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Parametric optimization of a neutral system with two delays and PD-controller

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In this paper a parametric optimization problem for a linear neutral system with two delays with an integral quadratic performance index is formulated and solved. The method of computing of a performance index value bases on determining of a Lyapunov functional defined on a state space such that its value for an initial state is equal to a performance index value. In the paper a form of a Lyapunov functional is assumed and a method of computing its coefficients is given. An example illustrating the application of discussed theory is presented. It concerns the system with a PD-controller designed to control a plant with two delays both retarded and neutral type. For such system a value of considered performance index is determined.

Key words: parametric optimization, Lyapunov functional, time delay system, neutral system

1. Introduction

The Lyapunov quadratic functionals are used to: test the stability of the systems, in computation of the critical delay values for time delay systems, in computation of the exponential estimates for the solutions of the time delay systems, in calculation of the robustness bounds for the uncertain time delay systems, to calculate of a quadratic performance index of quality for a process of parametric optimization for the time delay systems. One constructs the Lyapunov functionals for a system with a time delay with a given time derivative. For the first time such Lyapunov functional was introduced by Repin [14] for a case of the retarded time delay linear systems with one delay. Repin [14] delivered also a procedure for determination of the functional coefficients. Duda [1] used a Lyapunov functional, which was proposed by Repin, for the calculation of a quadratic performance index value in the process of parametric optimization for the systems with a time delay of retarded type and extended the results to a case of a neutral type time delay system in [2]. Duda [3] presented a method of determining of a

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Lyapunov functional for a linear dynamic system with two lumped retarded type time delays in a general case with no-commensurate delays and presented a special case with commensurate delays in which a Lyapunov functional could be determined by solving a set of the ordinary differential equations. Duda [4] introduced a method of determining of a Lyapunov functional for a linear dynamic system with two delays both retarded and neutral type time delay and in the paper [5] presented a method of determining of a Lyapunov quadratic functional for a linear time-invariant system with *k*-non- commensurate neutral type time delays. Duda [6] considered the parametric optimization problem for a neutral system with two delays with a P-controller. Infante and Castelan's [9] construction of a Lyapunov functional is based on a solution of a matrix differential-difference equation on a finite time interval. This solution satisfies the symmetry and boundary conditions. Kharitonov and Zhabko [13] extended the Infante and Castelan's results and proposed a procedure of construction of the quadratic functionals for the linear retarded type time delay systems which could be used for the robust stability analysis of the time delay systems. This functional was expressed by means of a Lyapunov matrix which depended on a fundamental matrix of a time delay system. Kharitonov [10] extended some basic results obtained for a case of the retarded type time delay systems to a case of the neutral type time delay systems, and in [11] to the neutral type time delay systems with a discrete and distributed delay. Kharitonov and Plischke [12] formulated the necessary and sufficient conditions for the existence and uniqueness of a Lyapunov matrix for a case of a retarded system with one delay.

The paper deals with a parametric optimization problem for a system with two delays both retarded and neutral type time delay with a PD-controller. To the best of author's knowledge, such a parametric optimization problem has not been reported in the literature. An example illustrating this method is also presented.

2. Formulation of a parametric optimization problem

Let us consider a linear system with two delays both retarded and neutral type whose dynamics is described by equations

$$\begin{cases} \dfrac{dx(t)}{dt} - D\dfrac{dx(t-\tau)}{dt} = Ax(t) + Bx(t-\tau) + Cu(t-r) \\ x(t_0+\theta) = \Phi(\theta) \\ u(t) = -Kx(t) - T_d\dfrac{dx(t)}{dt} \end{cases} \tag{1}$$

where: $t \geqslant t_0$, $\theta \in [-r, 0]$, $r \geqslant \tau > 0$ and are rationally independent; A, B, $D \in \mathbb{R}^{n\times n}$, D is nonsingular, $x(t) \in \mathbb{R}^n$, $\Phi \in C^1([-r,0],\mathbb{R}^n)$, $C \in \mathbb{R}^{n\times m}$, K, $T_d \in \mathbb{R}^{m\times n}$, $u(t) \in \mathbb{R}^m$, $C^1([-r,0],\mathbb{R}^n)$ is a space of continuously differentiable functions defined on an interval $[-r,0]$ with values in $\mathbb{R}^n$.

We can reshape an equation (1) to a form

$$\begin{cases} \dfrac{dx(t)}{dt} - D\dfrac{dx(t-\tau)}{dt} + CT_d\dfrac{dx(t-r)}{dt} = Ax(t) + Bx(t-\tau) - CKx(t-r) \\ x(t_0+\theta) = \Phi(\theta) \end{cases} \tag{2}$$

where: $t \geqslant t_0$, $\theta \in [-r,0]$, $r \geqslant \tau > 0$ and are rationally independent, A, B, $D \in \mathbb{R}^{n\times n}$, D is nonsingular, $C \in \mathbb{R}^{n\times m}$, K, $T_d \in \mathbb{R}^{m\times n}$, $\Phi \in C^1([-r,0],\mathbb{R}^n)$.

A solution of the equation (2) is a continuously differentiable function defined for $t \geqslant t_0 - r$ except at suitable multiples of the time delays.

If the condition

$$\frac{d\Phi(0)}{d\theta} = D\frac{d\Phi(-\tau)}{d\theta} - CT_d\frac{d\Phi(-r)}{d\theta} + A\Phi(0) + B\Phi(-\tau) - CK\Phi(-r) \tag{3}$$

holds then the solution of equation (2) has a continuous derivative for all $t \geqslant t_0 - r$.

A difference equation associated with (2) is given by the following term

$$x(t) = Dx(t-\tau) - CT_dx(t-r), \quad t \geqslant t_0. \tag{4}$$

The eigenvalues of a difference equation (4) play a fundamental role in the asymptotic behavior of the solutions of a neutral equation (2).

Definition 1 *The **spectrum** $\sigma(G)$ is a set of complex numbers λ for which a matrix $\lambda I - G$ is not invertible. The **spectral radius** of a matrix G is given by a form*

$$\gamma(G) = \sup\{|\lambda| : \lambda \in \sigma(G)\}. \tag{5}$$

We assume that matrices D and CT_d satisfy the hypotheses given in Theorem 9.6.1 of Hale and Verduyn Lunel [7]

$$\begin{cases} r \geqslant \tau > 0 \quad \text{and are rationally independent} \\ \sup\left\{\gamma\left(e^{i\theta_1}D - e^{i\theta_2}CT_d\right) : \theta_1, \theta_2 \in [0,2\pi]\right\} < 1 \end{cases} \tag{6}$$

and in this case a difference equation (4) is stable.

We introduce a new function y, defined as follows

$$y(t) = x(t) - Dx(t-\tau) + CT_dx(t-r) \quad for\, t \geqslant t_0. \tag{7}$$

Thus an equation (2) takes the form

$$\begin{cases} \dfrac{dy(t)}{dt} = Ay(t) + (AD+B)x(t-\tau) - (ACT_d + CK)x(t-r) \\ y(t) = x(t) - Dx(t-\tau) + CT_dx(t-r) \\ y(t_0) = \Phi(0) - D\Phi(-\tau) + CT_d\Phi(-r) \\ x(t_0+\theta) = \Phi(\theta) \end{cases} \tag{8}$$

The condition (6) guarantees asymptotic stability of the closed-loop system (2). The asymptotic stability of the closed-loop system (2) implies the asymptotic stability of the closed-loop system (8).

State of the system (8) forms the folowing vector

$$S(t) = \begin{bmatrix} y(t) \\ x_t \end{bmatrix} \qquad \text{for} \quad t \geqslant t_0 \tag{9}$$

where $x_t \in C^1([-r,0],\mathbb{R}^n)$, $x_t(\theta) = x(t+\theta)$ for $\theta \in [-r,0]$.

The state space is defined by a formula

$$X = \mathbb{R}^n \times C^1([-r,0],\mathbb{R}^n) \tag{10}$$

We search for the matrices K and T_d which minimize the integral quadratic performance index

$$J = \int_{t_0}^{\infty} y^T(t)y(t)dt \tag{11}$$

and matrix T_d fulfills the condition (6).

3. A method of determining of a performance index value

On the state space X one defines a Lyapunov functional, positively defined, differentiable, with derivative computed on a trajectory of the system (3) being negatively defined.

$$\begin{aligned} V(S(t)) &= y^T(t)\alpha y(t) + \int_{-r}^{0} y^T(t)\beta(\theta)x(t+\theta)d\theta + \\ &+ \int_{-r}^{0}\int_{\theta}^{0} x^T(t+\theta)\delta(\theta,\sigma)x(t+\sigma)d\sigma d\theta \end{aligned} \tag{12}$$

for $t \geqslant t_0$ where $\alpha = \alpha^T \in \mathbb{R}^{n\times n}$, $\beta \in C^1([-r,0],\mathbb{R}^{n\times n})$, $\delta \in C^1(\Omega,\mathbb{R}^{n\times n})$, $\Omega = \{(\theta,\varsigma) :\ \theta \in [-r,0],\ \varsigma \in [\theta,0]\}$. C^1 is the space of continuous functions with continuous derivative.

We identify the coefficients of the Lyapunov functional (12), assuming that its derivative computed on the system (3) trajectory satisfies the following relationship

$$\frac{dV(S(t))}{dt} = -y^T(t)y(t) \quad \text{for} \quad t \geqslant t_0. \tag{13}$$

If the relationship (13) holds and the closed-loop system (8) is asymptotically stable, one can easily determine the value of square indicator of the quality for the parametric optimization problem because

$$J=\int_{t_0}^{\infty} y^T(t)y(t)dt = V(S(t_0)). \tag{14}$$

4. Determination of the Lyapunov functional coefficients

Derivative of the functional (12) is computed on trajectory of the system (3) according to the formula

$$\frac{dV(S(t_0))}{dt}=\lim_{h\to 0}\sup\frac{V\left(y\left(t_0+h\right),x_{t_0+h}\right)-V\left(y_0,\Phi\right)}{h}. \tag{15}$$

The time derivative of the functional (12) calculated on the basis of (15) is given by the following

$$\begin{aligned}\frac{dV(S(t))}{dt}&=y^T(t)\left[A^T\alpha+\alpha A+\frac{\beta(0)+\beta^T(0)}{2}\right]y(t)+\\&+y^T(t)\left[2\alpha\left(B+AD\right)+\beta(0)D\right]x(t-\tau)+\\&+y^T(t)\left[-2\alpha\left(ACT_d+CK\right)-\beta(0)CT_d-\beta(-r)\right]x(t-r)+\\&+\int_{-r}^{0}y^T(t)\left[A^T\beta(\theta)-\frac{d\beta(\theta)}{d\theta}+\delta^T(\theta,0)\right]x(t+\theta)d\theta+\\&+\int_{-r}^{0}x^T(t-\tau)[(B+AD)^T\beta(\theta)+D^T\delta^T(\theta,0)]x(t+\theta)d\theta+\\&-\int_{-r}^{0}x^T(t-r)\left[(ACT_d+CK)^T\beta(\theta)+T_d^TC^T\delta^T(\theta,0)+\delta(-r,\theta)\right]x(t+\theta)d\theta+\\&-\int_{-r}^{0}\int_{\theta}^{0}x^T(t+\theta)\left[\frac{\partial\delta(\theta,\sigma)}{\partial\theta}+\frac{\partial\delta(\theta,\sigma)}{\partial\sigma}\right]x(t+\sigma)d\sigma d\theta.\end{aligned} \tag{16}$$

From equations (4) and (13) one obtains the following set of equations

$$A^T\alpha + \alpha A + \frac{\beta(0) + \beta^T(0)}{2} = -I \tag{17}$$

$$2\alpha(B + AD) + \beta(0)D = 0 \tag{18}$$

$$2\alpha(ACT_d + CK) + \beta(0)CT_d + \beta(-r) = 0 \tag{19}$$

$$A^T\beta(\theta) - \frac{d\beta(\theta)}{d\theta} + \delta^T(\theta, 0) = 0 \tag{20}$$

$$(B + AD)^T\beta(\theta) + D^T\delta^T(\theta, 0) = 0 \tag{21}$$

$$(ACT_d + CK)^T\beta(\theta) + T_d^T C^T\delta^T(\theta, 0) + \delta(-r, \theta) = 0 \tag{22}$$

$$\frac{\partial\delta(\theta,\sigma)}{\partial\theta} + \frac{\partial\delta(\theta,\sigma)}{\partial\sigma} = 0 \tag{23}$$

for $\theta \in [-r, 0]$, $\sigma \in [-r, 0]$.

From an equation (18) it follows

$$\beta(0) = -2\alpha\left(A + BD^{-1}\right). \tag{24}$$

We now substitute (24) into (17). After some calculations one obtains the result

$$\alpha P + P^T\alpha = I \tag{25}$$

where

$$P = BD^{-1}. \tag{26}$$

From an equation (25) one obtains the matrix α. Taking into account the relations (19) and (24) we get the formula

$$\beta(-r) = 2\alpha(PCT_d - CK). \tag{27}$$

From equation (21) one obtains

$$\delta^T(\theta, 0) = -(A + P)^T\beta(\theta). \tag{28}$$

We now put (28) into (20) receiving after some calculations

$$\frac{d\beta(\theta)}{d\theta} = -P^T\beta(\theta) \tag{29}$$

for $\theta \in [-r, 0]$.

Solution of differential equation (29) is as follows

$$\beta(\theta) = \exp\left(-P^T(\theta+r)\right)\beta(-r) \tag{30}$$

for $\theta \in [-r,0]$. After putting (27) into (30) one obtains

$$\beta(\theta) = 2\exp\left(-P^T(\theta+r)\right)\alpha\left(PCT_d - CK\right). \tag{31}$$

Solution of equation (23) is as below

$$\delta(\theta,\sigma) = \varphi(\theta-\sigma) \tag{32}$$

where $\varphi \in C^1\left([-r,r],\mathbb{R}^{n\times n}\right)$.

From equations (32) and (22) one obtains

$$\delta(-r,\theta) = \varphi(-r-\theta) = -\left(ACT_d + CK\right)^T\beta(\theta) - T_d^T C^T \delta^T(\theta,0). \tag{33}$$

After putting (28) into (33) one obtains the following

$$\varphi(-r-\theta) = \left(PCT_d - CK\right)^T\beta(\theta) \tag{34}$$

for $\theta \in [-r,0]$. Hence

$$\varphi(\xi) = \left(PCT_d - CK\right)^T\beta(-\xi-r). \tag{35}$$

Taking into account (32) and (35) we obtain

$$\delta(\theta,\sigma) = \left(PCT_d - CK\right)^T\beta(\sigma-\theta-r). \tag{36}$$

Finally taking the relation (31) into account one gets a formula

$$\delta(\theta,\sigma) = 2\left(PCT_d - CK\right)^T\exp\left(-P^T(\sigma-\theta)\right)\alpha\left(PCT_d - CK\right). \tag{37}$$

In this way we obtained all parameters of a Lyapunov functional.

5. Determination of a performance index value

According to formula (14) the performance index value is given by a term

$$\begin{aligned} J &= V\left(y(t_0),\Phi\right) = y^T(t_0)\alpha y(t_0) + \\ &+ \int_{-r}^{0} y^T(t_0)\beta(\theta)\Phi(\theta)d\theta + \int_{-r}^{0}\int_{\theta}^{0}\Phi^T(\theta)\delta(\theta,\sigma)\Phi(\sigma)d\sigma d\theta. \end{aligned} \tag{38}$$

After putting the relations (31) and (37) into (38) one gets

$$
\begin{aligned}
J &= y^T(t_0)\alpha y(t_0) + 2\int_{-r}^{0} y^T(t_0)\exp\left(-P^T(\theta+r)\right)\alpha\left(PCT_d - CK\right)\Phi(\theta)d\theta + \\
&+ 2\int_{-r}^{0}\int_{\theta}^{0}\Phi^T(\theta)\left(PCT_d - CK\right)^T\exp\left(-P^T(\sigma-\theta)\right)\alpha\left(PCT_d - CK\right)\Phi(\sigma)d\sigma d\theta.
\end{aligned}
\tag{39}
$$

To obtain the optimal values of the parameters K and T_d we compute the performance index derivatives with respect to K_{ij} and $T_{d_{ls}}$ and then we equal them to zero

$$
\begin{cases}
\dfrac{\partial J(K_{ij}, T_{d_{ls}})}{\partial K_{ij}} = 0 \\
\dfrac{\partial J(K_{ij}, T_{d_{ls}})}{\partial T_{d_{ls}}} = 0
\end{cases}
\quad \text{for} \quad i \leqslant m,\ j \leqslant n,\ l \leqslant m,\ s \leqslant n.
\tag{40}
$$

In this case one obtains a set of algebraic equations with unknown variables K_{opt} and $T_{d_{opt}}$.

6. An example

Let us consider a system described by the equation

$$
\begin{cases}
\dfrac{dx(t)}{dt} - d\dfrac{dx(t-\tau)}{dt} + cT_d\dfrac{dx(t-r)}{dt} = ax(t) + bx(t-\tau) - ckx(t-r) \\
x(t_0+\theta) = \Phi(\theta)
\end{cases}
\tag{41}
$$

where: $t \geqslant t_0$, $x(t) \in \mathbb{R}$, $\theta \in [-r, 0]$, $a, b, c, d, k \in \mathbb{R}$, $d \neq 0$, $r \geqslant \tau > 0$.

The elements d and cT_d should satisfy the condition (6), which takes the following form

$$
|\, d \,| + |\, cT_d \,| < 1.
\tag{42}
$$

One can rewrite equation (41) to the form

$$
\begin{cases}
\dfrac{dy(t)}{dt} = ay(t) + (b + ad)x(t-\tau) - (acT_d + ck)x(t-r) \\
y(t) = x(t) - dx(t-\tau) + cT_d x(t-r) \\
y(t_0) = \Phi(0) - d\Phi(-\tau) + cT_d\Phi(-r) \\
x(t_0+\theta) = \Phi(\theta)
\end{cases}
\tag{43}
$$

where: $t \geqslant t_0$, $x(t) \in \mathbb{R}$, $\theta \in [-r, 0]$, $a, b, c, d, k, T_d \in \mathbb{R}$, $d \neq 0$, $r \geqslant \tau > 0$. We search for the parameters k and T_d which minimize an integral quadratic performance index

$$J = \int_{t_0}^{\infty} y^T(t)y(t)dt = V\left(y(t_0), \Phi\right). \tag{44}$$

A Lyapunov functional V is defined by the formula

$$V(S(t)) = \alpha y^2(t) + \int_{-r}^{0} y(t)\beta(\theta)x(t+\theta)d\theta + \int_{-r}^{0}\int_{\theta}^{0} x(t+\theta)\delta(\theta,\sigma)x(t+\sigma)d\sigma d\theta \tag{45}$$

where according to (25) α is given by

$$\alpha = \frac{1}{2p} = \frac{d}{2b}. \tag{46}$$

According to (31) the coefficient β is given by the formula

$$\beta(\theta) = 2\alpha\left(pcT_d - ck\right)\exp\left(-\frac{b(\theta+r)}{d}\right) = \left(cT_d - \frac{cdk}{b}\right)\exp\left(-\frac{b(\theta+r)}{d}\right). \tag{47}$$

It follows from (37) that the element δ can be expressed as

$$\begin{aligned} \delta(\theta,\sigma) &= 2\alpha\left(pcT_d - ck\right)^2\exp\left(-\frac{b(\sigma-\theta)}{d}\right) = \\ &= \left(\frac{bcT_d}{d} - ck\right)^2\frac{d}{b}\exp\left(-\frac{b(\sigma-\theta)}{d}\right). \end{aligned} \tag{48}$$

Value of the performance index is given by

$$\begin{aligned} J &= \frac{d}{2b}\left(\Phi(0) - d\Phi(-\tau) + c\Phi(-r)T_d\right)^2 + \\ &+ \left(cT_d - \frac{cdk}{b}\right)\left(\Phi(0) - d\Phi(-\tau) + c\Phi(-r)T_d\right)\int_{-r}^{0}\Phi(\theta)\exp\left(-\frac{b(\theta+r)}{d}\right)d\theta + \\ &+ \left(\frac{bcT_d}{d} - ck\right)^2\frac{b}{d}\int_{-r}^{0}\int_{\theta}^{0}\Phi(\theta)\Phi(\sigma)\exp\left(-\frac{b(\sigma-\theta)}{d}\right)d\sigma d\theta. \end{aligned} \tag{49}$$

A performance index is a quadratic function with respect to the variables k and T_d.

To attain the optimal values of the parameters k and T_d we compute the performance index derivatives with respect to k and T_d

$$\frac{\partial J(k,T_d)}{\partial k} = -\frac{cd}{b}\left(\Phi(0)-d\Phi(-\tau)+c\Phi(-r)T_d\right)\int_{-r}^{0}\Phi(\theta)\exp\left(-\frac{b(\theta+r)}{d}\right)d\theta+ \\ - \frac{2bc^2}{d}\left(\frac{bT_d}{d}-k\right)\int_{-r}^{0}\int_{\theta}^{0}\Phi(\theta)\Phi(\sigma)\exp\left(-\frac{b(\sigma-\theta)}{d}\right)d\sigma d\theta \tag{50}$$

$$\frac{\partial J(k,T_d)}{\partial T_d} = \frac{cd\Phi(-r)}{b}\left(\Phi(0)-d\Phi(-\tau)+c\Phi(-r)T_d\right)+ \\ + c\left(\Phi(0)-d\Phi(-\tau)+c\Phi(-r)T_d\right)\int_{-r}^{0}\Phi(\theta)\exp\left(-\frac{b(\theta+r)}{d}\right)d\theta+ \\ + c^2\Phi(-r)\left(T_d-\frac{dk}{b}\right)\int_{-r}^{0}\Phi(\theta)\exp\left(-\frac{b(\theta+r)}{d}\right)d\theta+ \\ + \frac{2b^2c^2}{d^2}\left(\frac{bT_d}{d}-k\right)\int_{-r}^{0}\int_{\theta}^{0}\Phi(\theta)\Phi(\sigma)\exp\left(-\frac{b(\sigma-\theta)}{d}\right)d\sigma d\theta. \tag{51}$$

Now we equal the derivatives to zero and we obtain a set of linear equations with unknown variables k_{opt} and $T_{d_{opt}}$.

$$\begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}\begin{bmatrix} k_{opt} \\ T_{d_{opt}}. \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \tag{52}$$

where

$$q_{11} = -\frac{2bc}{d}\int_{-r}^{0}\int_{\theta}^{0}\Phi(\theta)\Phi(\sigma)\exp\left(-\frac{b(\sigma-\theta)}{d}\right)d\sigma d\theta \tag{53}$$

$$q_{12} = \frac{cd\Phi(-r)}{b}\int_{-r}^{0}\Phi(\theta)\exp\left(-\frac{b(\theta+r)}{d}\right)d\theta+ \\ +\frac{2b^2c}{d^2}\int_{-r}^{0}\int_{\theta}^{0}\Phi(\theta)\Phi(\sigma)\exp\left(-\frac{b(\sigma-\theta)}{d}\right)d\sigma d\theta \tag{54}$$

$$q_{21} = -\frac{cd\Phi(-r)}{b}\int_{-r}^{0}\Phi(\theta)\exp\left(-\frac{b(\theta+r)}{d}\right)d\theta + \\ -\frac{2b^2c}{d^2}\int_{-r}^{0}\int_{\theta}^{0}\Phi(\theta)\Phi(\sigma)\exp\left(-\frac{b(\sigma-\theta)}{d}\right)d\sigma d\theta \tag{55}$$

$$q_{22} = \frac{cd\Phi^2(-r)}{b} + 2c\Phi(-r)\int_{-r}^{0}\Phi(\theta)\exp\left(-\frac{b(\theta+r)}{d}\right)d\theta + \\ +\frac{2b^3c}{d^3}\int_{-r}^{0}\int_{\theta}^{0}\Phi(\theta)\Phi(\sigma)\exp\left(-\frac{b(\sigma-\theta)}{d}\right)d\sigma d\theta \tag{56}$$

$$h_1 = -\frac{b}{d}\left(\Phi(0) - d\Phi(-\tau)\right)\int_{-r}^{0}\Phi(\theta)\exp\left(-\frac{b(\theta+r)}{d}\right)d\theta \tag{57}$$

$$h_2 = \left(\Phi(0) - d\Phi(-\tau)\right)\left(-\frac{d\Phi(-r)}{b} - \int_{-r}^{0}\Phi(\theta)\exp\left(-\frac{b(\theta+r)}{d}\right)d\theta\right). \tag{58}$$

Presented below is the numerical solutions.

1. According to following set of parameters

$$\Phi(\theta) = \text{constans},\ b = 0.516,\ c = 1,\ d = -0.4,\ r = 1$$

one obtains the optimal values

$$k_{opt} = 1,5451, \quad T_{d_{opt}} = 0,4253$$

for which the condition (42) is satisfied

$$\mid d \mid + \mid cT_d \mid = 0.8253 < 1.$$

2. According to following set of parameters

$$\Phi(\theta) = \text{constans},\ b = 0.516,\ c = 1,\ d = -0.45,\ r = 1$$

one obtains the optimal values

$$k_{opt} = 2,0391, \quad T_{d_{opt}} = -1,2100$$

for which the condition (42) is not satisfied because

$$| d | + | cT_d | = 1,66 > 1 \tag{59}$$

From (59) it follows that the optimization procedure may lead to instability of the closed-loop system and therefore the constraints for elements d and T_d given by a formula (42) are necessary.

7. Conclusions

The paper presents a parametric optimization problem for a system with two delays both retarded and neutral type time delay with a PD-controller. The method for computing of the performance index value bases on determining a Lyapunov functional defined on the state space which value for initial state is equal to the performance index. In the paper form of the Lyapunov functional is assumed and the method for computing its coefficients is presented. An example shows that the optimization procedure may lead to instability of the closed-loop system and therefore the matrices D and T_d need to be constrained by the formula (6).

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