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REALIZATION OF COORDINATION TECHNOLOGY OF HIERARCHICAL SYSTEMS IN DESIGN OF ACTIVE MAGNETIC BEARINGS SYSTEM

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A cybernetic technology of mechatronic design of active magnetic bearings systems (AMB) originated from theory of systems is suggested in the paper. Traditional models of artificial intelligence and mathematics do not allow describing mechatronic systems being designed on all its levels in one common formal basis. They do not describe the systems structure (the set of dynamic subsystems with their interactions), their control units, and do not treat them as dynamic objects operating in some environment. They do not describe the environment structure either. Therefore, the coordination technology of hierarchical systems has been chosen as a theoretical means for realization of design and control. The theoretical basis of the given coordination technology is briefly considered. An example of technology realization in conceptual and detailed design of AMB system is also presented.

Keywords: hierarchical systems, design, coordination, mechatronic, magnetic bearings

1. Introduction

In the design process of active magnetic bearings (AMB) we deal with mechatronic objects which contain connected mechanical, electromechanical, electronic and computer subsystems. Various methods and models which are used for each system coordination (design and control) cannot describe all subsystems in common theoretical basis and, at the same time, describe the mechanism with all interactions in the structure of a higher level and the system as a unit in its environment. It is important to define the common theoretical means which will describe all subsystems of a mechatronic object being designed (AMB systems) and its coordination (design and control) system in a common formal basis. This task is topical for the systems of computer aided design (CAD). Besides, theoretical means of the coordination technology must allow performing the design and control tasks under condition of any information uncertainty, i.e. (1) to create and change mechatronic system construction and technology by selecting units of lower levels and settling their interactions to make the state and activity of the system in higher levels (environment) best coordinated with environmental aims (selection stratum); (2) to change the ways (strategies) of the design task performing when the designed unit is multiplied and the knowledge uncertainty is removed (learning stratum); (3) to change the above mentioned strata when new knowledge is created (self-coordination stratum).

The coordination technology must also cohere with traditional forms of information representation in mechatronics, i.e. numerical and geometrical systems. The theoretical basis of the design process in agreement with these requirements must be a hierarchical construction connecting any level unit with its lower and higher levels. Mathematical and cybernetic theories based on the set theory are incoherent with the above design requirements since the set theory describes one-level world outlook.

In this paper, the coordination technology of Hierarchical System by Mesarovich *et al.* (1970) with its standard block *aed* (ancient Greek word) by Novikava *et al.* (1990, 1995, 1997) Miatliuk

(2003), Novikava and Miatliuk (2007) has been chosen as the theoretical basis for performing a mechatronic design task. In comparison with traditional methods, *aed* technology allows presentation of the designed object structure, its dynamic representation as a unit in the environment, the environment itself and the control system in common formal basis together with easy formalization of the design process. In the paper, the *aed* formal basis and coordination technology of hierarchical systems are described. AMB system construction and the system conceptual and detailed design are presented as practical examples of the proposed technology. Finally, the developed technology for the design of exemplary AMB mechatronic systems is analysed.

2. Formal basis of design technology

The *aed* model S^ℓ considered below unites the codes of the two level system (Measarovic *et al.*, 1970) and general systems theory by Mesarovic and Takahara (1990), the number code L^S, geometry and cybernetics methods. The dynamic representation $(\overline{\rho}, \overline{\varphi})$ is the main means of the description of the named codes. *Aed* is a standard element of hierarchical systems (Novikava *et al.*, 1990, 1995, 1997; Miatliuk, 2003; Novikava and Miatliuk, 2007), which realizes the general laws of systems organization on each level and the inter-level connections. *Aed* S^ℓ contains ω^ℓ and σ^ℓ models which are connected by the coordinator S_0^ℓ

$$S^\ell \leftrightarrow \{\omega, S_0, \sigma\}^\ell \tag{2.1}$$

where ω^ℓ is a dynamic representation of any level $\ell \in L^S$ system in its environment, σ^ℓ is the system structure, S_0^ℓ is coordinator. The structure diagram of *aed* S^ℓ is presented in Fig. 1.

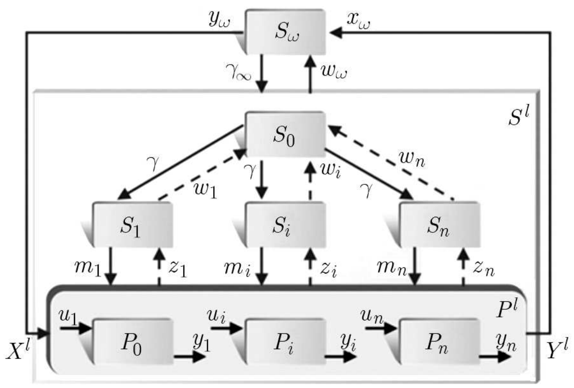

Fig. 1. Structure diagram of *aed* – standard block of Hierarchical Systems. S_0 is the coordinator, S_ω is the environment, S_i are subsystems, P_i are subprocesses, P^l is the process of level ℓ, X^l and Y^l are the input and output of the system S^l; m_i, zi, γ, w_i, u_i, y_i are interactions

Aggregated dynamic representations ω^ℓ of all *aed* connected elements, i.e. the object ${}_oS^\ell$, processes ${}_oP^\ell$, ${}_\omega P^\ell$ and environment ${}_\omega S^\ell$ are presented in form of the dynamic system $(\overline{\rho}, \overline{\varphi})^\ell$

$$\begin{aligned} \overline{\rho}^\ell &= \{\rho_t: \quad C_t \times X_t \to Y_t \quad \wedge \quad t \in T\}^\ell \\ \overline{\varphi}^\ell &= \{\phi_{tt'}: \quad C_t \times X_{tt'} \to C_{t'} \quad \wedge \quad t, t' \in T \quad \wedge \quad t' > t\}^\ell \end{aligned} \tag{2.2}$$

where C^ℓ is the state, X^ℓ – input, Y^ℓ – output, T^ℓ – time of level ℓ, $\overline{\rho}^\ell$ and $\overline{\varphi}^\ell$ are the reactions and state transition functions, respectively. Dynamic representations ω^ℓ of the object ${}_oS^\ell$, the processes ${}_oP^\ell$, ${}_\omega P^\ell$ and the environment ${}_\omega S^\ell$ are connected by their states, inputs and outputs.

The model of the system structure is defined as follows

$$\sigma^\ell = \{S_0^\ell, \{\overline{\omega}^{\ell-1}, {}_\sigma U^\ell\}\} = \{S_0^\ell, \widetilde{\sigma}^\ell\} \tag{2.3}$$

where S_0^ℓ is the coordinator, $\overline{\omega}^{\ell-1}$ are aggregated dynamic models of the subsystems $\overline{S}^{\ell-1} = \{S_i^{\ell-1} : i \in I^\ell\}$ of the lower level $\ell - 1$, ${}_\sigma U^\ell$ are structural connections ${}_\sigma U^\ell \supset {}_\omega\overline{U}^{\ell-1} = \{{}_\omega U_i^{\ell-1} : i \in I^\ell\}$ of the subsystems $\overline{S}^{\ell-1}$. $\widetilde{\sigma}^\ell$ is the connection of the dynamic systems $\overline{\omega}^{\ell-1}$ and their structural interactions ${}_\sigma U^\ell$ coordinated with the external ones ${}_\omega U^\ell = {}_\sigma U^{\ell+1}|S^\ell$.

The coordinator S_0^ℓ is the main element of hierarchical systems which realizes the processes of systems design and control (Novikava *et al.*, 1995; Miatliuk, 2003). It is defined according to *aed* presentation of Eq. (2.1) in the following form

$$S_0^\ell = \{\omega_0^\ell, S_{00}^\ell, \sigma_0^\ell\} \tag{2.4}$$

where ω_0^ℓ is the aggregated dynamic realization of S_0^ℓ, σ_0^ℓ is the structure of S_0^ℓ, S_{00}^ℓ is the coordinator control element. S_0^ℓ is defined recursively. The coordinator S_0^ℓ constructs its aggregated dynamic realization ω_0^ℓ and the structure σ_0^ℓ by itself. S_0^ℓ performs the design and control tasks on its selection, learning and self-organization strata (Miatliuk, 2003). All metric characteristics μ of systems being coordinated (designed and controlled) and the most significant geometry signs are determined in the frames of *aed* informational basis in the codes of numeric positional system L^S (Miatliuk, 2003; Novikava and Miatliuk, 2007).

The external connections ${}_\omega U^\ell$ of ω^ℓ with other objects are its coordinates in the environment ${}_\omega S^\ell$. The structures have two basic characteristics: ξ^ℓ (connection defect) and δ^ℓ (constructive dimension); μ^ℓ, ξ^ℓ and δ^ℓ are connected and described in the positional code of the L^S system (Miatliuk, 2003; Novikava and Miatliuk, 2007). For instance, the numeric characteristic (constructive dimension) $\delta^\ell \in \Delta^\ell$ of the system S^ℓ is presented in the L^S code as follows

$$\begin{aligned} &\widetilde{\delta}^\ell = (n_3, \dots, n_0)_\delta && \widetilde{\delta}^\ell \in \{\delta_\sigma^\ell, \delta_\omega^\ell\} \\ &(n_i)_\delta = (n_{3-i})_\xi && (n_i)_\delta \in N \qquad i = 0,1,2,3 \end{aligned} \tag{2.5}$$

where δ_ω^ℓ and δ_σ^ℓ are constructive dimensions of σ^ℓ and ω^ℓ, respectively. This representation of geometrical information allows execution of all operations with geometric images on the computer as operations with numeric codes.

The *aed* technology briefly described above presents a theoretical basis for AMB systems design and control. In comparison with the two-level system proposed by Mesarovic *et al.* (1970), the presented informational model of *aed* S^ℓ has new positive characteristic features (Novikava *et al.*, 1990, 1995, 1997; Miatliuk, 2003; Novikava and Miatliuk, 2007). Formalization, availability of the environment block ${}_\omega S^\ell$, description of the inter-level relations, coordination technology and information aggregation make the *aed* technology more efficient in the design tasks.

3. Coordination technology realization in the design of AMB system

3.1. Conceptual formal model of an AMB system

Formal description of the Active Magnetic Bearing (AMB) system in *aed* form is an example of the Hierarchical System (HS) (aed) coordination technology realization in the conceptual design of a mechatronic system. The AMBs systems are usually used in rotating machinery, flywheels, industrial turbomachinery, etc. (Schweitzer and Maslen, 2009). In this paper we focus on an AMB system which is a part of the experimental stand of a suspension system (Fig. 2) developed at Automation and Robotics Department, Bialystok University of Technology (Mystkowski and Gosiewski, 2007, Gosiewski and Mystkowski, 2006, 2008).

The AMB system is presented in *aed* form as follows

$$_M S^\ell \leftrightarrow {}_M\{\omega, S_0, \sigma\}^\ell \tag{3.1}$$

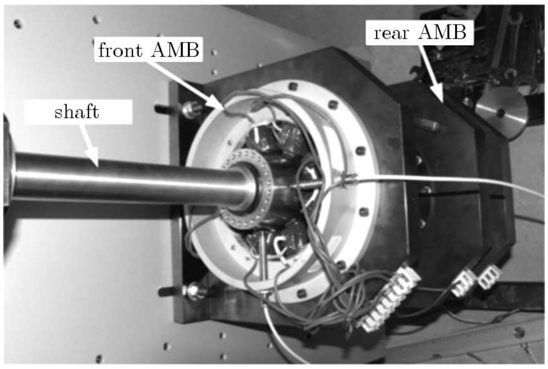

Fig. 2. AMB-beam test rig

where ${}_M\omega^\ell$ is an aggregated dynamic representation of the AMB system ${}_MS^\ell$, see Eq. (2.2), ${}_M\sigma^\ell$ is the system structure, ${}_MS_0^\ell$ is coordinator, i.e. design and control system, ℓ is the index of level.

The AMB system construction ${}_M\sigma^\ell$ contains the set of sub-systems $\overline{\omega}^{\ell-1}$ and their structural connections ${}_\sigma U^\ell$. Thus, according to Eq. (2.3), the structural subsystems presented in aggregated dynamic form $\overline{\omega}^{\ell-1}$ are:

- front AMB – ${}_M\omega_1^{\ell-1}$
- rear AMB – ${}_M\omega_2^{\ell-1}$
- thrust passive magnetic bearing (PMB) – ${}_M\omega_3^{\ell-1}$
- shaft – ${}_M\omega_4^{\ell-1}$.

In their turn, each subsystem has its own structural elements – the lower level $\ell-1$ subsystems. In the AMB subsystem ${}_M\omega_1^{\ell-1}$, these are eight $i=8$ electromagnetic coils ${}_M\omega_{1i}^{\ell-2}$ and the displacement sensors assembly ${}_M\omega_{1,9}^{\ell-2}$ which creates the external part of the AMB. The internal part is the magnetic core ${}_M\omega_{1,10}^{\ell-2}$ attached to the shaft. The subsystems ${}_M\overline{\omega}^{\ell-1}$ are connected by their common parts – the structural connections ${}_\sigma U^{\ell-1}$ that are elements of lower levels. For instance, the shaft ${}_M\omega_4^{\ell-1}$ and the front AMB ${}_M\omega_1^{\ell-1}$ are connected by their common element – the magnetic core ${}_\sigma U_{1,4}^{\ell-1} \leftrightarrow {}_M\omega_{1,10}^{\ell-2} \leftrightarrow {}_M\omega_{4,1}^{\ell-2}$, where ${}_M\omega_{1,10}^{\ell-2}$ is aggregated dynamic realization of the magnetic core being the subsystem of the front AMB ${}_M\omega_1^{\ell-1}$, and ${}_M\omega_{4,1}^{\ell-2}$ the realization of the magnetic core being the subsystem of the shaft ${}_M\omega_4^{\ell-1}$.

Aggregated dynamic realizations ${}_M\overline{\omega}^{\ell-1}$, i.e. dynamic models ${}_i(\overline{\rho},\overline{\varphi})^{\ell-1}$, Eq. (2.2), of the subsystems ${}_M\overline{S}^{\ell-1}$, are formed after definition of their inputs-outputs concerning each concrete sub-process they execute. Thus, for the shaft ${}_M\omega_4^{\ell-1}$ concerning its rotation process, the input ${}_MX_4^{\ell-1}$ is the torque M obtained from the loading system (motor), and the output ${}_MY_4^{\ell-1}$ is the angular velocity Ω of the shaft (Fig. 2). The shaft dynamic model ${}_M\omega_4^{\ell-1}$ in this case is presented at the detailed design stage in form of the differential equation described by Gosiewski and Mystkowski (2006, 2008).

The environment ${}_\omega S^\ell$ of the AMB system has its own structure and contains:

ω_1^ℓ – measuring and signal conditioning system (electronic),

ω_2^ℓ – loading system – motor/generator (electromechanical),

ω_3^ℓ – control systems in feedback loop of the general control AMB system (computer system).

Thus, the object being controlled ${}_MS^\ell$ (AMB system), environment subsystems, i.e. measuring ${}_\omega S_1^\ell$ (sensors, filters, estimators), loading ${}_\omega S_2^\ell$ (electromotor, generator, clutch) and control systems ${}_\omega S_3^\ell$ in the feedback loop (computer, processor, converters DAC and ADC) create the general control AMB system. The immediate input ${}_MX^\ell$ for the AMB system (which is at the same time the output ${}_{\omega}Y^{\ell}_{M} = {}_{M}X^{\ell}$ of the environment of the AMB system) are signals from the loading system – the motor torque and control signal, i.e. the voltage/current or flux which come from internal or external controllers of the control system. The output of the AMB system is the axial displacement of the shaft in the plane orthogonal to the shaft symmetry axis, measured currents, flux, rotor angular speed, coil temperature, etc. The output ${}_{M}Y^{\ell}$ of the AMB system ${}_{M}S^{\ell}$, i.e. the displacement of the shaft, is at the same time the input ${}_{\omega}X^{\ell}_{M} = {}_{M}Y^{\ell}$ of the environment which is measured by eddy-current sensors or optical (laser) sensors. The states ${}_{M}C^{\ell}_{i}$ of the AMB system ${}_{M}S^{\ell}$ are:

${}_{M}c^{\ell}_{1}$ – displacements,

${}_{M}c^{\ell}_{2}$ – velocities,

${}_{M}c^{\ell}_{3}$ – accelerations,

${}_{M}c^{\ell}_{4}$ – magnetic forces.

The dynamic representation ${}_{M}\omega^{\ell}$ of the AMB system is constructed in form of Eq. (2.2) by the inputs ${}_{M}X^{\ell}$, states ${}_{M}C^{\ell}$ and outputs ${}_{M}Y^{\ell}$ mentioned above. The dynamic representation at the conceptual stage can be given in $(\overline{\rho}, \overline{\varphi})$, which is transformed into the state-space matrix form at the detailed design stage

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \qquad \mathbf{y} = \mathbf{C}\mathbf{x} \tag{3.2}$$

The first state equation in Eq. (3.2) corresponds to the state transition function $\overline{\varphi}$ in Eq. (2.2), and the second output equation corresponds to the reaction $\overline{\rho}$. Vectors $\mathbf{x}$, $\mathbf{y}$, $\mathbf{u}$ and matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ of the equations are defined by Gosiewski and Mystkowski (2006). Therefore, Eq. (2.2) is the dynamic representation ${}_{M}\omega^{\ell}$ of the AMB system at the stage of conceptual design, and Eq. (3.2) is the AMB model which is used at the detailed design stage of the AMB system life circle (Ulman, 1992).

The AMB system process P^{ℓ} is a part of the higher-level process $P^{\ell+1}$ in the environment ${}_{\omega}S^{\ell}$, i.e. the general control AMB system. This process contains:

P^{ℓ}_{1} – control of the shaft displacement, vibration damping and machine diagnostics (by the AMB system ${}_{M}S^{\ell}$),

P^{ℓ}_{2} – measuring of output values of the AMB system by the measuring and signal conditioning system,

P^{ℓ}_{3} – reading of measured values and converting by the Digital Signal Processor (DSP) or any other real-time digital processor,

P^{ℓ}_{4} – processing and estimating,

P^{ℓ}_{5} – creation of the simulation model and sending it to DSP memory,

P^{ℓ}_{6} – sending control signals to the AMB system in real time,

P^{ℓ}_{7} – AMB system loading realized by the electromotor or generator that causes rotation of the shaft or convertion of the kinetic energy.

P^{ℓ}_{8} – shaft rotation.

P^{ℓ}_{1} and P^{ℓ}_{7} are realized by electromechanical subsystems of the general mechatronic system (general control AMB system), P^{ℓ}_{2}-P^{ℓ}_{6} are realized by the computer subsystem, and P^{ℓ}_{8} by the mechanical one. The general process is composed of sub-processes $\overline{P}^{\ell}$ executed by the general control AMB system, which includes the ABM system ${}_{M}S^{\ell}$ and its environment ${}_{\omega}S^{\ell}$.

So, all the subsystems of the general control AMB system, i.e. mechanical (shaft $S^{\ell-1}_{4}$), electromechanical (AMB system ${}_{M}S^{\ell}$ and motor ${}_{\omega}S^{\ell}_{2}$), computer-electronic (measuring ${}_{\omega}S^{\ell}_{1}$ and control system ${}_{\omega}S^{\ell}_{3}$) have their aggregated dynamic ω^{ℓ} and structural σ^{ℓ} descriptions. All the connected descriptions of the subsystems $\overline{S}^\ell$ and processes $\overline{P}^\ell$ are presented in the informational resources (data bases) of the coordinator which realizes the design process connecting in this way the structure ${}_M\sigma^\ell$ and the functional dynamic realization ${}_M\omega^\ell$ of the AMB system being designed.

The coordinator ${}_MS_0^\ell$ in our case is realized in form of an automated design and control system of the AMB, which maintains its functional modes by the control system and realizes the design process by a higher level computer aided design (CAD) system (general supervisor) if necessary. The AMB control system is designed according to the hierarchical concept and contains low-level and high-level controllers (Fig. 4).

All metrical characteristics of the subsystems and processes described above are presented in form of numeric positional systems L^S (Novikava *et al.*, 1990, 1995, 1997; Miatliuk, 2003; Novikava and Miatliuk, 2007).

3.2. System architecture

The hierarchical system coordination technology allows one to describe active magnetic bearings (AMBs) coupled architecture and its coordination, i.e. design and control (Schweitzer and Maslen, 2009; Miatliuk *et al.*, 2010a). This technology enables one to allocate the inter-subsystems in the AMB structure. In this case, by using a novel approach, the conceptual design of the AMB system is considered as a multilevel model which enables introduction of further necessary changes into AMB construction and technology. This approach supports the design and assembling of AMB parts and can be considered as a self-optimization process. The main AMB model layers reflects AMB mechatronic subsystems, i.e. the mechanical subsystem, electrical subsystem and control software (supervisory intelligence), see Fig. 3. These subsystems can be

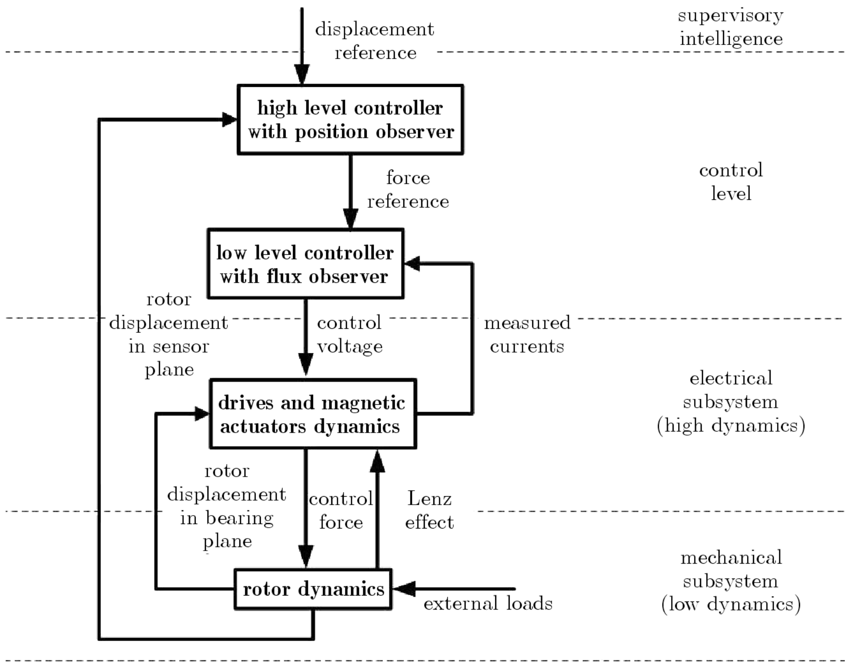

Fig. 3. Structure diagram of the AMB hierarchical system

constructed due to machine demands by selecting parts $\overline{\omega}^{\ell-1}$ and setting their interactions ${}_\sigma U^\ell$, see Eq. (2.3). Thus, the whole design process can be divided into engineering departments according to due knowledge. For example, high dynamics of the electrical AMB subsystem (at a low level) is faster than the mechanical one and requires different controller/actuators/sensors with a suitable bandwidth. Thus, these subsystems should be designed with taking into account their specified performances according to the whole system functional requirements. According to the hierarchical control structure (see Fig. 1), the design technology realization steps are as follows. First, the low level (inner) closed-loop sub-system is designed in which the inner controller provides a fast response of the control loop with respect to the model of the electrical part of the AMB system (Schweitzer and Maslen, 2009). Here, since the electrical subsystem dynamics of the AMB model has uncertainties and consists of nonlinearities, the nonlinear control low is realized with robust controller (Gosiewski and Mystkowski, 2006, 2008). The robust controller overcomes control plant uncertainties and provides a fast response due to variations of the desired signals from the high level controller. Second, the high level control sub-system is designed based on the outer measured signals in the AMB mechanical sub-system. This high level control loop works slower than the inner controller since the dynamics of the AMB mechanical part refers to the significant inertia of AMB position control. The design process is formally presented in form of coordination strategies realized on the selection layer of the coordinator and described by the output functions λ of the coordinator canonical model (φ, λ) (Miatliuk, 2003). The change of coordination strategies in the coordinator learning and self-organization layers is described by the state transition functions φ.

3.3. Control structure

The hierarchical structure of the AMB control system consists of (at least) three layers. The first one (high level) consist of a complex AMB dynamic model (nonlinear) which refers to the concrete plant system. This plant model after simplification is used for controller synthesis and refers to the abstract system S^ℓ, Eq. (3.1). The second layer consists of the low level controller presented in form of the coordinator S_0^ℓ, Eq. (2.4), responding to the low level control task by direct impact on AMB dynamics and it is strongly nonlinear. The low level ℓ control subsystems represent a decentralized (local) control loop based on command signals from the high level $\ell+1$ control system. The last layer represents a high level controller (global) given in from of $S_0^{\ell+1}$ coordinator which performs high order tasks. The main advantage of such approaches is the decoupling of control laws for simpler evaluation by the designing engineers. For such a control structure, the high level controller is not dependent on the nonlinearities located in the low level layer. This enables designing a linear high level controller. However, the refinement of inter-couplings due to the nonlinear nature of this dynamic system is the main challenge. Referring to the two-level control architecture as shown in Fig. 4, the plant S^ℓ behaviour is assumed to be described by the ${}_M\omega^\ell$ model built on the relation of AMB inputs X^ℓ, outputs Y^ℓ and states C^ℓ, see Eq. (2.2). C^ℓ is defined by the control inputs $G^{\ell-1}$ from the low level controller, i.e. the coordinator S_0^ℓ. The measured plant outputs $W^{\ell-1}$ are the feedback from the plant S^ℓ to the low level controller S_0^ℓ. The low level controller S_0^ℓ is directly connected by its input $X_0^\ell = \{G^l, W^{l-1}\}$ and output $Y_0^\ell = \{G^{l-1}, W^1\}$ with the plant model and with the high level controller $S_0^{\ell+1}$ where $\{G^{l-1}, W^{l-1}\}$ and $\{G^l, W^l\}$ are low level and high level signals, respectively. Similarly, the high level controller $S_0^{\ell+1}$ has its inputs $X_0^{\ell+1} = \{G^{l+1}, W^l\}$ and outputs $Y_0^{\ell+1} = \{G^l, W^{1+1}\}$ as well.

Ccontrol signals of the controllers are presented in form of coordinator strategies described by the output functions $\widehat{\lambda}_0^\ell$ of the coordinator canonical models $(\overline{\widehat{\varphi}}, \overline{\widehat{\lambda}})_0^\ell$ (Miatliuk, 2003) built on its inputs, outputs and states as follows

$$\widehat{\lambda}_{0t}^\ell : \quad C_0^\ell \times \widehat{\widetilde{X}}_0^\ell \to \widehat{\widetilde{Y}}_0^\ell \tag{3.3}$$

For instance, the control signal from the low-level $\ell/(\ell-1)$ controller S_0^ℓ to the plant is presented in form of the coordinator S_0^ℓ output function $\widehat{\lambda}_{0t}^{\ell/(\ell-1)}$

$$\overline{\widehat{\lambda}}_0^{\ell/(\ell-1)} = \Big\{\widehat{\lambda}_{0t}^{\ell/(\ell-1)} : \quad \widehat{\widetilde{C}}_0^\ell \times \widetilde{W}^{\ell-1} \to \widehat{\widetilde{G}}^{\ell-1}\Big\} \tag{3.4}$$

where $\widehat{\widetilde{C}}_0^\ell$ is the controller (coordinator) states space.

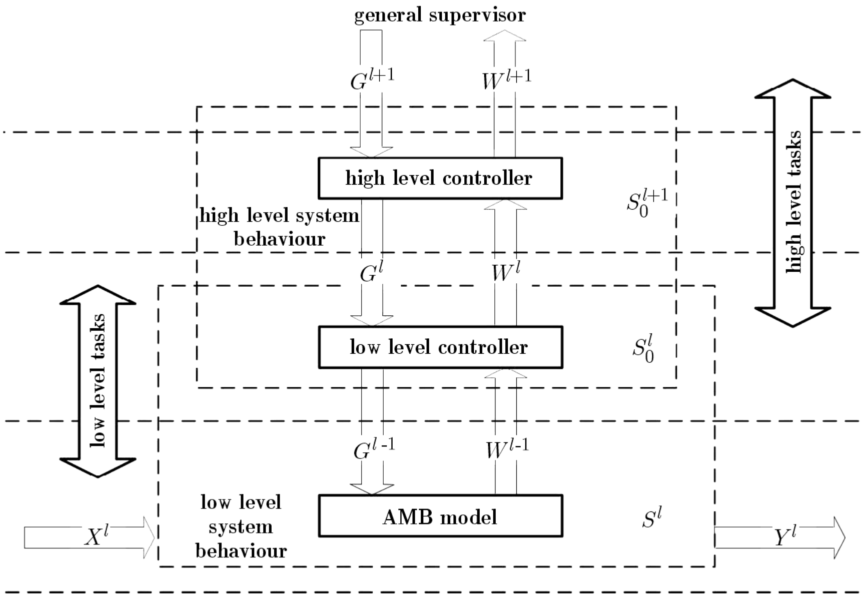

Fig. 4. Hierarchical AMB control architecture

The change of controller states is described by the state transition function $\overline{\widehat{\varphi}}_0^\ell$ of the coordinator canonic model (Miatliuk, 2003)

$$\overline{\widehat{\varphi}}_0^\ell = \{\widehat{\varphi}_{0tt'}^\ell : \quad C_0^\ell \times X_{0tt'}^\ell \to C_0^\ell\} \tag{3.5}$$

For the current (or flux) controlled AMB, the high level controller provides the vector of 4 control currents which after biasing the vector of 8 reference currents (reference forces) are presented by the signals G^ℓ (Fig. 4). The reference forces are provided to the low level control loops. The referenced voltages $G^{\ell-1}$ are input to the drives and actuators of the AMB system. The rotor displacements in the bearing planes ($W^{\ell-1}$) are estimated based on the measured rotor displacements in the sensor planes ($W^{\ell-1}$). They are provided to the low level controller. The desired rotor position is the reference signal of the high level (rotor position) controller and the desired electromagnetic force is the reference signal of the low level (current/flux) controller, respectively.

In order to simplify the design of the control system, the one-degree-of-freedom (1 DOF) AMB dynamic control model (Fig. 4) is considered as the hierarchical system. Its control model is considered as a cascade of two simple systems consisting of high level (electrical) and low level (mechanical) mechatronic subsystems with their coordinators. In this case, the AMB controller structure is coupled to the position and flux feedback, which refers to global and local control loops, respectively. The given conceptual model of the AMB system is concretized at its detailed design stage.

4. Exemplary detailed design of an AMB system

4.1. Simplified AMB model

At the detailed design stage which follows the conceptual one in the AMB system life circle (Ullman, 1992) the simplified 1 DOF (one degree of freedom) AMB model is used. The AMB consists of two opposite and identical magnetic actuators (electromagnets), which are generating the attractive forces F_1 and F_2, on the rotor (Schweitzer and Maslen, 2009). To control the position x of the rotor of mass m to the equilibrium state $x = 0$, the voltage inputs of the electromagnets V_1 and V_2 are used to design the control law, see Fig. 5.

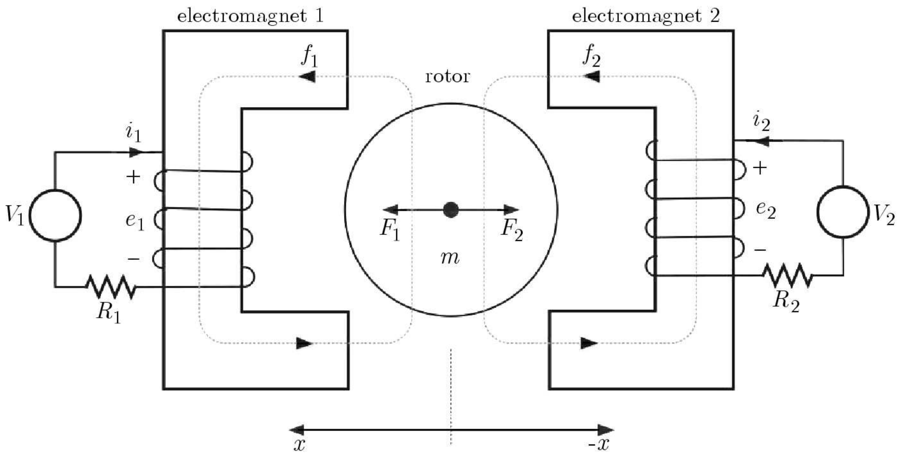

Fig. 5. A simplified one-dimensional AMB (Schweitzer and Maslen, 2009)

The simplified mechatronic model of the AMB is nonlinear and coupled with mechanical and electrical dynamics. Referring to Fig. 5, neglecting gravity, the dynamic equation is given by Schweitzer and Maslen (2009)

$$m\frac{d^2x}{dt^2} = \frac{\Phi|\Phi|}{\mu_0 A} = F(\Phi) \tag{4.1}$$

where Φ is the total magnetic flux through each active coil, A is the cross area of each electromagnet pole and μ_0 is the permeability of vacuum ($4\pi \cdot 10^{-7}$ Vs/Am). Equation (4.1) corresponds to the dynamic representation $(\overline{\rho}, \overline{\varphi})$ given at the AMB conceptual design stage.

The system nonlinearity in Eq. (4.1) is given by the function $\eta(\Phi) = \Phi|\Phi|$, and it is non-decreasing. The total flux generated by the i-th electromagnet is $\Phi_i = \Phi_0 + \phi_i$. In the case of zero-bias operation, the bias flux Φ_0 equals zero and the total flux is equal to the control flux ϕ_i. Then, we define the generalized flux which is given by

$$\phi := \phi_1 - \phi_2 = \frac{1}{N}\Big(\int (V_1 - Ri_1)\, dt - \int (V_2 - Ri_2)\, dt\Big) \qquad i = 1, 2 \tag{4.2}$$

where N is the number of turns of the coil of each electromagnet, V is applied control voltage, and i is current in the electromagnet with resistance R.

4.2. Low level controller

The fast inner controller (low level coordinator S_0^{ℓ}) generates the required fluxes in the AMB structure due to nonlinear characteristics of the controlled flux ϕ versus the generated force F. Since the magnetic flux sensors may complicate significantly the electrical and mechanical structure of the AMB system, a low level flux observer can be applied. The low level observer estimates the flux ϕ based on current measurements in the electrical part of the AMB system. The low level control loop consists of the electrical dynamics of the AMB system. The governing equations for this dynamics are given by Schweitzer and Maslen (2009)

$$\frac{d}{dt}\phi_1 = \frac{1}{N}(V_1 - Ri_1) \qquad \frac{d}{dt}\phi_2 = \frac{1}{N}(V_2 - Ri_2) \tag{4.3}$$

After neglecting the resistance in Eq. (4.3), the electrical dynamics is simplified

$$\dot{\phi}_i = \frac{V_i}{N} \qquad i = 1, 2 \tag{4.4}$$

The low level controller works in the inner flux loop. The reference force signal f_r for the low level flux controller is provided by the high level position controller. Thus, the transform function for the low level control feedback rule in the s-domain

$$G_l(s) = \frac{f_c(s)}{f_r(s)} := \frac{\phi_c(s)}{\phi_r(s)} \tag{4.5}$$

The control force f_c depends on the control flux ϕ_c which fulfils the condition of switching scheme:
— when $\phi_c \geqslant 0$

$$\phi_c = \phi_1 \qquad \phi_2 = 0$$

— when $\phi_c < 0$

$$\phi_c = -\phi_2 \qquad \phi_1 = 0$$

The low level control law $u_\phi = -f_\phi(\phi_r - \phi_c)$, where f_ϕ is a nonlinear control function which also ensures the bounds of ϕ_i, i.e. $\lim_{t\to\infty} \phi_i(t) = \min\{\phi_1(0), \phi_2(0)\}$.

Equations (4.3)-(4.5) correspond to the dynamic representation $(\widehat{\varphi}, \widehat{\lambda})_0^\ell$ of the low level coordinator S_0^ℓ given at the AMB conceptual design stage.

4.3. High level controller

Now, with respect to the outer controller (high level coordinator $S_0^{\ell+1}$), since the AMB model from the force f to the position x is linear, no linearization is needed and, therefore, the position control law can be linear. Moreover, the high level controller is not coupled with the low level control loop. The high level control loop provides the reference force f_r and consists the mechanical dynamics of the AMB system. The high level position feedback control rule in s-domain is based on the measured rotor displacement x_{mat} at the magnetic bearing plane and the referenced displacement x_r

$$G_h(s) = \frac{x_m(s)}{x_r(s)} \tag{4.6}$$

where the displacement x_m is estimated (by the linear high level position observer) based on the measured mass displacement x.

In order to provide the equilibrium state of dynamics Eq. (4.1) the time derivatives in Eq. (4.1) go to zero

$$\frac{d^2x}{dt^2} = \frac{\Phi|\Phi|}{\mu_0 m A} \to 0 \tag{4.7}$$

If the static gain of the control loop of G_h is defined as the state feedback controller (static gain matrix $\mathbf{K}$), then

$$\lim_{s\to 0} G_h = K \qquad \text{when} \qquad \frac{d^2x}{dt^2} \to 0 \tag{4.8}$$

Therefore, Eqs. (4.1)-(4.8) present detailed design models of the AMB system and its controllers. Equation (4.1) corresponds to the dynamic model $(\overline{\rho}, \overline{\varphi})$ of the AMB given at the AMB conceptual design stage, and Eqs. (4.3)-(4.5) and Eqs. (4.6)-(4.8) correspond to the dynamic models of the low-level and high-level controllers, respectively.

5. Conclusions

The realization of the coordination technology for AMB mechatronic systems (design and control) in the formal basis of hierarchical systems is briefly given in the paper. In comparison with traditional methods of mathematics and artificial intelligence, the proposed formal model contains connected descriptions of the designed object structure, its aggregated dynamic representation as a unit in its environment, the environment model and the control system. All the descriptions are connected by the coordinator which performs the design and control tasks on its strata. Besides, the proposed *aed* technology coheres with traditional systems of information presentation in mechatronics: numeric, graphic and natural language forms (Novikava and Miatliuk, 2007). The technology is also coordinated with general requirements of the design and control systems (Novikava *et al.*, 1990, 1995) as it considers mechatronic subsystems of different nature (mechanical, electromechanical, electronic, computer) in common theoretical basis.

The presentation of the AMB system in the formal basis of HS allows creation of the AMB conceptual model necessary for its transition to concrete mathematical models used at the detailed design stage of the AMB. At the detailed design stage, the low level and high level control loops of the AMB control structure are introduced. Each sub-system consists of the controller and observer structures which provide reference signals to each other. In this approach, the high level control loop is not dependent on the low level one. Thus, the magnetic force field nonlinearities in the low level sub-subsystem are not dependent on the high level position control loop. In the proposed approach, the electromagnetic nonlinearities are shifted from the high level control loop into the low level control loop. At the detailed design stage, the AMB (control) subsystems are described by traditional DE. At the conceptual design stage, the subsystems are presented in form of (ρ, φ) which are generalizations of DE and algebra systems. So, the transition from the conceptual to the detailed design stage in frames of the proposed technology is convenient and requires concretisation of the abstract dynamic system only.

The given technology brings new informational means for the conceptual and detailed design of mechatronic systems and AMB systems in particular. The described *aed* technology has been also applied to the design and control of other engineering objects (Miatliuk and Siemieniako, 2005; Miatliuk *et al.*, 2006; Miatliuk and Diaz-Cabrera, 2013), in biomechanics (Miatliuk *et al.*, 2009a,b) and mechatronics (Miatliuk *et al.*, 2010a; Miatliuk and Kim, 2013).

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