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### DETERMINING THE OPTIMUM CUT-OFF GRADES IN SULFIDE COPPER DEPOSITS

#### OKREŚLANIE OPTYMALNEJ WARTOŚCI ODCIĘCIA ZAWARTOŚCI PROCENTOWEJ PIERWIASTKA UŻYTECZNEGO W ZŁOŻACH SIARCZKU MIEDZI

Optimum cut-off grades determination in mining life affects production planning and ultimate pit limit and it is also important from social, economical and environmental aspects. Calculation of optimum cut-off grades has been less considered for mines containing various mineral processing methods. In this paper, an optimization technique is applied to obtain optimum cut-off grades for both concentration and heap leaching processes. In this technique, production costs and different recoveries of heap leaching method directed into modeling different annual cash flows in copper mines. Considering the governing constraints, the Lagrange multiplier method is practiced to optimize the cut-off grades in which the objective function is supposed to maximize Net Present Value. The results indicate the effect of heap leaching process on the optimum cut-off grades of primary and secondary sulfide deposits.

**Keywords:** optimum cut-off grades, heap leaching, concentration, sulfide deposits

Określanie optymalnego poziomu odcięcia dla zawartości procentowej pierwiastka użytecznego ma poważny wpływ na planowanie produkcji, określanie ostatecznych limitów zasobów złoża; jest to także ważna kwestia z punktu widzenia kwestii społecznych, ekonomicznych i środowiskowych. Obliczanie optymalnego poziomu odcięcia dla zawartości procentowej pierwiastka użytecznego nie było zwykle szeroko rozważane w przypadku kopalni prowadzących ciągły system przeróbki. W pracy tej przedstawiono technikę optymalizacji określania poziomu zawartości procentowej pierwiastka użytecznego z uwzględnieniem zarówno procesów koncentracji jak i ługowania. W metodzie uwzględniono koszty produkcji i różne wskaźniki odzysku rudy, wielkości te wykorzystane zostały do modelowania rocznych przepływów gotówki w kopalniach miedzi. Uwzględniając narzucone ograniczenia, zastosowano metodę mnożników Lagrange'a w celu optymalizacji określania poziomu zawartości procentowej pierwiastka użytecznego, gdzie przyjętą funkcją celu jest maksymalizacja wartości bieżącej netto. Wyniki wskazują

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wpływ procesów ługowania na zawartość procentową pierwiastka użytecznego w rudzie pochodzącej z pierwotnych lub wtórnych złóż siarczku miedzi.

**Słowa kluczowe:** określanie optymalnego poziomu odcięcia dla zawartości pierwiastka użytecznego w rudzie, ługowanie, koncentracja, złoża siarczku miedzi

## 1. Introduction

Lane was among the first pioneers who tried to determine the optimum cut-off grades (Lane, 1964). He reconsidered optimal cut-off grades in correspondence with opportunity costs being oriented to maximize Net Present Value (Lane, 1988). Moreover, in regard to aforementioned methodology, having applied an iterative algorithm, he measured optimum mining cut-off grades. The algorithm proposed by Lane almost was one of the complete models with regard to optimum cut-off grades determination. However, one of the deficits of his methods is contemplating only the concentration processing method which couldn't consider hydrometallurgical methods.

Lane defined Net Present Value maximization as the objective function. NPV maximization is not the only objective of the cut-off grades determination and other target function can be used as Internal Rate of Return (Rendu, 2009) and Output Rates (Khodayari et al., 2012.).

Ever since, there has been countless efforts to determination of optimum cut-off grades which mostly focused on facilitating and developing optimum cut-off grades calculation methods (Asad, 2007; Johnson et al., 2011; Rashidinejad et al., 2008b). In the other hand, mineral processing methods play a key role in the optimum cut-off grades policy as the mining operation dominated by several processing methods. The works, which was done by Asad et al. (2011), Osanloo et al. (2008), Rashidinejad, et al. (2008a), Bascetin et al. (2007) and He, et al. (2009), completed the optimum cut-off grades determination model but, the effects of mineral processing methods were not considered on their models.

Some of the researchers have calculated optimum cut-off grades by several processing methods and multi metal deposits (Ataei et al., 2003; Davey, 1979). The cut-off grade between two processing methods is calculated by using a relation of recovery-grade. Utilizing mineral processing terminology, fixed tails limit is defined in order to determine mined materials destinations. Considering material destinations, the waste dump- Leach dump cut-off grade is a certain amount and no copper recovery is expected when the copper content is less than it (Davey, 1979). Obviously, this fixed grade couldn't lead to maximum NPV of mining operation. From another point of view, using grade- recovery relation needs more experimental works in regard to complexity of hydrometallurgical processes.

Osanloo, et al. (2003) employed equivalent grade factor to calculate optimum cut-off grades of multi metal deposits. To demonstrate the application, they considered a copper mine which involves copper and molybdenum minerals. The final copper and molybdenum products, extracted from copper concentrate. They didn't consider different methods of copper processing. As a whole, different mineral processing methods have been less considered in calculating optimum cut-off grades.

Different types of hydrometallurgical methods are remarkably practiced in mineral processing for the sake of their several advantages. Some of the characteristic of these methods are: low investment and operation costs (Dreisinger, 2006; Watling, 2006), environmentally friendly (Simpson et al., 1998; Wu et al., 2008; Rahimi et al., 2014), less sensitivity to grade variations and mineralogy (Dopson et al., 2003), capacity expansion (Rawlings et al., 2003), uncomplicated

process (Watling, 2006), material reclamation (Wu et al., 2008) and advanced degree of natural liberation of the copper content (Habashi, 1999).

The general principle of copper hydrometallurgical methods includes a solvent which makes it possible to solve a valuable mineral from crushed ores (Davenport et al., 2002). In fact, choosing a suitable solvent and adjusting it with ore conditions and metal content should be considered (Habashi, 1999).

Heap leaching is one of the most high-ranking and applicable hydrometallurgical methods extracting copper from its ores which is mainly employed for any types of oxide ores and copper sulfides in industrial and semi-industrial ranges (Brierley et al., 2001; Renman et al., 2006; Sukla et al., 2009). It is very significant to fathom the mechanisms of heap leaching and also their influence on economy (Padilla et al., 2008). Hence, a number of experimental studies have been carried out (Bartlett, 1997; Bouffard et al., 2001; Dixon, 2000) because it is subjected into precisely choosing cut-off grades of copper mines.

The average grade of minerals sent to mineral processing plants has been remarkably decreased by extracting high grade ores. Furthermore, the conventional copper processing methods are technically more adjusted with high grade ores (Bartlett, 1997). So, it seems necessary to apply other methods to low grade ores processing. Different production capacities and economical parameters of such methods make the NPV of mines change (Rendu, 2008). As a result, it is significant to adopt a complete and optimum policy to determine cut-off grades in order to access the NPV maximization of these projects. In this study, an optimum cut-off grades modeling of heap leaching and concentration methods are presented and in the next step, a constrained optimization method is employed to compute the optimum cut-off grades by remarking different governing constraints.

## 2. Methodology

The notable characteristics of hydrometallurgical methods are their huge variety and vast applications which sometimes are more noticeable than low operating costs of these methods. Consequently, these methods have been facilitated to apply in all parts of copper industries by these characteristics.

Heap leaching is recognized as one of the simplest applicable technologies with lowest costs of hydrometallurgical methods which practiced in copper industry so far. Since this method has specific benefits, it is often chosen by copper large mines owners for oxide and low grade sulfide ores treatment. In the other hand, the heap leaching method is recommended because the concentration method is less successful in copper oxide ores. For the sake of shorter payback period, the small mines owners think of this method for the first place. Designing and engineering of this method is mostly relied on the characteristics of input ores to the heap. As a matter of fact, simple changes of ore mineralogy can make general changes in nature of process. So, it is necessary to plan an extensive mineralogical and metallurgical tests program before selecting a mineral processing method and even before investment on mines exploitation.

One of the main concerns of heap leaching method is its low recovery in comparison to concentration method. Thus, this method is usually recommended for copper low grade ores treatment. From another point of view, the conventional concentration method does not achieve high recovery in low grade copper ores. Hence, in the studies, it is assumed that the average grade of heap leaching method is less than concentration method.

Fig. 1 illustrates flowchart to determine various material destinations based on ore type and mineralogy. It can also be contemplated in order to optimum cut-off grades modeling.

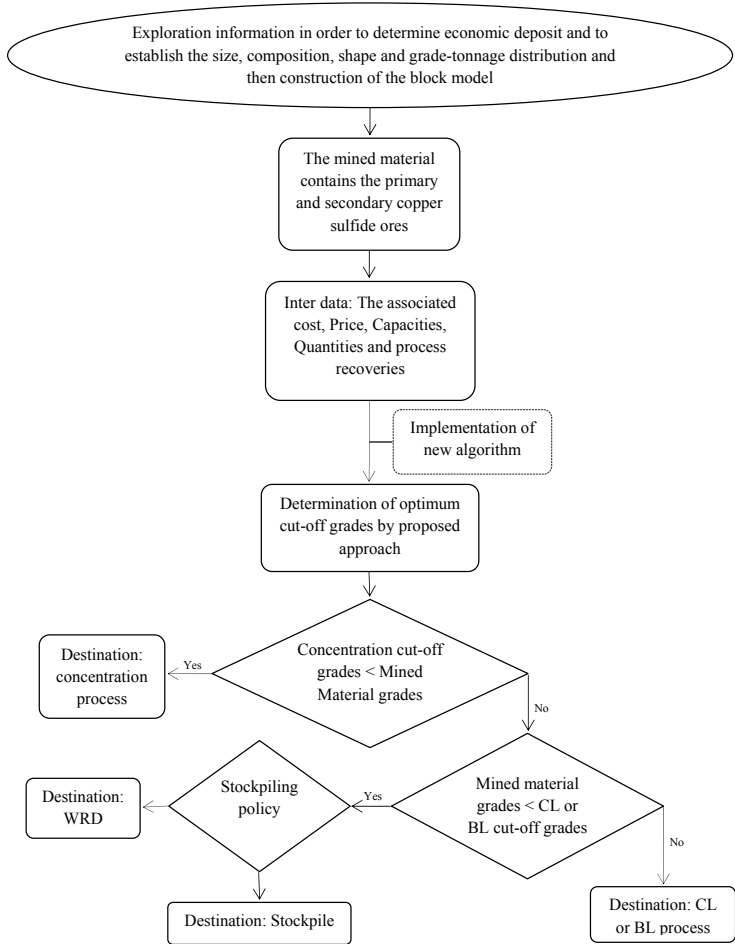


Fig. 1. Flowchart for finding optimum cut-off grades and material mined destinations (CL: Chemical Leaching, BL: Bacteria Leaching, WRD: Waste Rock Dump)

The flowchart which is shown in Fig. 1, indicates a copper mine embracing primary and secondary<sup>1</sup> copper sulfide ores. As follows, there are some options to ascertain material destinations. In this case, to exactly determine optimum cut-off grades, an economical model with an objective function is required. In the current essay, the mined material destinations are assessed

<sup>1</sup> Chalcopyrite (CuFeS<sub>2</sub>) is by far the biggest copper source. Chalcopyrite and Bornite (Cu<sub>5</sub>FeS<sub>4</sub>) is considered as the primary sulfide minerals which is generally treated by Pyrometallurgical methods (Davenport, et al., 2002). Chalcocite (Cu<sub>2</sub>S) and Covellite (CuS) are prevalent secondary sulfide minerals that are easily leached with sulfuric acid if an oxidant is present (Bartlett, 1992). Secondary copper sulfide minerals are also treated by pyrometallurgical methods.

and a model shall be provided to maximize the NPV of future cash flows. In this model, the Net Present Value can be calculated from expected future cash flows by using production and economical parameters. In the other hand, the model introduced for the primary and secondary copper sulfide ores is simply applicable for another copper processing methods including oxide heap leaching or copper sulfide concentration. This analytical model contains an optimization of multi-constraints and variables function. In addition, mine and plants production capacity limitations are considered for this model. The optimum amounts of variables are calculated through Lagrange multiplier constrained method in order to maximize the NPV.

### 3. Model construction

Calculating maximum net present value needs annual cash flows modeling based on mine conditions. It is essential to determine average ore grades and total ore tonnage sent to different mineral processing plants in order to model annual cash flows. Although average grades calculation needs grade-tonnage distribution, the completed model includes opportunity costs and other economical parameters. To complete the model, limitations of the mine and plants capacities are considered. Table 1, describes the parameters practiced in the model.

TABLE 1

Exercised parameters for model construction and their units

Parameters	Notation	Unit	Parameters	Notation	Unit
1	2	3	4	5	6
The length of production period	$T$	year	Average grade of material sent to concentration plant	$\bar{a}_C$	%
Discount Rate	$\delta$	%	Average grade of material sent to heap	$\bar{a}_H$	%
Concentrated Ore	$\sigma_c$	Ton	Pyrometallurgical refined material	$\sigma_s$	$T_p$
Year indicator	$i$	–	Concentration recovery	$\eta_C$	%
Maximum Ore Grade	$G$	%	Smelting and electrorefining recovery	$\eta_S$	%
Concentration* Cut-off grade	$g^c$	%	Pyrometallurgical refining Capacity	$Cap_S$	$T_p/\text{Year}$
Material tonnage	$q$	Ton	Hydrometallurgical refined materials	$\sigma_X$	$T_p$
Grade category indicator	$\zeta$	–	Heap leaching recovery	$\eta_H$	%
Concentration Capacity	$Cap_C$	$T_o/\text{Year}$	Solvent Extraction and electrowinning recovery	$\eta_X$	%
Leached Ore	$\sigma_H$	$T_o$	Hydrometallurgical refining Capacity	$Cap_X$	$T_p/\text{Year}$
Heap leaching Cut-off Grade	$g^h$	%	Commodity price	$P$	$\$/T_p$
Heap leaching Capacity	$Cap_H$	$T_o/\text{Year}$	Smelting and electrorefining cost	$s$	$\$/T_p$

\* Concentration cut-off grades are also named in some references as Milling Cut-off grades.

1	2	3	4	5	6
Material mined	$\sigma_E$	Tons	Solvent Extraction and electrowinning cost	$x$	$\$/T_p$
Dumped materials	$\gamma(W, S)$	Tons	Concentrating cost	$c$	$\$/T_o$
Mining capacity	$Cap_\varepsilon$	Ton/Year	Heap leaching cost	$h$	$\$/T_o$
Fixed cost	$f$	$\$/T_m$	Mining cost	$e$	$\$/T_m$

$T_p$ : Ton of product,  $T_o$ : Ton of ore,  $T_m$ : Ton of material,  $W$ : Waste dump,  $S$ : Stockpile

The overall NPV is obtained from the equation below in copper mines as continuous series of cash flow.

$$NPV = \int_{t_o}^{T_{life}} Cash\ Flow \cdot e^{-\hat{c}t} dt \tag{1}$$

That  $t_o$  is the time of operation beginning. The tonnage of annual extracted materials is calculated by the following relation along with grade – tonnage curve.

$$\sigma_{Ei}(g) = \int_{g^h}^{g^c} q_i(g) dg + \int_{g^c}^G q_i(g) dg + \gamma_i(W, S) \leq Cap_\varepsilon \tag{2}$$

$g$  is the grade indicator. Sometimes, the net present value can be in the discrete time approximation form with grade-tonnage table which is indicated in equation 3.

$$\begin{aligned} \sigma_{Ei}(g) &= \left( \frac{g_{up}^{\zeta(g^c)} - g^c}{g_{up}^{\zeta(g^c)} - g_{down}^{\zeta(g^c)}} \right) \times q_i^{\zeta(g^c)} + \sum_{\zeta=\zeta_g^{c+1}}^{\zeta_G} q_i^\zeta + \sum_{\zeta=\zeta_g^{h+1}}^{\zeta=\zeta_g^{c-1}} q_i^\zeta \\ &+ \left( \frac{g_{up}^{\zeta(g^h)} - g^h}{g_{up}^{\zeta(g^h)} - g_{down}^{\zeta(g^h)}} \right) \times q_i^{\zeta(g^h)} + \left( \frac{g^c - g_{down}^{\zeta(g^c)}}{g_{up}^{\zeta(g^c)} - g_{down}^{\zeta(g^c)}} \right) \times q_i^{\zeta(g^c)} + \gamma_i(W, S) \\ &= \sum_{\zeta=e_g^{h+1}}^{\zeta_G} q_i^\zeta + \left( \frac{g_{up}^{\zeta(g^h)} - g^h}{g_{up}^{\zeta(g^h)} - g_{down}^{\zeta(g^h)}} \right) \times q_i^{\zeta(g^h)} + \gamma_i(W, S) \leq Cap_\varepsilon \end{aligned} \tag{3}$$

$g_{up}$  and  $g_{down}$  show respectively the upper and lower limit of the grade category in grade- tonnage table.

The amounts of annual marketable products from concentration and heap leaching methods are calculated by the following relations.

$$\sigma_{Si}(g) = \eta_C \eta_S \int_{g^c}^G g \times q_i(g) dg \quad \text{and} \quad \sigma_{Xi}(g) = \eta_H \eta_X \int_{g^h}^{g^c} g \times q_i(g) dg \tag{4}$$

That in relation 4:

$$\sigma_{Si}(g) \leq Cap_S, \quad \sigma_{Xi}(g) \leq Cap_X \tag{5}$$

In these relations, the Pyrometallurgical process recovery is calculated through the recoveries of flotation, smelting and refinery processes. In addition, leaching recovery is obtained from heap, Solvent Extraction (SX) and Electrowinning (EW) recoveries. In the other hand, the average grades of material sent to leaching and concentration plant can be obtained from relation 6.

$$\bar{a}_C(g) = \frac{\int_{g^c}^G g \times q_i(g) dg}{\int_{g^c}^G q_i(g) dg} \quad \text{and} \quad \bar{a}_H(g) = \frac{\int_{g^h}^{g^c} g \times q_i(g) dg}{\int_{g^h}^{g^c} q_i(g) dg} \quad (6)$$

And in the form of discrete time approximation with grade-tonnage table:

$$\bar{a}_C(g) = \frac{1}{2} \left[ \left( \frac{\left( g_{up}^{\zeta(g^c)} \right)^2 - \left( g^c \right)^2}{g_{up}^{\zeta(g^c)} - g_{down}^{\zeta(g^c)}} \right) \times q_i^{\zeta(g^c)} + \sum_{\zeta=\zeta_g^{c+1}}^{\zeta_G} \left( g_{up}^{\zeta} + g_{down}^{\zeta} \right) q_i^{\zeta} \right] \quad (7)$$

and

$$\bar{a}_H(g) = \frac{1}{2} \left[ \left( \frac{\left( g_{up}^{\zeta(g^h)} \right)^2 - \left( g^h \right)^2}{g_{up}^{\zeta(g^h)} - g_{down}^{\zeta(g^h)}} \right) \times q_i^{\zeta(g^h)} + \sum_{\zeta=\zeta_g^{h+1}}^{\zeta_g^{c-1}} \left( g_{up}^{\zeta} + g_{down}^{\zeta} \right) q_i^{\zeta} + \left( \frac{\left( g^c \right)^2 - g_{down}^{\zeta(g^c)}}{g_{up}^{\zeta(g^c)} - g_{down}^{\zeta(g^c)}} \right) q_i^{\zeta(g^c)} \right] \quad (8)$$

Finally, the copper price, costs and the relations 2, 4, 6 are added to the relation 1 and then with considering grade-tonnage curve, the overall NPV of continues series of cash flows is obtained as equation 9:

$$NPV = \max : \left[ \int_{i=t_0}^{T_{life}} \int_{g=g^c}^G (P_i - s_i) \eta_C \eta_S \cdot g \cdot q_i(g) \cdot e^{-\hat{\alpha}i} dg di + \int_{i=t_0}^{T_{life}} \int_{g=g^h}^{g^c} (P_i - x_i) \eta_H \eta_X \cdot g \cdot q_i(g) \cdot e^{-\hat{\alpha}i} dg di - \int_{i=t_0}^{T_{life}} \int_{g^c}^G c_i \cdot q_i(g) \cdot e^{-\hat{\alpha}i} dg di - \int_{i=t_0}^{T_{life}} \int_{g^h}^{g^c} h_i \cdot q_i(g) \cdot e^{-\hat{\alpha}i} dg di - \int_{i=t_0}^{T_{life}} \int_{g^c}^{g^c} e_i \cdot q_i(g) \cdot e^{-\hat{\alpha}i} dg di - \int_{i=t_0}^{T_{life}} \int_{g^c}^G e_i \cdot q_i(g) \cdot e^{-\hat{\alpha}i} dg di - \int_{i=t_0}^{T_{life}} \int_{g^c}^G e_i \gamma_i(W, S) \cdot e^{-\hat{\alpha}i} di - \int_{i=t_0}^{T_{life}} \int_{g^c}^G (f_i + \partial \cdot NPV_i) \cdot e^{-\hat{\alpha}i} di \right] \quad (9)$$

Hence, sum of the discounted yearly cash flows made during mining life should be calculated in order to maximize the overall NPV. This maximization process is done by Lagrange multiplier method.

The lagrangian function is written as:

$$\begin{aligned} \mathcal{L} = (g^h, g^c, \lambda) = & \int_{g=g^c}^G (P_i - s_i) \eta_C \eta_S \cdot g \cdot q_i(g) dg + \\ & \int_{g=g^h}^{g^c} (P_i - x_i) \eta_H \eta_X \cdot g \cdot q_i(g) dg - \int_{g=g^c}^G c_i \cdot q_i(g) dg - \int_{g=g^h}^{g^c} h_i \cdot q_i(g) dg - \\ & \left( \int_{g=g^h}^{g^c} e_i \cdot q_i(g) dg + \int_{g=g^c}^G e_i \cdot q_i(g) dg + e_i \gamma_i(W, S) \right) - \\ & (f_i + \partial \cdot NPV_i) \cdot T + \sum_{k=1}^m \ddot{e}_k T_{k(g)} \end{aligned} \quad (10)$$

Where,  $\lambda$  is as lagrangian multiplier,  $\mathcal{L}$ : Lagrangian indicator,  $m$ : the number of constraint,  $k$ : The constraint number indicator. In the cash flow lagrangian function, the length of production period (Mining, concentration, heap, SX and EW, Smelting and electrorefining) is shown by  $T$ . Remarking the limiting of the capacity of an individual stage in the annual Cash Flows is resulted into limiting cut-off grades. In this method, five different limitations are defined. 1 – Mining production capacity due to limitation of hauling and loading equipment, drilling and blasting, mine design, dewatering and, etc. 2 – Production capacity of concentration plant due to limitation of grinding and milling capacities, flotation cells capacity, thickener capacity, tailing dam and, etc. 3 – Smelting, refinery and marketing limitations due to limitation of dryer capacity, furnaces, casting wheel capacity, electrolyzes cells capacity, Acid plant capacity and Electrostatic Precipitator (ESP), and etc. 4 – Heap capacity due to limitation of hauling and loading equipment, bacterial culture limitation, heap stability, Ionic Liquid Solution (ILS), Pregnant Liquid Solution (PLS) and raffinate ponds capacities and, etc. 5 – SX and EW capacities limitation, marketing limitations due to limitation of agitator capacity, settlers, extractor, Flotation cells, EW cells, power consumption and, etc. The Times “ $T$ ” is controlled by the mining rate and plants production capacity.

Based on the described limitations and using lagrangian method, limiting cut-off grades is obtained according to the following relations.

Likewise, if mine throughput is the governing limitation, the optimum cut-off grade is indicated as relation 11.

$$\text{Exploitation limitation: } \begin{cases} g_E^h = \frac{h_i}{\eta_H \eta_X (P_i - x_i)} \\ g_E^c = \frac{c_i - h_i}{(P_i - s_i) \eta_C \eta_S - \eta_H \eta_X (P_i - x_i)} \end{cases} \quad (11)$$



Then, if heap capacity is the limiting factor, the optimum cut-off grade is indicated as below:

$$\text{Heap limitation} : \begin{cases} g_H^h = \frac{\partial \cdot NPV_i + Cap_{\mathcal{H}} h_i + f_i}{Cap_{\mathcal{H}} \eta_H \eta_X (P_i - x_i)} \\ g_H^c = \frac{(c_i - h_i) Cap_{\mathcal{H}} - f_i - \partial \cdot NPV_i}{((P_i - s_i) \eta_C \eta_S - \eta_H \eta_X (P_i - x_i)) Cap_{\mathcal{H}}} \end{cases} \quad (12)$$

If SX, EW plants and marketing outputs are the limiting factors, the optimum cut-off grade is obtained as below:

$$\begin{matrix} \text{solvent extraction} \\ \text{electrowinning} \\ \text{marketing} \\ \text{limitation} \end{matrix} : \begin{cases} g_X^h = \frac{Cap_X h_i}{\eta_H \eta_X (P_i - x_i) Cap_X - f_i - \partial \cdot NPV_i} \\ g_X^c = \frac{(c_i - h_i) Cap_X}{\eta_H \eta_X ((x_i - P_i) Cap_X - f_i - \partial \cdot NPV) + \eta_C \eta_S Cap_X (P_i - s_i)} \end{cases} \quad (13)$$

Also, if Concentration throughput is the governing limitation, the optimum cut-off grade is given by:

$$\text{Concentration limitation} : \begin{cases} g_C^h = \frac{h_i}{\eta_H \eta_X (P_i - x_i)} \\ g_C^c = \frac{(c_i - h_i) Cap_C + \partial \cdot NPV_i + f_i}{((P_i - s_i) \eta_C \eta_S - \eta_H \eta_X (P_i - x_i)) Cap_C} \end{cases} \quad (14)$$

And lastly, if smelting, refinery or marketing throughputs are the governing limitations, the optimum cut-off grade is calculated by:

$$\begin{matrix} \text{Smelting} \\ \text{electrorefining} \\ \text{marketing} \\ \text{limitation} \end{matrix} : \begin{cases} g_S^h = \frac{h_i}{\eta_H \eta_X (P_i - x_i)} \\ g_S^c = \frac{(c_i - h_i) Cap_S}{((P_i - s_i) Cap_S - f_i - \partial \cdot NPV_i) \cdot \eta_C \eta_S - \eta_H \eta_X (P_i - x_i) Cap_S} \end{cases} \quad (15)$$

In separate conditions, two limiting components might be simultaneously in balance. The capacities of these stages are balanced to use the maximum capacity of each stage. In this case, 2-combination of applicable constraints from 5, gives 10 combinations of governing constraints which consequently lead to 10 balancing cut-off grades. These cut-off grades are named dual balancing cut-off grades and they are influenced by governing limitations and distribution of grades of material mined. Dual balancing cut-off grades are geometrically defined as the maximum point of two cash flow surfaces intersecting curves which come from the following:

$$\left\{ \frac{\sigma_{Ci}}{\sigma_{Ei}} = \frac{Cap_C}{Cap_\varepsilon}, \frac{\sigma_{Hi}}{\sigma_{Ei}} = \frac{Cap_{\mathcal{H}}}{Cap_\varepsilon}, \dots, \frac{\sigma_{Si}}{\sigma_{Xi}} = \frac{Cap_S}{Cap_\varepsilon} \right. \quad (16)$$

Considerably, three limiting components can be in balance. Hence, 3-combination of operating constraints from 5, gives 10 combinations of governing constraints and 10 cut-off grades can be calculated. 10 more balancing cut-off grades are called triple balancing cut-off grades that can be obtained by relation 17. Triple balancing cut-off grades geometrically happens on three cash flow surfaces intersecting points. In extreme cases, 4 or 5 components can be in balance position with together simultaneously. To be able to calculate cut-off grades with advance programming language needs to know the distribution of grades of the mined material and economical parameters.

$$\left\{ \frac{\sigma_{Ci}}{Cap_C} = \frac{\sigma_{Hi}}{Cap_H} = \frac{\sigma_{Ei}}{Cap_\epsilon}, \frac{\sigma_{Si}}{Cap_S} = \frac{\sigma_{Ci}}{Cap_C} = \frac{\sigma_{Ei}}{Cap_\epsilon}, \dots, \frac{\sigma_{Si}}{Cap_S} = \frac{\sigma_{Hi}}{Cap_H} = \frac{\sigma_{Xi}}{Cap_X} \right. \quad (17)$$

Fig. 2 schematically shows the cash flow surface in light of governing limitations. In the other hand, limiting cut-off grade and dual and triple balancing cut-off grades are specified. The cash flow surfaces can be modified based upon capacities, costs and commodity price. The *X* and *Y* axes can indicate the ore tonnages or ore grades.

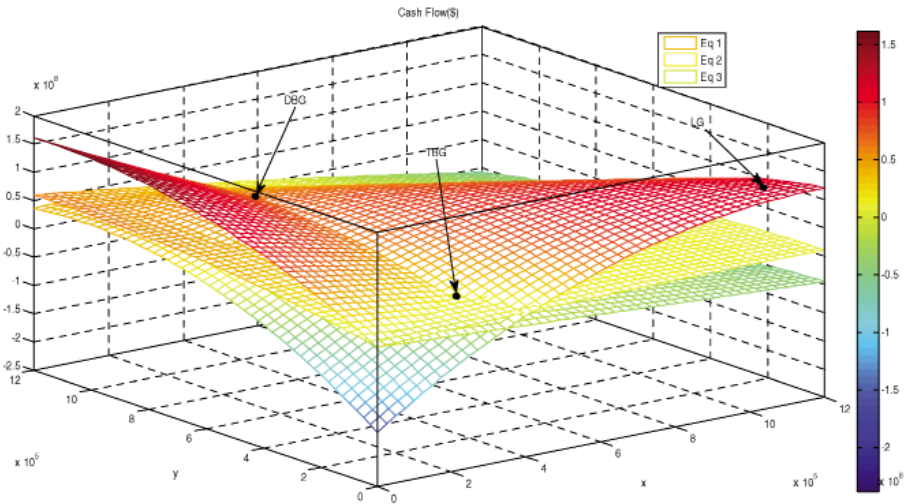


Fig. 2. The cash flow surface in light of governing limitations and limiting (LG), dual balancing (DBG) and triplet (TBG) cut-off grades are specified

Optimum cut-off grades, which are calculated as limiting, duals balancing or triples balancing are geometrically located in maximum of the surface, passing through the minimum of all cash flow surfaces.

The obtained optimum cut-off grade is remarked as the first assumption and entered into the iteration algorithm in order to maximize *NPV*. Matlab programming is applied to calculate maximum *NPV* and optimum cut-off grades under different circumstances. This algorithm is implemented by Matlab language programming which is not only used for iterative steps of the approach, but also is developed to solve the system of nonlinear equations and Lagrange multipliers.

## 4. Results and Discussions:

A hypothetical mine with demonstrated grade-tonnage distribution (Table 2) is analyzed to scrutinize the approach. The Rayleigh distribution is used to model the hypothetical mine. The adjusted R-square value of the model is about 0.98 which indicate a good fit for this distribution. The copper mineralogy is predominantly Chalcocite ( $\text{Cu}_2\text{S}$ ) and Bornite ( $\text{Cu}_5\text{FeS}_4$ ) with minor enargite ( $\text{Cu}_3\text{AsS}_4$ ) and covellite ( $\text{CuS}$ ). The pit outline contains 2.5 million tons of materials.

TABLE 2

Grade-tonnage distribution of the hypothetical mine

Grade (%)	Tonnage (Ton)
0-0.1	293,760
0.1-0.2	689,920
0.2-0.3	704,700
0.3-0.4	473,290
0.4-0.5	228,500
0.5-0.6	82,070
0.6-0.7	22,304
0.7-0.8	4,630
0.8<G	839

Other descriptive parameters of this mine are featured out in the following table (Table 3).

TABLE 3

Economic parameters, operational capacities and plants recovery of case study

Component	Value	Unit	Component	Value	Unit	Component	Value	Unit
$\partial$	10	%	$e$	2.5	$\$/T_m$	$\eta_H$	59	%
$Cap_C$	300	$TT/\text{year}$	$h$	2.2	$\$/T_o$	$\eta_X$	95	%
$Cap_H$	100	$TT/\text{year}$	$\eta_C$	85	%	$Cap_X$	1	$TT/\text{year}$
$Cap_E$	500	$TT/\text{year}$	$\eta_S$	95	%	$P$	6.6	$T\$/T_p$
$f$	100	$T\$/\text{year}$	$Cap_S$	3	$TT/\text{year}$	$s$	1.2	$T\$/T_p$
$x$	200	$\$/T_p$	$c$	5	$\$/T_o$			

$T\%$ : Thousand dollars,  $TT$ : Thousand ton,  $T_p$ : Ton of product,  $T_o$ : Ton of ore,  $T_m$ : Ton of material

The ore mineralogy demonstrates that this mine contains of primary and secondary copper sulfide minerals. So, the optimum cut-off grades of concentration and leaching (chemical or bacteria) methods are calculated by the objective function of  $NPV$  maximization.

Based on the information presented in this stage, the cash flow functions and their optimum amounts are calculated for the production years. As mentioned, these functions form surfaces throughout governing limitations (Fig. 3). The limiting cut-off grades (surfaces maximum), dual balancing cut-off grades (the maximum point of two cash flow surfaces intersecting curves) and the triples balancing cut-off grades (three cash flow surfaces intersecting points) of a hypothetical mine displayed in Fig. 3. Furthermore, these points are projected on the  $X$  and  $Y$  axes can illustrate concentration and leaching cut-off grades.

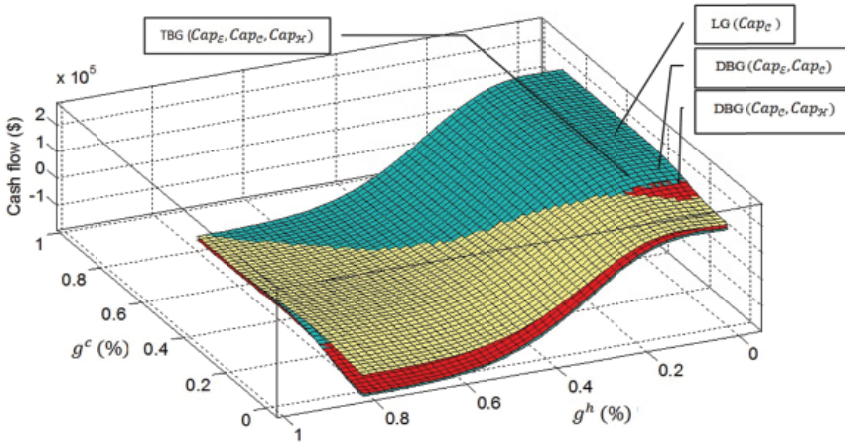


Fig. 3. The limiting, dual and triple balancing cut-off grades of the hypothetical mine

Moreover, Fig. 4 schematically features out the geometric location of optimum cut-off grade in the given example. As can be seen, this optimum cut-off grade is geometrically located in maximum of the surface, passing through the minimum of all cash flow surfaces. As observed, this grade is one of the triple and dual balancing cut-off grades and it has been calculated using Matlab programming.

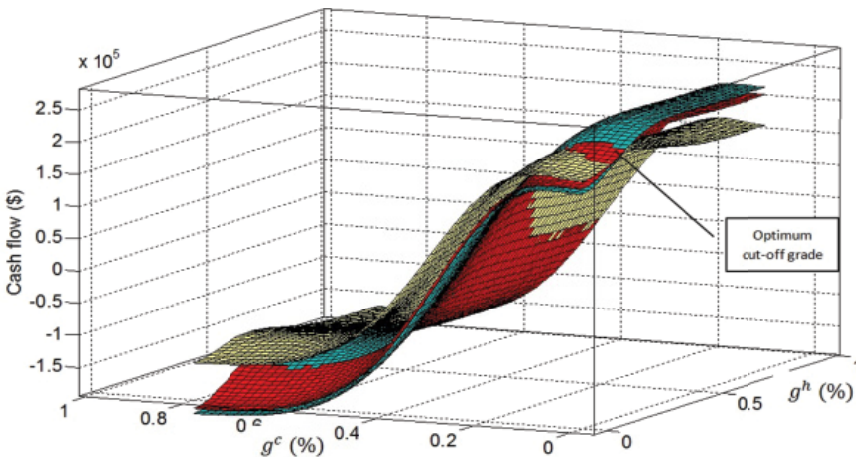


Fig. 4. The geometric location of optimum cut-off grade

The two mineral processing cut-off grades calculated and carried out by iterative operation and lagrangian multiplier optimization method. The *NPV* changes and annual cash flows calculated within different years are indicated in Table 4. Also as indicated in table 4, the leaching and concentration optimum cut-off grade are dropped to 0.1 and 0.18 percent respectively in year 6.

TABLE 4

Optimum cut-off grades policy of the hypothetical mine

Year	Total material (T)	$g^h$ (%)	$g^c$ (%)	$\sigma_{Ei}(T_m)$	$\sigma_{Si}(T_p)$	$\sigma_{Xi}(T_p)$	$\gamma_i(w,s)(T_m)$	Life (year)	Cash Flows (\$)	Overall NPV (\$)
1	2,500,000	0.13	0.2	500,000	798	94	100,000	5.00	1,844,050	6,990,400
2	2,000,000	0.13	0.2	500,000	798	94	100,000	4.00	1,844,050	5,845,480
3	1,500,000	0.13	0.2	500,000	798	94	100,000	3.00	1,844,050	4,586,060
4	1,000,000	0.12	0.19	478,000	784	88	78,211	2.09	1,782,600	3,221,170
5	522,000	0.11	0.19	465,000	775	84	65,237	1.12	1,740,740	1,764,840
6	57,000	0.10	0.18	57,000	96	10	6,568	0.13	211.98	25,106

As observed, the cut-off grades reduction policy can end into accessing the maximum *NPV* value. This decreasing policy is perceived in both leaching and concentration cut-off grades. Holistically, the optimum cut-off grades are one of the triple or dual balancing ones while mining, concentration or heap are the limiting factors.

The commodity price of final product changes the mines cut-off grades and their annual benefit. If it analyses copper price changes within the last 5 recent years, some volatility and inflation price will be observed. Fig. 5 reveals variations of concentration optimum cut-off grades versus copper price when the mining, concentrator, heap, SX and EW, Smelter and electrorefining capacities are the controlling constraints for the above example. As observed the concentration optimum cut-off grades are descending functions of copper price.

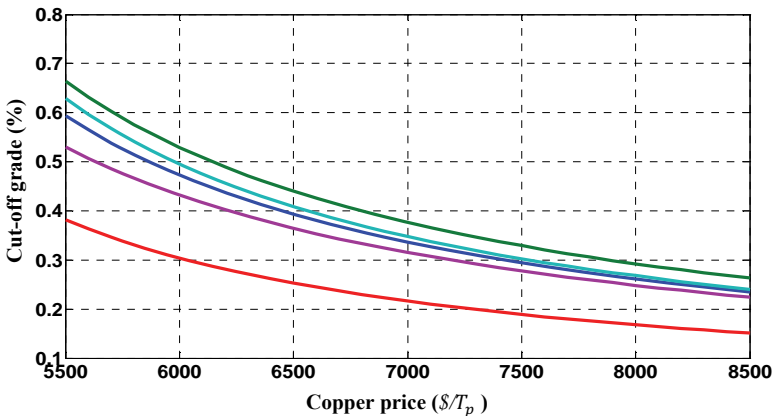


Fig 5. Variations of concentration optimum cut-off grades versus copper price under different circumstances, mining limitation (—), concentrator limitation (—), heap leaching limitation (—), SX, EW and marketing (—), Smelter, Electrorefining and marketing limitation (—)

As indicated in Fig. 5, the \$8500 price of copper leads to lower limiting cut-off grades. These low grades are required to take the technical precautions in concentrator plants. Also, Fig. 6 shows that the copper price changes have an essential effect on leaching and concentration optimum cut-off grades in case mining capacity is the applicable constraint.

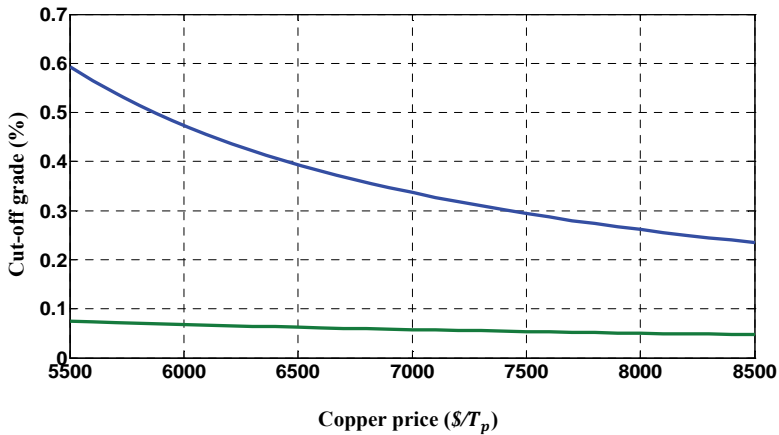


Fig. 6. The effect of copper price variations on optimum cut-off grades (mining capacity is the limiting factor), concentration cut-off grade (—), Heap leaching cut-off grade (—)

As a result, while the objective is to maximize the net present value, leaching and concentration optimum cut-off grades of the hypothetical mine should decrease as copper price rises. However the rate of leaching cut-off grades reduction is less than concentration cut-off grades. This relation is approximately seen in the other optimum cut-off grades when the other process capacities (concentrator, smelter and electrorefining, heap, SX and EW) are controlling factors.

## 5. Conclusions

Several mineral processing methods are employed in copper mines. In this paper, an optimization technique used to obtain concentration and heap leaching optimum cut-off grades. The objective function is considered maximize net present value. It is observed that using heap leaching method in processing of low grades copper ores not only can effects on the net present value of the mines but also can decreases the complexity of wastes management and the adverse environmental impacts of mining projects. The overall net present value of the case study calculated \$6,990,400 and the leaching and concentration optimum cut-off grades obtained 0.1 and 0.18 percent respectively in year 6. The results indicate that the copper price changes have an essential effect on leaching and concentration optimum cut-off grades in case mining capacity is the applicable constraint. Leaching and concentration optimum cut-off grade of the case study decrease as copper price rises. However the rate of leaching cut-off grades reduction is less than concentration cut-off grades.

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