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FINITE-AMPLITUDE ACOUSTIC WAVES IN A LIQUID-FILLED TUBE

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The paper is focussed on description finite-amplitude waves in fluid-filled in tubes. The model equations for study of nonlinear waves are presented here. The used model equations take into account boundary layer effects which cause not only losses but also dispersion of waves.

INTRODUCTION

The analyze of nonlinear sound waves in fluid-filled waveguides represents interesting problem of physical acoustics particularly from the point of view parametric amplification and generation of sound. This task was solved by many authors but a lot of them was focussed on study nonlinear waves in waveguides which are not axisymmetric. On the base of results presented in many works it is obvious that it is necessary to take into account dispersion effects which are caused by the presence of lateral boundaries and a boundary layer.

MODEL EQUATIONS

Before writing the basic equations of "waveguide nonlinear acoustics" let us clarify, using elementary examples, the nature of the mode synchronism. Consider a re-reflection of a plane wave between rigid boundaries. On reflection from such boundary the wave phase does not change, and propagation along a broken line between the boundaries takes place in a way very similar to that along an appropriate straight path, the only difference being that the effective velocity of the field energy propagation along the waveguide axis x (group velocity) turns out to be smaller than the sound velocity c_0. In the case of a finite amplitude wave the harmonic generation, discontinuity formation, etc., are essentially the same as in a plane wave (the interaction of noncollinearly propagating waves, i. e. the incident and reflected waves, proves insignificant within the quadratic approximation). In this sense a waveguide with rigid walls has no meaningful distinction from free space.
However, in the case of high-frequency nonlinear waves we can observe the jagged wave

shape in the shock region which is caused by higher-modes excited by the harmonics above the cutoff frequency of the first mode.
We can use for description nonlinear waves in tubes the following model equations:

The modified Burgers equation

$$\frac{\partial v}{\partial x} - \frac{\beta}{c_0^2} v \frac{\partial v}{\partial \tau} + \sqrt{2} K \frac{\partial^{\frac{1}{2}} v}{\partial \tau^{\frac{1}{2}}} = \frac{b}{2c_0^3} \frac{\partial^2 v}{\partial \tau^2}, \tag{1}$$

where v is the particle velocity, $\tau = t - x/c_0$ is the retarded time, t is time, x is distance from a source, ρ_0 is density, $b = 4/3\eta' + \eta'' + \kappa(1/c_p - 1/c_V)$ is dissipative coefficient, η', η'' are the shear and bulk viscosity coefficients, κ is the heat-conductivity coefficient, c_p and c_V are the specific heats at constant pressure and volume, $\beta = (\gamma + 1)/2$ is the coefficient of nonlinearity for gases, $\gamma = c_p/c_V$, $K = \sqrt{2\nu}[1 + (\gamma - 1)/\sqrt{Pr}]/2c_0 R_0$, $\nu = \eta'/\rho_0$, Pr is the Prandtl's number, R_0 is inner diameter of the tube. The suffix "0" is used to indicate equilibrium values.
The fractional derivate represents following the integrodifferential operator for the acoustic boundary layer (e.g. see [1]):

$$\frac{\partial^{\frac{1}{2}} v}{\partial \tau^{\frac{1}{2}}} = \sqrt{\frac{1}{\pi}} \int_{-\infty}^{\tau} \frac{\partial v(\tau', x, r = R_0)}{\partial \tau'} \frac{\mathrm{d}\tau'}{\sqrt{\tau - \tau'}}, \tag{2}$$

where r is the radial spatial variable.
The boundary condition:

$$v(0, t) = v_m \sin(\omega \tau). \tag{3}$$

The Khokhlov-Zabolotskaya-Kuznetzov (KZK) equation (see [2])

$$\frac{\partial}{\partial \tau}\left(\frac{\partial v}{\partial x} - \frac{\beta}{c_0^2} v_x \frac{\partial v}{\partial \tau}\right) - \frac{c_0}{2}\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r}\right) = \frac{b}{2\rho_0 c_0^3} \frac{\partial^3 v}{\partial \tau^3}, \tag{4}$$

$$c_0 \frac{\partial v}{\partial r} + \frac{\partial w}{\partial \tau} = 0, \tag{5}$$

where v and w are the axial and radial velocity, respectively.
The boundary conditions:

$$v(0, r, t) = v_m(r) \sin(\omega \tau). \tag{6}$$

$$\begin{aligned} \frac{\partial v}{\partial r} &= -\frac{R_0 K}{c_0} \sqrt{\frac{2}{\pi}} \frac{\partial}{\partial \tau} \int_{-\infty}^{\tau} \frac{\partial v(\tau', x, r = R_0)}{\partial \tau'} \frac{\mathrm{d}\tau'}{\sqrt{\tau - \tau'}} = \\ &= -\sqrt{2} \frac{R_0 K}{c_0} \frac{\partial^{\frac{3}{2}} v}{\partial \tau^{\frac{3}{2}}}; \quad r = R_0. \end{aligned} \tag{7}$$

The following conditions must be satisfied for the Stokes boundary layer thickness $\delta = \sqrt{2\nu/\omega}$ in order the model equations (1) and (4) could be used:

$$\delta/R_0 \approx \mu << 1 \quad \text{and} \quad \delta/\lambda \approx \sqrt{\mu} << 1, \tag{8}$$

where μ is a small parameter that is equal in nonlinear acoustics to the peak Mach number of the source and λ is the wavelength.

In the case of the model equation (1) it is supposed that the main harmonics of a wave must be below the cutoff frequency of the first mode. We can obtain from the condition for the axisymmetrical transversal modes

$$\left(\frac{\partial \mathrm{J}_1(nkr)}{\partial r}\right)_{r=R_0} = 0$$

the cutoff frequency as

$$nkR_0 < 1,841\ . \tag{9}$$

We can suppose higher harmonics can be so small in the shock region that the condition from (8) has not to be satisfied. However, in this case the dominant loss mechanism shifts from wall losses to Navier-Stokes losses, increasing as the square of the frequency. At high frequencies, boundary layer dispersive effects would still remain, but should be small enough to allow the approximation of phase speed by c_0. Then, the requirement on the ratio of boundary layer thickness to wavelength would become less important because of the unimportance of the boundary layer effects.

The exact solution of given equations is not possible in this moment. There are a large number of approximate solutions. For a brief analysis it is suitable to use, in the case of sinusoidal waves, the approximate solution presented in [1]. If we need to appreciate all effects included in the equations, we must use the numerical solution.

The widely used method is the solution of these equations in the frequency domain. When the source is periodic in time, it is possible to find the solution in the form of Fourier series

$$v(\tau, x, r) = \sum_{n=1}^{\infty}(g_n(x,r)\sin(n\tau)) + h_n(x,r)\cos(n\tau) \tag{10}$$

In order to do numerical integration, the infinite series (10) has to be truncated. This means that the flow of energy from lower to higher harmonics stops with the last harmonic retained in the series. So the calculated value for this last harmonic cab become large in comparison to the lower harmonics. Then one can observe the energy transfer also from higher to lower harmonics. In some cases, this can cause the numerical instability. We can circumvent this problem when we filter all harmonics by numerical filter, which guarantees that the amplitude of any harmonics never exceeds the amplitude of the next lower harmonics.

The stability of numerical solution is also dependent on the absorption term. Exploring ideal liquids it is necessary to add an fictitious attenuation term. The solution becomes unstable with increasing Goldberg number to values that are greater than 100 -150 when the integration exceeds the discontinuity distance.

It is possible to solve the Burgers equation in the frequency domain by means of the standard Runge-Kutta method of fourth order. But this equation does not give any information about changes of velocity on the radius of the tube.

Widely used method of numerical solution of KZK equation is described in [3] (Bergen Code). But this method is for solution of sound beam propagation in the free space. To solve the case of a tube it is necessary to add an boundary condition which takes into account the boundary layer which takes into account the reflections of sound waves from the duct walls and dispersion, which does not arise in the free space propagation

CONCLUSION

The numerical analysis shows that it is necessary to take into account also the members which represent the boundary layer effect in model equations. Therefore the standard Burgers equation can be used only in the case of the negligible boundary layer effects. The boundary layer significantly can influence the form of sound waves, especially due to the dispersion. The dispersion make the asymmetry of the primary symmetrical waves; the peak is rounded while the trough remains sharp. The effects of dispersion grow stronger with distance: the harmonics get more and more out of step with each other.
Experimental and numerical results show that it is necessary to use for description nonlinear waves in tubes the model equations which take into account boundary layer effects.

REFERENCES

[1] Bednařík, M.: *Description of nonlinear waves in gas-filled tubes by the Burgers and the KZK equations with a fractional derivative*, Proc. of the 16th ICA, p. 2307-2308, Seattle (1998).

[2] Zhileikin, Ya. M., Zhuravleva, T. M., Rudenko, O. V., *Nonlinear effects in the propagation of high-frequency sound waves in tubes*, Sov. Phys. Acoust., 26, 32-34 (1980).

[3] S. I. Aanonsen, T. Barkve, J. Naze Tjøta, S. Tjøta: *Distortion and harmonic generation in the nearfield of a finite amplitude sound beam.*, J. Acoust. Soc. Am. 75, 749-768 (1984).

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