_{mr}. It can be formulated as follows – refer to Figure 2:

$$PE = \begin{cases} kd_a \\ kd_b \\ \Phi \end{cases} \tag{6}$$

where:

$$d_a = \sqrt{\frac{s_E^2 + s_N^2}{2} + \sqrt{\left(\frac{s_E^2 - s_N^2}{2}\right)^2 + s_{EN}^2}} \tag{7}$$

$$d_b = \sqrt{\frac{s_E^2 + s_N^2}{2} - \sqrt{\left(\frac{s_E^2 - s_N^2}{2}\right)^2 + s_{EN}^2}} \tag{8}$$

$$\Phi = \frac{\pi}{2} - \frac{1}{2}\operatorname{atan2}\left(2s_{EN}, s_E^2 - s_N^2\right) \tag{9}$$

Φ – a clockwise angle of rotation from North either of the semi-major ellipse's axis (if $s_E > s_N$) or of the semi-minor axis (if $s_N > s_E$);

atan2 – the four-quadrant inverse tangent (arctangent) function of the real parts of two arguments (y, x) in Cartesian reference frame; and the formulae (7), (8) and (9) are derived from the square root of the eigenvalues of the covariance matrix (4) confined to:

$$\mathbf{C_{PA}} = \begin{bmatrix} s_E^2 & s_{EN} \\ s_{EN} & s_N^2 \end{bmatrix} \tag{10}$$

and oriented in the direction of the eigenvectors of (5).

The elliptical presentation of a protection area provides a navigator with additional benefits coming from information of N-E variances and covariances, resulting in changes of the orientation and shape of the ellipse.

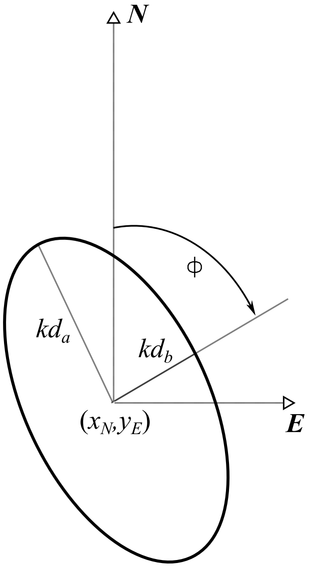

Figure 2. Elliptical representation of an SBAS point positioning protection area

Marine Vessel Protection Area Determination Based on SBAS Data

The mathematical model describing how the vessel is presented on the ECS display can be expressed by two observation equations:

$$x_{j,N} = x_N - x_{GPS} + d_j \cos(\psi + \alpha_j) \tag{11}$$

$$y_{j,E} = y_E - y_{GPS} + d_j \sin(\psi + \alpha_j) \tag{12}$$

where:

$$d_j = \sqrt{x_j^{\,2} + y_j^{\,2}} \tag{13}$$

$$\alpha_j = \frac{\pi}{2} - \operatorname{atan2}\left(x_j, y_j\right) \tag{14}$$

x_j, y_j – the calculated coordinates of consecutive points j of the ship contour in the body-fixed reference frame. This means it is fixed to the marine vessel at the common reference point of aft perpendicular with positive x axis to fore, y axis to starboard, following the convention used in marine craft hydrodynamics and simulations – see Figure 3;

x_{GPS}, y_{GPS} – the coordinates (offsets from 0 at aft perpendicular) of EGNOS-augmented GPS receiver antenna in the body-fixed reference frame;

$x_{j,N}$, $y_{j,E}$ – the calculated coordinates of consecutive points j of ship contour in the local reference frame (ENU);

x_N, y_E – the recorded positions of EGNOS-augmented GPS receiver antenna in the local reference frame (ENU);

ψ – the heading of marine vessel counted clockwise from North in the local reference frame (ENU);

d_j – the j^{th} distance between the GPS antenna and j^{th} point of the ship contour;

α_j – the j^{th} angle between the GPS antenna and j^{th} point of the ship contour, counted clockwise from the x-axis in the body-fixed reference frame.

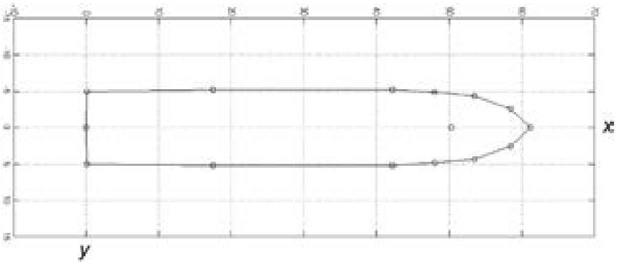

Figure 3. The model marine vessel contour consisting of 14 points in the body-fixed reference metric frame. The GNSS antenna position point is located at the fore part

The errors of parameters in equations (11) and (12) will propagate to the final MVPA according to the Gauss's Error Propagation Law. The statistical evaluation of this propagation effect has been presented in (Zalewski & Tomczak, 2005) and fully elaborated in (Zalewski, 2013).

The systematic errors of x_{GPS}, y_{GPS}, d_j and α_j can be minimized to a negligible magnitude by a precise dimensional control. Therefore, only the propagation of other parameter errors (x_N, y_E, ψ) are taken into account in the MVPA determination according to the formula:

$$\mathbf{C_{j,PA}} = \mathbf{J_j}\,\mathbf{C}\,\mathbf{J_j}^{\mathbf{T}} \tag{15}$$

where:

$\mathbf{C_{j,PA}}$ – the covariance matrix of derived quantities:

$$\mathbf{C_{j,PA}} = \begin{bmatrix} s_{j,E}^2 & s_{j,EN} \\ s_{j,EN} & s_{j,N}^2 \end{bmatrix} \tag{16}$$

$s^2_{j,E}$ – the Easting variance of consecutive points j of the ship contour in the local reference frame (ENU) [m^2];

$s^2_{j,N}$ – the Northing variance of consecutive points j of the ship contour in the local reference frame (ENU) [m^2];

$s_{j,EN}$ – the covariance of points j respective coordinates [m^2];

$\mathbf{J_j}$ – the Jacobian matrix containing first-order partial derivatives of equations (11) and (12), excluding x_{GPS}, y_{GPS} due to their negligible errors:

$$\mathbf{J_j} = \begin{bmatrix} 1 & 0 & \sin(\psi+\alpha_j) & d_j\cos(\psi+\alpha_j) \\ 0 & 1 & \cos(\psi+\alpha_j) & -d_j\sin(\psi+\alpha_j) \end{bmatrix} \tag{17}$$

C – the covariance matrix of observations:

$$\mathbf{C} = \begin{bmatrix} s_E^2 & s_{EN} & 0 & 0 \\ s_{EN} & s_N^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & s_\psi^2 \end{bmatrix} \tag{18}$$

where s^2_ψ is the marine vessel heading variance, relevant to the marine-specific attitude/heading equipment (the typical values for marine gyros in transport vessels are in range 0.5°–1°);

$\mathbf{J_j}^{\mathbf{T}}$ – the transposed Jacobian matrix (17).

The estimated error of each j^{th} contour point involves the errors of two jointly distributed variables of $x_{j,N}$ and $y_{j,E}$ coordinates. Thus, the positional error follows a bivariate normal distribution. By taking this into account to fully describe the estimated error of each j^{th} point, it is necessary to determine the orientation Φ_j and lengths of the semi-major axis $d_{j,a}$ and semi-minor axis $d_{j,b}$ of the j^{th} error ellipses, according to the formulas similar to (7)–(9) given here:

$$d_{j,a} = \sqrt{\frac{s_{j,E}^2 + s_{j,N}^2}{2} + \sqrt{\left(\frac{s_{j,E}^2 - s_{j,N}^2}{2}\right)^2 + s_{j,EN}^2}} \tag{19}$$

$$d_{j,b} = \sqrt{\frac{s_{j,E}^2 + s_{j,N}^2}{2} - \sqrt{\left(\frac{s_{j,E}^2 - s_{j,N}^2}{2}\right)^2 + s_{j,EN}^2}} \tag{20}$$

$$\Phi_j = \frac{\pi}{2} - \frac{1}{2}\left(\operatorname{atan2}\left(2s_{j,EN}, s_{j,E}^2 - s_{j,N}^2\right) + \psi\right) \tag{21}$$

where Φ_j is a clockwise angle of rotation from the ship's body-fixed x-axis either of the semi-major ellipse axis (if $s_{j,E} > s_{j,N}$) or the semi-minor axis (if $s_{j,E} < s_{j,N}$).

Each of the determined ellipses j can be further enlarged to the established confidence level by multiplying $d_{j,a}$ and $d_{j,b}$ by a coverage factor k, in a similar calculation to formula (6).

Knowing the parameters (19)–(21) of uncertainty ellipses centred on contour points j, the next step is to find the extreme outer points of these ellipses in order to construct the MVPA. In order to do this the maximum vertical values for a generalized, rotated ellipse in the Cartesian reference frame, fixed to the j^{th} segment of ship contour (x_c-axis between consecutive j and j+1 points), i.e., the upper bounding line or tangent of such an ellipse has to be calculated (see Figure 4).

The algorithm is as follows:

1) The angle β_j is the clockwise angle between the line leading through points j and j+1 and the x-axis. In the case where the maximum number for j is reached, j+1 is set to j=1. The points are counted clockwise from the x-axis in the body-fixed reference frame. The angle β_j is determined according to the formula:

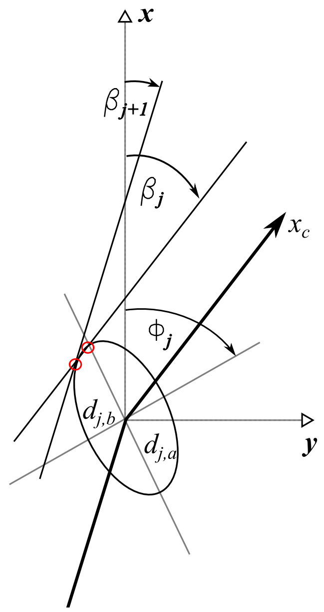

Figure 4. Construction of two tangent points highlighted with red circles for the *j*-th error ellipse

$$\beta_j = \frac{\pi}{2} - \text{atan}\,2\left(x_j - x_{j+1}, y_j - y_{j+1}\right) \tag{22}$$

2) Tangent points of ellipses with lines of slope β_j are determined according to the formulas:

$$\Phi_{j,c} = \frac{\pi}{2} - \Phi_j + \beta_j \tag{23}$$

$$\mathbf{R_j} = \begin{bmatrix} \cos\beta_j & -\sin\beta_j \\ \sin\beta_j & \cos\beta_j \end{bmatrix} \tag{24}$$

$$A_{j,1} = d_{j,b}^2 \sin^2 \Phi_{j,c} + d_{j,a}^2 \cos^2 \Phi_{j,c} \tag{25}$$

$$A_{j,2} = d_{j,b}^2 \cos^2 \Phi_{j,c} + d_{j,a}^2 \sin^2 \Phi_{j,c} \tag{26}$$

$$t_{j,1} = \sqrt{\frac{-A_{j,1} d_{j,a}^2 d_{j,b}^2}{\cos^2 \Phi_{j,c} \sin^2 \Phi_{j,c} \left(d_{j,b}^2 - d_{j,a}^2\right)^2 - A_{j,1} A_{j,2}}} \tag{27}$$

$$\mathbf{T_{j,2}} = \begin{bmatrix} t_{j,1} & \dfrac{-t_{j,1} \cos^2 \Phi_{j,c} \sin^2 \Phi_{j,c} \left(d_{j,b}^2 - d_{j,a}^2\right)}{A_{j,1}} \\ -t_{j,1} & \dfrac{t_{j,1} \cos^2 \Phi_{j,c} \sin^2 \Phi_{j,c} \left(d_{j,b}^2 - d_{j,a}^2\right)}{A_{j,1}} \end{bmatrix} \tag{28}$$

$$\mathbf{T_{j,3}} = \mathbf{T_{j,2}}\, R \tag{29}$$

$$\begin{bmatrix} y_{j,tp} & x_{j,tp} \\ y_{j,tn} & x_{j,tn} \end{bmatrix} = \begin{bmatrix} T_{j,3}(1,1) + y_j & T_{j,3}(1,2) + x_j \\ T_{j,3}(2,1) + y_j & T_{j,3}(2,2) + x_j \end{bmatrix} \tag{30}$$

where: $\Phi_{j,c}$ is the counter-clockwise angle of the j^{th} ellipse rotation to the x-axis, in a standard Cartesian $0xy$ reference frame, and $x_{j,tp}, y_{j,tp}, x_{j,tn}, y_{j,tn}$ are the coordinates of consecutive tangent points j in body-fixed reference frame. The extreme outer points are either $x_{j,tp}$, $y_{j,tp}$ if $\beta_j > 0$ or $x_{j,tn}$, $y_{j,tn}$ if $\beta_j \leq 0$.

The MVPA is constructed by linear connection of the resultant tangent points. This way the bounding spline representing the furthest points of ellipses with respect to the ship's hull are found. In order to achieve an acceptable level of coverage by ellipse

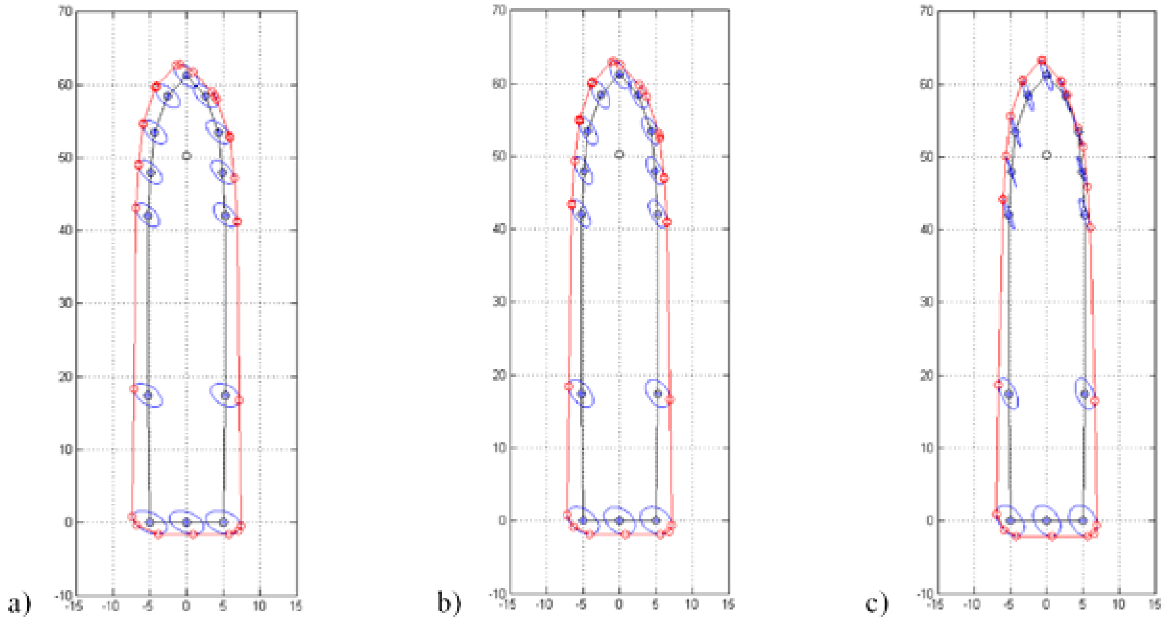

Figure 5. Examples of MVPAs around a ship heading to 45° in the body-fixed reference metric frame. The antenna position point is located in fore part

Figure 6. Presentation of a MVPA based on SBAS integrity data in an ECS display

areas by the MVPA, or to minimize the linear spline approximation error, the number of tangent points can be increased by adding additional tangent lines of slope angles in the range between β_j and β_{j+1}.

In the Figure 5a the resultant MVPA is presented in red and was built from three tangent points for each uncertainty ellipse in blue, for input parameters: $\psi = 45°$, $s_E = 1$ m, $s_N = 2$ m, $s_{EN} = 0$ m^2, $s_\psi = 2°$, $k = 1$. It should be noticed that in the body-fixed reference frame the ellipses are rotated according to the ship's heading. Specifically, the ellipses' semi-major axes are approximately oriented towards North, i.e. the direction where the position error has the largest variance (s_N). The semi-major axes are bigger in the aft section due to heading error propagation, as the GPS antenna is assumed to be in the forepart of the vessel.

Assuming non-zero covariances of x_N, y_E the MVPA will change gradually, as presented in Figures 5b and 5c. The resultant MVPA in Figure 5b is built from three tangent points for each uncertainty ellipse, for input parameters: $\psi = 45°$, $s_E = 1$ m, $s_N = 2$ m, $s_{EN} = 1$ m^2, $s_\psi = 2°$, $k = 1$. The ellipses have changed their dimensions and rotation in comparison to Figure 5a due to the East-North (EN) positive covariance.

In the Figure 5c the MVPA is built for input parameters: $\psi = 45°$, $s_E = 1$ m, $s_N = 2$ m, $s_{EN} = 2$ m^2, $s_\psi = 2°$, $k = 1$. This gives a perfect positive correlation of position estimation errors in the East and the North directions. This 100% covariance between E-N parameters of GNSS EGNOS corrected position, while heading to 45°, results in a very small MVPA margin close to the ship's starboard bow. There is very low probability of an erroneous ship contour position in this area.

It must be emphasized that in Figures 5a, 5b, 5c the k-factor corresponds to 1σ uncertainty ellipse (39.3% confidence level). In reality the MVPA should be based on ellipses of confidence derived from the accepted risk level. For example, for 5% risk, the confidence level of the ellipses will rise to 95%, meaning the ellipses forming the MVPA will be approximately 2.45 times larger.

A presentation of a MVPA based on SBAS integrity data in an ECS display is shown in Figure 6 as a black envelope around the red ship's contour.

Conclusions

The MVPA concept has been developed for marine ECS or ECDIS where a vessel is shown as a two-dimensional spatial object as a contour model of a ship. Utilization of the heading variance and the elliptical components of SBAS-estimated GNSS errors, in the protection area model, provide a navigator with an additional benefit coming from the information of changes in the MVPA orientation and shape.

Two types of alerts can be generated by the MVPA:

1. Alert based on the fixed limit values: activated if circular protection level HPL, derived solely from GNSS/SBAS, exceeds the horizontal alert limit (HAL) set for a specific marine area and operation.
2. Alert based on the domain methodology: activated if there are dangers to navigation inside the constructed MVPA. The MVPA can be of variable size depending on the k-factor which is set for a specific marine area and operation and relevant to the integrity risk.

Taking the geometric properties of MVPA into account, the best location for the installation of the GNSS antenna is the centre of ship's waterplane contour in the body-fixed reference frame. This location for the GNSS antenna leads to similar values of heading errors when propagated to the MVPA envelope at the fore and aft. The realization of this condition is usually impracticable in the majority of marine transport vessels, so mariners must be aware of a larger uncertainty in the ship's position at the fore or aft depending on the construction of the vessel. For example, when the navigation bridge and GNSS antenna are located in the forepart then the MVPA is larger in the aft, and in case of the navigation bridge and GNSS antenna in the aft, the MVPA is larger at the fore.

References

1. ESA, CNES (2009) User Guide for EGNOS Application Developers, Ed. 1.1, 07/30/2009.
2. ICAO (2006) Standards and Recommended Practices (SARPs), Volume 1 – Annex 10, Amendments 1-81.
3. IMO (2001) Revised Maritime Policy and Requirements for a Future Global Navigation Satellite System (GNSS), Resolution A.915(22) from 22nd Session of the Assembly of International Maritime Organization, adopted on 29 November 2001, London.
4. RTCA, Inc. SC-159 (2013) RTCA DO-229D Minimum Operational Performance Standards (MOPS) for Global Positioning System / Satellite–Based Augmentation System Airborne Equipment, 2006, Change 1.
5. Zalewski, P. & Tomczak, A. (2005) Method of Probabilistic Evaluation of Ship's Contour Inclusive Area for a Pilot Navigation System. *2nd International Congress of Seas and Oceans*, Szczecin – Świnoujście, Poland, 20–24 September 2005. pp. 229–239.
6. Zalewski, P. (2013) *Systemy autonomiczne w procesie oceny bezpieczeństwa jednostek pływających na akwenie ograniczonym*. Szczecin: Wydawnictwo Naukowe Akademii Morskiej w Szczecinie.
7. Pisonero Berges, C. (2006) *The EGNOS SBAS Message Format Explained* [Online] 2006. Available from: http://www.navipedia.net/index.php/The_EGNOS_SBAS_Message_Format_Explained [Accessed: 20 September 2015].