Spectral analysis of parametric two degrees-of-freedom system

Juliusz Krzyżkowski*, Yuriy Pyryev**

* Research and Development Centre for the Graphic Arts (COBRPP)
Miedziana 11, 00-958 Warsaw, Poland
e-mail: j.krzyzkowski@cobrpp.com.pl
**Warsaw University of Technology, Department of Production Engineering
Institute of Mechanics and Printing, Division of Graphic Arts
Konwiktorska 2, 02-217 Warsaw, Poland,

In this paper model of a vibrating system of two rotating, stressed to each other and periodically losing contact cylinders is presented. The model is described by two-degrees-of-freedom Hill's differential equations system, which was numerically solved. Spectra of the vibrations were analyzed in here.
Keywords: parametric vibrations, Hill equations, TDOF system, spectral analysis

Introduction

In vibrating parametric systems oscillations are induced by periodic variations of the systems properties, such as stiffness, damping, moment of inertia etc. Disturbances in the system are excited by the system itself, not due to outer forces. The simplest examples of one-degrees-of-freedom parametric systems are pendulum with fluctuating length or a child playing on the swing. Parametric systems are described in general by Hill's differential equations or by Mathieu equations in case of sinusoidal variations of the system's parameters.

In the parametric systems, in which stiffness or damping are changing, eigenfrequencies are also varying. However, similarly to the vibrating systems with constant parameters, there also resonance phenomenon can be observed. It is called parametric resonance and occurs when parameters vary with certain frequencies. However conditions, which need to be satisfied to induce parametric resonance are more complicated than in forced vibrations. For one-degrees-of-freedom parametric systems and sinusoidal excitation are widely described in literature [1-3]. Analysis of stability of two degrees-of-freedom (TDOF) was performed in [4,5]. In the works [6,7] parametric excitations were used to suppress vibrations of TDOF systems. The spectra of parametric systems was presented inter alia in [8].

In this article the results of spectral analyses of that system are presented.

Model

We will consider two rotating cylinders, which are stressed to each other (fig.1). One of them has metallic surface and the second one is covered with rubber material. On the sur-

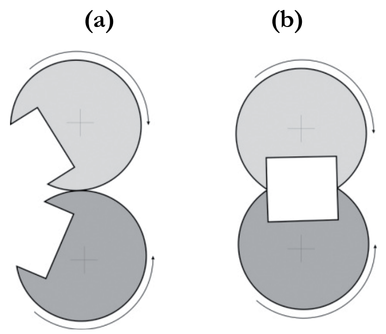


Figure 1. Considered cylinders system: cylinders in contact (a) and the moment of canals encounter (b)

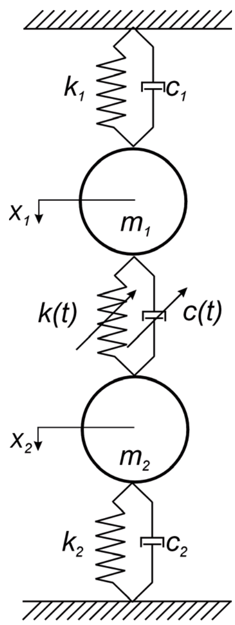


Figure 2. Model of a system

face of each cylinder a rectangular channel is placed. The canals encounter after each cylinders' revolution and in that moment the contact between cylinders disappear (fig. 1b), which results in declining to the minimum the stiffness and damping properties of the system. After a while, when the cylinders are again in contact, both parameters grow up and reach their maximum value. Such periodical fluctuations of the parameters excite parametric vibrations.

A model of the cylinders system is presented in figure 2. It can be described by system of Hill's equations (1).

$$\widehat{\mathbf{M}}\ddot{\mathrm{x}}+\widehat{\mathbf{C}}(t)\dot{\mathrm{x}}+\widehat{\mathbf{K}}(t)\mathrm{x}=0$$

$$\widehat{\mathbf{M}}=\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix},\ \mathbf{x}=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$$

$$\widehat{\mathbf{C}}(t)=\begin{bmatrix} c_1+c(t) & -c(t) \\ -c(t) & c_2+c(t) \end{bmatrix}, \qquad (1)$$

$$\widehat{\mathbf{K}}(t)=\begin{bmatrix} k_1+k(t) & -k(t) \\ -k(t) & k_2+k(t) \end{bmatrix}$$

where:
$\widehat{\mathbf{M}}$ – the mass matrix, $\widehat{\mathbf{C}}$ – the damping matrix , $\widehat{\mathbf{K}}$ – the stiffness matrix, m_1 m_2 – the masses of the cylinders; c_1, c_2 – the damping coefficients of the bearings in which cylinders are mounted; k_1, k_2,– the stiffness coefficients of the bearings in which the cylinders are mounted; $k(t)=kf(t)$ – the stiffness of the system in the cylinders' contact zone; $c(t)=cf(t)$ – the damping of the system in the cylinders' contact zone; *f(t)* – the function of parameters' variations.

The changes of variations of the parameters *k(t)* and *c(t)* are defined by function *f(t)* (2), which is presented in figure 3. Function *f(t)* also defines the form of the exciting force.

$$f(t)=\begin{cases} 1, & T_n<t<T_{2\varepsilon} \\ \tau^2(3-2\tau), & T_{2\varepsilon}<t<T_{\varepsilon} \\ \tau^2(3+2\tau), & T\ <t<T\ \end{cases} \qquad (2)$$

where:
n=0,1,2…; $\tau={(T_\varepsilon-t)}/{\varepsilon}$; $T_\varepsilon=T_{n+1}-\varepsilon$; $T_{2\varepsilon}=T_{n+1}-2\varepsilon$; $T_n=nT$;, $T={2\pi}/{\omega}$ – period of the cylinders' load variations; $f(t+T)=f(t)$; 2ε – time in which the cylinders are not in contact (see Fig. 3).

For computations purposes the order of differential equations system (1) was reduced and the new variables were introduced: $x_1=z_1$; $\dot{x}_1=z_2$; $x_2=z_3$; $\dot{x}_2=z_4$. In that way four differential equations were obtained in the form (3).

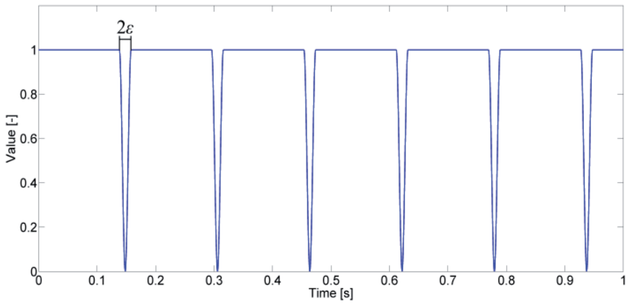


Figure 3: Character of system's stiffness and damping variations (function f(t))

$$\dot{\mathbf{z}}(t)=\widehat{\mathbf{A}}(t)\mathbf{z}(t) \qquad (3)$$

where:

$$\widehat{\mathbf{A}}(t)=\begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1+k(t)}{m_1} & -\frac{c_1+c(t)}{m_1} & \frac{k(t)}{m_1} & \frac{c(t)}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k(t)}{m_2} & \frac{c(t)}{m_2} & -\frac{k_2+k(t)}{m_2} & \frac{c_2+c(t)}{m_2} \end{bmatrix}$$

$$\mathbf{z}(t)=\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}$$

The systems of two rotating cylinders with canals on their surfaces, like the one described above, are commonly used in printing presses. The presented model lets specify critical speed of working machine in which the system is stable and parametric resonance does not occur.

Results

System of equations (3) was solved in Matlab® (license number: 69266). Integration of the system of four equations was performed with the fourth order Runge-Kutta method. The parameters of the system (masses, stiffness and damping coefficients) used in calculations were close to the real ones, which occur in the printing presses (Tab. 1).

Table 1. Parameters of the system [4]

Parameter	Value
m_1	85,0 [kg]
m_2	105,0 [kg]
k_1	2.94e9 [Nm^{-1}]
k_2	3.16e9 [Nm^{-1}]
k	2.05e8 [Nm^{-1}]
c_1	4,90e2[$Ns^{-1}m^{-1}$]
c_2	4,90e2[$Ns^{-1}m^{-1}$]
c	9,21e2[$Ns^{-1}m^{-1}$]

Parametric systems are instable for frequencies of the exciting force *f* (the force induced by variation of the system's parameters) equal to

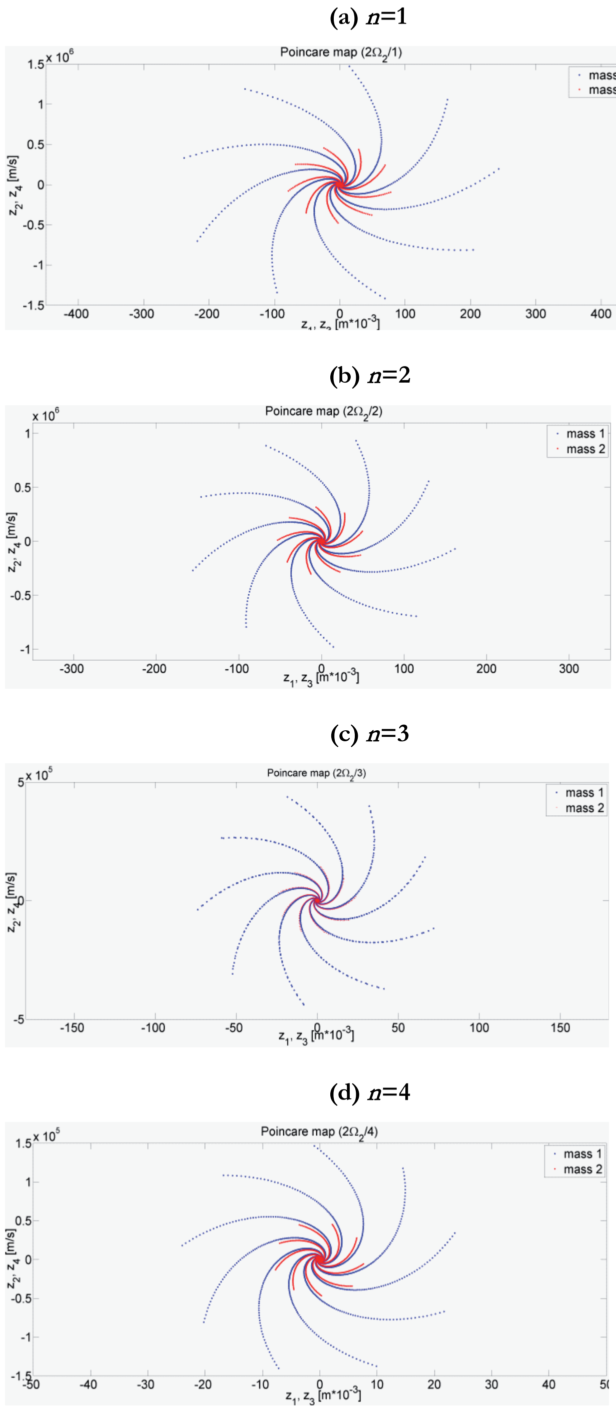
$$\omega = {}^{2\Omega_i}/_{n}, \qquad (4)$$

where: n=1,2,…,7 and Ω_i are the system's natural frequencies (calculated as in [4]) of the modified system with constant parameters.

Considered TDOF system is instable when it is excited with the frequency satisfying equation (4), in which Ω_i is equal to Ω_2 – the larger one of two system's natural frequencies. Parametric resonance for all cases of system's instability is shown in phase space in figure 4.

Parametric resonance does not occur if the system is excited with the frequencies, which are in relation (4) to the lower of the system's natural frequency Ω_1.

Spectral analysis was performed in the following manner: System of equations (3) was solved for time period of [0;10] seconds. From the vibratory signal obtained in the time domain the last 3 seconds were taken into account when performing FFT transformation. The reason of such procedure was to avoid peaks in the spectrograms, which are related to vibrations appearing only just after exciting the system.



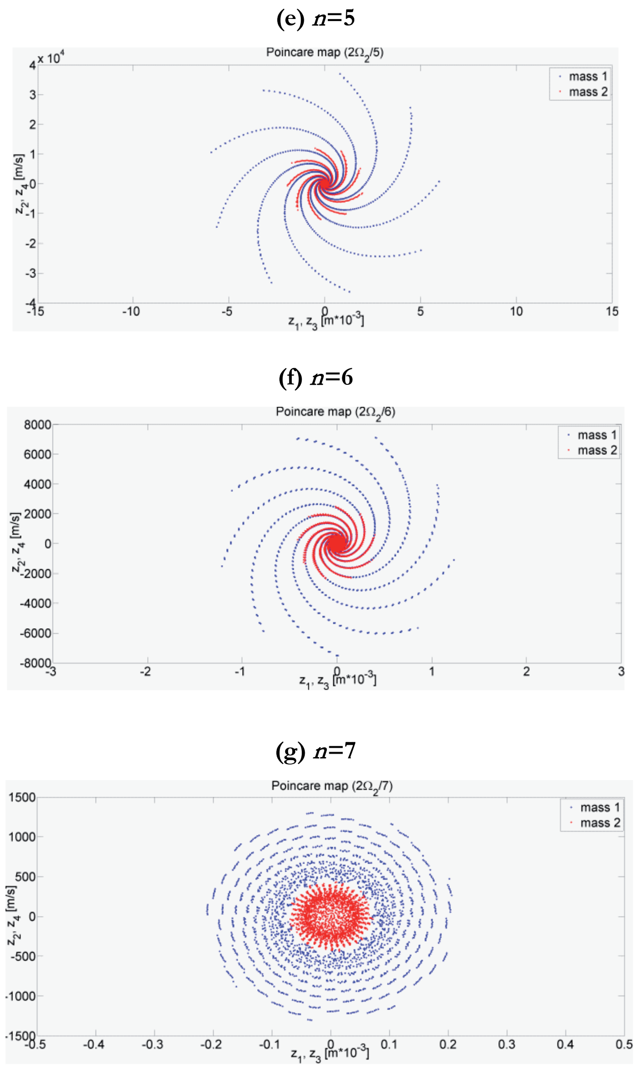


Figure 4: Poincaré section for parametric resonance of the system for different excitation frequencies (z_1, z_3 – masses displacements; z_2, z_4 – masses velocities)

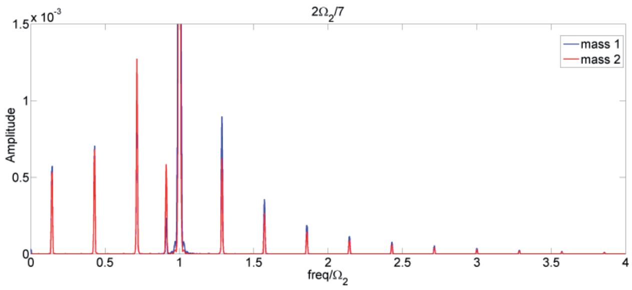


Figure 5. Comparison of masses m1 and m2 vibrations spectra

Fourier transformation was conducted with Matlab's built in algorithm. In order to optimize work of the algorithm, length of the input signal was enlarged to the closest power of 2 by adding zeros at the end of the data sequence. Frequencies on the spectrograms are relative values with respect to Ω_2.

Spectrograms of exciting force $f(t)$ were prepared with the same procedure.

Spectral analysis of the resonating masses showed, that their spectra are similar and the only difference occurs in the values of certain peaks. Thus analysis of only one mass (m_1) vibrations are further presented.

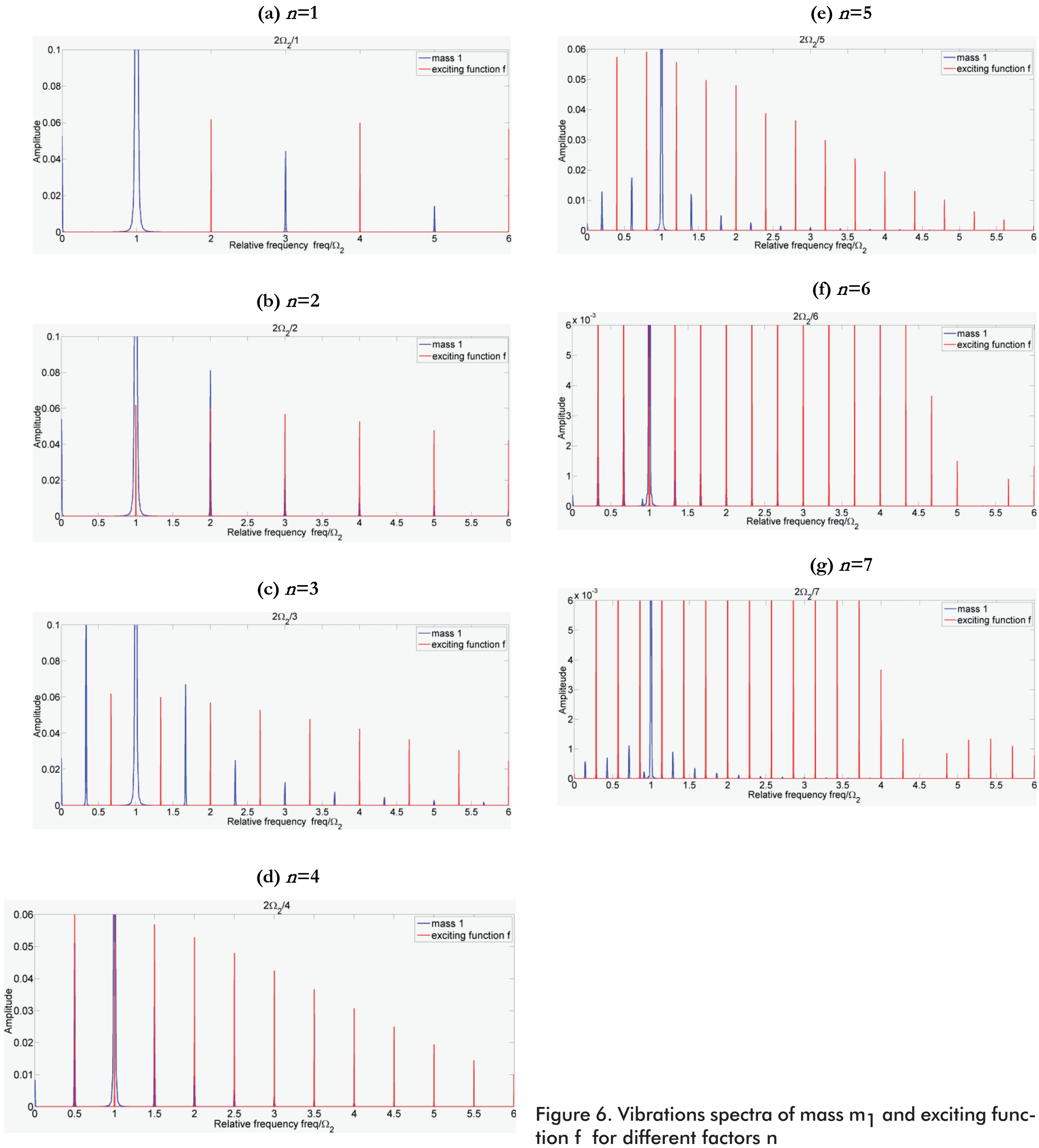


Figure 6. Vibrations spectra of mass m_1 and exciting function f for different factors n

Figure 5 shows an exemplary comparison of the vibrations spectra of masses m_1 and m_2 for exciting frequency equal to $\omega = {}^{2\Omega_2}/_{7}$

In figure 6 spectra of vibrations of m_1 and exciting force f are depicted.

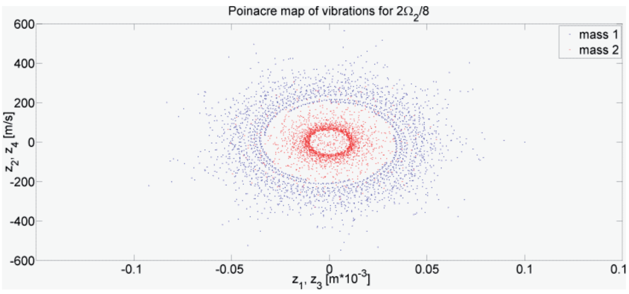
As one can see there does not exist simple relation between frequencies of exciting force f and those of system body's vibrations. There is visible correlation only for even values of factor n, i.e. n=2, 4, 6. If we consider odd values of n (1, 3, 5, 7), peaks of the exciting force spectrum are located exactly in between of body's vibrations frequencies.

All spectra presented in figure 6 indicate the largest peak for the relative frequency equal to ${}^{\Omega_2}/_{\Omega_2}$. Next to it there is noticeable peak corresponds to lower of natural frequencies ${}^{\Omega_1}/_{\Omega_2}$. It is visible only in figure 6f and 6g, because of the scale used in the figures. Other graphs show that harmonics of ${}^{\Omega_2}/_{\Omega_2}$ dominate over it. With the growth of factor n (decrease of excitation frequency) there appear larger number of peaks in the spectrum. The relation between frequencies of the peaks and values of parameter n are collected in table 2.

Odd values of factor n imply appearing of peaks in frequencies equal to ${}^{2k-1}/_{n}$ whereas even n values induce peaks in frequencies ${}^{2k}/_{n}$ (k=1, 2, 3…).

If we consider vibrations of the stable system, correlation between spectrum of vibrations and exciting function are similar to those in the case of parametric resonance spectra. The difference is that peak representing frequency ${}^{\Omega_1}/_{\Omega_2}$, which becomes dominating over harmonics of frequency Ω_2. An example of these relations for n=8 is presented in figure 7.

(a)



(b)

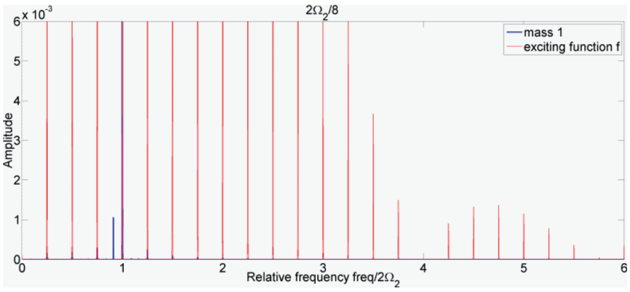


Figure 7. Poincaré section (a) and spectra of mass m1 vibrations and exciting function f for n=8 (b)

Table 2. Relation between frequencies

n	Relative frequencies
1	1, 3, 5
2	$^2/_2$, $^4/_2$, $^6/_2$, $^8/_2$, $^{10}/_2$
3	$^1/_3$, 1, $^5/_3$, $^7/_3$
4	$^1/_2$, 1, $^3/_2$, $^4/_2$, $^5/_2$, $^6/_2$, $^7/_2$, $^8/_2$, $^9/_2$, $^{10}/_2$, $^{11}/_2$, $^{12}/_2$
5	$^1/_5$, $^3/_5$, 1, $^7/_5$, $^9/_5$, $^{11}/_5$, $^{13}/_5$, $^{15}/_5$, $^{17}/_5$, $^{19}/_5$, $^{21}/_5$
6	$^2/_6$, $^4/_6$, 1, $^8/_6$, $^{10}/_6$, $^{12}/_6$, $^{14}/_6$, $^{16}/_6$, $^{18}/_6$, $^{20}/_6$, $^{22}/_6$, $^{24}/_6$, $^{26}/_6$
7	$^1/_7$, $^3/_7$, $^5/_7$, 1, $^9/_7$, $^{11}/_7$, $^{13}/_7$, $^{15}/_7$, $^{17}/_7$, $^{19}/_7$, $^{21}/_7$, $^{23}/_7$, $^{25}/_7$, $^{27}/_7$

Conclusions

In the paper the analysis of the model of two rotating cylinders with variable stiffness and damping was investigated. The conducted spectral analysis of the system's vibrations shown, that spectrum of the instable system consists mainly of the frequency corresponding to the larger one of two its natural frequencies and its super- and subharmonics. The frequencies of the harmonics are fractions of parameter n, which defines the exciting force frequency. However, the spectra of system's bodies vibrations and the exciting force do not simply correlate to each other. Such correlation of both spectra is noticeable only in the case of even values of parameter n.

If the system is stable, in its spectrum sub- and superharmonics of the larger one of the natural frequencies decrease significantly and more meaningful becomes the peak corresponding to the smaller one of two natural frequencies.

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Author(s):

MSc Juliusz Krzyżkowski *– MSc in Papaermaking in Printing at Warsaw University of Technology (2009), Poland, where is PhD student. Research areas: vibration of printing presses and their influence on printing quality. Since 2013 works for Research and Development Centre for the Graphic Arts (COBRPP), Poland, where is involved in research on printed electronics.*

assoc. Prof., DSc Yuriy Pyryev *– MSc – Faculty of Mechanics and Mathematics of Ivan Franko National University in L'viv (1975); Doctor (1986 – Podstryhach Institute of Applied Problems of Mechanics and Mathematics); Habilitation, Dr. of Science (2004 – Technical University of Łódź). Research areas: Coupled thermodynamics, Nonlinear Contact Problems, Nonlinear Mechanics. Since 2009 (Warsaw University of Technology, Institute of Mechanics and Printing, Division of Graphic Arts) works on modelling of phenomena occurring in printing presses.*

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