# Positive time-varying continuous-time linear systems and electrical circuits

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**Abstract.** The positivity of time-varying continuous-time linear systems and electrical circuits are addressed. Necessary and sufficient conditions for the positivity of the systems and electrical circuits are established. It is shown that there exists a large class of positive electrical circuits with time-varying parameters. Examples of positive electrical circuits are presented.

**Key words:** positive, linear, time-varying, system, electrical circuit, stability, test.

## 1. Introduction

A dynamical system is called positive if its trajectory starting from any nonnegative initial state remains forever in the positive orthant for all nonnegative inputs. An overview of state of the art in positive theory is given in the monographs [1, 2]. Variety of models having positive behavior can be found in engineering, economics, social sciences, biology and medicine, etc..

The Lyapunov, Bohl and Perron exponents and stability of time-varying discrete-time linear systems have been investigated in [3–8]. The positivity and stability of fractional time varying discrete-time linear systems have been addressed in [9–13] and the stability of continuous-time linear systems with delays in [14]. The fractional positive linear systems have been analyzed in [15–20]. The positive electrical circuits and their reachability have been considered in [21] and the controllability and observability in [22]. The stability and stabilization of positive fractional linear systems by state-feedbacks have been analyzed in [19]. The normal positive electrical circuits has been introduced in [23].

In this paper positivity of time-varying continuous-time linear systems and electrical systems is addressed.

The paper is organized as follows. In Sec. 2 the solution to the time-varying linear systems and their properties are recalled. Necessary and sufficient conditions for the positivity of time-varying continuous-time linear systems are established in Sec. 3. The positive electrical circuits with time-varying parameter are addressed in Sec. 4. Concluding remarks are given in Sec. 5.

The following notation is used: $\Re$ – the set of real numbers, $\Re^{n \times m}$ – the set of $n \times m$ real matrices, $\Re_+^{n \times m}$ – the set of $n \times m$ matrices with nonnegative entries and $\Re_+^n = \Re_+^{n \times 1}$, $M_n$ – the set of $n \times n$ Metzler matrices (real matrices with nonnegative off-diagonal entries), $I_n$ – the $n \times n$ identity matrix, $T$ – denotes the transposition of matrix (vector).

## 2. Preliminaries

Consider the matrix linear differential equation with time-varying coefficients [24, 25]

$$\frac{dX}{dt} = A(t)X, \tag{1}$$

where $X = X(t) \in \Re^{n \times n}$ and $A(t) \in \Re^{n \times n}$ with entries $a_{ij}$ being continuous-time functions of time $t \in [0, +\infty)$.

To solve Eq. (1) the Picard method is used

$$\frac{dX_k}{dt} = A(t)X_{k-1} \quad \text{for} \quad k = 1, 2, \dots \tag{2}$$

and $X_k$ is the $k$-th iteration of solution for differential Eq. (2).

From (2) we obtain

$$X_k = I_n + \int_{t_0}^{t} A(\tau)X_{k-1}(\tau)d\tau, \tag{3}$$

where $X(t_0) = I_n$.

Using (3) for $k = 1, 2, \dots$ and

$$\Omega_{t_0}^{t}(A) = I_n + \int_{t_0}^{t} A(\tau)d\tau + \int_{t_0}^{t} A(\tau) \int_{t_0}^{\tau} A(\tau_1)d\tau_1(\tau)d\tau + \dots \tag{4}$$

we can write the solution of (1) in the form

$$X(t) = \Omega_{t_0}^{t}(A)X_0 \tag{5}$$

and $X_0 = X(t_0)$ is the initial condition.

It is easy to show [24] that

$$\Omega_{t_0}^{t}(A) = \Omega_{t_0}^{t_1}(A)\Omega_{t_1}^{t}(A)$$
$$\text{for} \quad t_0 < t_1 < t \in [0, +\infty). \tag{6}$$

**Lemma 1.** If the matrix $A(t) \in \Re^{n \times n}$ satisfy the condition

$$A(t_1)A(t_2) = A(t_2)A(t_1)$$
$$\text{for} \quad t_1, t_2 \in [t_0, t] \in [0, +\infty) \tag{7}$$

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then

$$\Omega_{t_0}^t(A) = e^{\int_{t_0}^t A(\tau)d\tau}. \tag{8}$$

Proof is given in [24].

**Lemma 2.** If $A_1 = A_1(t) \in \Re^{n \times n}$ and $A_2 = A_2(t) \in \Re^{n \times n}$, $t \in [0, +\infty)$, then

$$\Omega_{t_0}^t(A_1 + A_2) = \Omega_{t_0}^t(A_1)\Omega_{t_0}^t(A), \tag{9}$$

where

$$A = A(t) = [\Omega_{t_0}^t(A_1)]^{-1}A_2\Omega_{t_0}^t(A_1). \tag{10}$$

**Proof.** Let

$$X = X(t) = \Omega_{t_0}^t(A_1), \quad Y = Y(t) = \Omega_{t_0}^t(A_1 + A_2). \tag{11}$$

Differentiating $Y = XZ$ we obtain

$$\frac{dY}{dt} = \frac{dX}{dt}Z + X\frac{dZ}{dt}$$

and taking into account that

$$\frac{dY}{dt} = (A_1 + A_2)Y \quad \text{and} \quad \frac{dX}{dt} = A_1 X$$

we have

$$(A_1 + A_2)XZ = A_1 XZ + X\frac{dZ}{dt}$$

or

$$A_2 XZ = X\frac{dZ}{dt}. \tag{12}$$

Solving (12) we obtain

$$\frac{dZ}{dt} = X^{-1}A_2 XZ \tag{13}$$

and

$$Z = \Omega_{t_0}^t(X^{-1}A_2 X). \tag{14}$$

This completes the proof.

**Lemma 3.** Let $A \in \Re^{n \times n}$ be a matrix with constant entries independent of time $t$. If $A(t) = A$ then

$$\Omega_{t_0}^t(A) = e^{A(t-t_0)}. \tag{15}$$

Now let us consider the time-varying system described by the equation

$$\dot{x}(t) = A(t)x + B(t)u, \tag{16}$$

where $x = x(t) \in \Re^n$, $u = u(t) \in \Re^m$ are the state and input vectors and $A(t) \in \Re^{n \times n}$, $B(t) \in \Re^{n \times m}$ are matrices with entries depending continuously on time $t \in [0, +\infty)$.

**Lemma 4.** The solution of Eq. (16) with given initial condition $x_0 = x(t_0) \in \Re^n$ and input $u(t) \in \Re^m$ has the form

$$x = \Omega_{t_0}^t(A)x(t_0) + \int_{t_0}^t K(t,\tau)B(\tau)u(\tau)d\tau \tag{17a}$$

where

$$K(t,\tau) = \Omega_{t_0}^t(A)[\Omega_{t_0}^\tau(A)]^{-1}. \tag{17b}$$

Proof is given in [25].

## 3. Positive time-varying continuous-time linear systems

Consider the time-varying linear system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t), \tag{18a}$$

$$y(t) = C(t)x(t) + D(t)u(t), \tag{18b}$$

where $x(t) \in \Re^n$, $u(t) \in \Re^m$, $y(t) \in \Re^p$ are the state, input and output vectors and $A(t) \in \Re^{n \times n}$, $B(t) \in \Re^{n \times m}$, $C(t) \in \Re^{p \times n}$, $D(t) \in \Re^{p \times m}$ are real matrices with entries depending continuously on time and $\det A(t) \neq 0$ for $t \in [0, +\infty)$.

**Definition 1.** The system (18) is called positive if $x(t) \in \Re_+^n$, $y(t) \in \Re_+^p$, $t \in [0, +\infty)$ for any initial conditions $x_0 \in \Re_+^n$ and all inputs $u(t) \in \Re_+^m$, $t \in [0, +\infty)$.

**Theorem 1.** Let $A(t) \in \Re^{n \times n}$, $t \in [0, +\infty)$. Then

$$\Omega_{t_0}^t(A) = I_n + \int_{t_0}^t A(\tau)d\tau$$

$$+ \int_{t_0}^t A(\tau)\int_{t_0}^\tau A(\tau_1)d\tau_1(\tau)d\tau + ... \in \Re_+^{n \times n} \tag{19}$$

$$\text{for} \quad t \geq t_0$$

if and only if $A(t) \in M_n$, $t \in [0, +\infty)$.

**Proof.** *Necessity.* From (19) it follows that $\Omega_{t_0}^t(A) \in \Re_+^{n \times n}$ for small value of $t > t_0$ only if $A(t) \in M_n$.

**Sufficiency.** Let choose constant $\lambda > 0$ such that

$$\lambda \geq \max_{1 \leq i \leq n}|a_{ii}(t)|, \tag{20}$$

where $a_{ii}(t)$ is the $i$-th $i = 1, 2, ..., n$ diagonal entry of $A(t)$. In this case if $A(t) \in M_n$ then $A(t) + I_n\lambda \in \Re_+^{n \times n}$ for $t \in [0, +\infty)$. Taking into account that $A(t) = [A(t) + I_n\lambda] - I_n\lambda$ and Lemmas 2, 3 for $A_1(t) = -I_n\lambda$ and $A_2(t) = A(t) + I_n\lambda$ we obtain

$$\Omega_{t_0}^t(A) = \Omega_{t_0}^t(-I_n\lambda)\Omega_{t_0}^t(\overline{A})$$

$$= e^{-I_n\lambda(t-t_0)}\Omega_{t_0}^t(A_2) \in \Re_+^{n \times n} \tag{21}$$

$$\text{for} \quad t \in [0, +\infty)$$

since

$$\Omega_{t_0}^t(-I_n\lambda) = e^{-I_n\lambda(t-t_0)}$$

$$\text{and} \quad \overline{A} = [\Omega_{t_0}^t(-I_n\lambda)]^{-1}A_2(t)\Omega_{t_0}^t(-I_n\lambda)$$

$$= A_2(t) \in \Re_+^{n \times n} \tag{22}$$

$$\text{for} \quad t \in [0, +\infty)$$

if $A(t) \in M_n$. This completes the proof.

**Theorem 2.** The time-varying linear system (18) is positive if and only if

$$A(t) \in M_n, \qquad B(t) \in \Re_+^{n \times m},$$

$$C(t) \in \Re_+^{p \times n}, \qquad D(t) \in \Re_+^{p \times m}, \tag{23}$$

$$t \in [0, +\infty).$$

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**Proof. Sufficiency.** By Lemma 4 the solution of Eq. (18a) is given by (17a) and if $x(t) \in \Re_+^n$, $t \in [0, +\infty)$ if $A(t) \in M_n$ and $B(t) \in \Re_+^{n \times m}$ for $t \in [0, +\infty)$ since form Theorem 1 we have $\Omega_{t_0}^t(A) \in \Re_+^{n \times n}$, $K(t, \tau) \in \Re_+^{n \times n}$ and by assumption $x(t_0) \in \Re_+^n$, $u(t) \in \Re_+^m$, $t \in [0, +\infty)$.

From Eq. (18a) we have $y(t) \in \Re_+^p$, $t \in [0, +\infty)$ since $C(t) \in \Re_+^{p \times n}$, $D(t) \in \Re_+^{p \times m}$ and $x(t) \in \Re_+^n$, $u(t) \in \Re_+^m$, $t \in [0, +\infty)$.

*Necessity.* Let $u(t) = 0$ for $t \in [0, +\infty)$ and $x(t_0) = e_i$ ($i$-th column of $I_n$). The trajectory does not leave the orthant $\Re_+^n$ only if $\dot{x}(t) = A(t)e_i \in \Re_+^n$ what implies $a_{ij} \geq 0$ for $i \neq j$ and $A(t) \in M_n$, $t \in [0, +\infty)$. From the same reason for $x(t_0) = 0$ we have $\dot{x}(t) = B(t)u(t) \in \Re_+^n$ what implies $B(t) \in \Re_+^{n \times m}$ for $t \in [0, +\infty)$ since $u(t) \in \Re_+^m$ can be arbitrary. From (18b) for $u(t) = 0$, $t \in [0, +\infty)$ we have $y(t_0) = C(t)x(t_0) \in \Re_+^p$ and $C(t) \in \Re_+^{p \times n}$, $t \in [0, +\infty)$ since $x(t_0) \in \Re_+^n$ can be arbitrary. In a similar way assuming $x(t_0) = 0$ we obtain $y(t_0) = D(t)u(t_0) \in \Re_+^p$ and $D(t) \in \Re_+^{p \times m}$ since $u(t_0) \in \Re_+^m$ is arbitrary.

**Example 1.** Consider the positive time-varying continuous-time linear system (18a) with the matrices

$$A(t) = \begin{bmatrix} -1 & 2 & 0 \\ 0 & -e^{-t} & 0 \\ 1 & e^{-t} & -e^{-t} \end{bmatrix},$$

$$B(t) = \begin{bmatrix} 1.1 + \sin t \\ e^{-t} \\ 1.2 + \cos t \end{bmatrix}.$$

(24)

The system described by (18a) with the matrices (21) is positive since $A(t) \in M_3$ and $B(t) \in \Re_+^3$ for $t \in [0, +\infty)$.

# 4. Positive time-varying linear electrical circuits

**Example 2.** Consider the time-varying electrical circuit shown in Fig. 1 with given nonzero resistances $R_1(t)$, $R_2(t)$, $R_3(t)$ inductances $L_1(t)$, $L_2(t)$ depending on time $t$, and source voltages $e_1(t)$, $e_2(t)$.
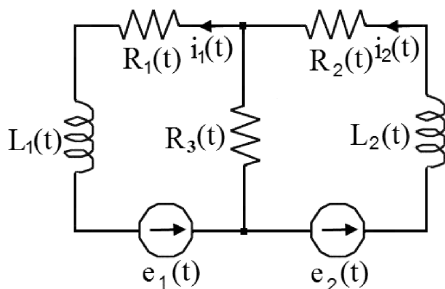


Fig. 1. Electrical circuit

Using Kirchhoff's laws, we can write the equation

$$e_1(t) = \left[ R_1(t) + \frac{dL_1(t)}{dt} \right] i_1(t) + L_1(t)\frac{di_1(t)}{dt}$$
$$+ R_3(t)[i_1(t) - i_2(t)],$$

$$e_2(t) = \left[ R_2(t) + \frac{dL_2(t)}{dt} \right] i_2(t) + L_2(t)\frac{di_2(t)}{dt}$$
$$+ R_3(t)[i_2(t) - i_1(t)],$$

(25)

which can be written in the form

$$\frac{d}{dt}\begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix} = A(t)\begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix} + B(t)\begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix}, \quad (26a)$$

where

$$A(t) = \begin{bmatrix} -\dfrac{R_1(t) + R_3(t) + a^*}{L_1(t)} & \dfrac{R_3(t)}{L_1(t)} \\ \dfrac{R_3(t)}{L_2(t)} & -\dfrac{R_2(t) + R_3(t) + b^*}{L_2(t)} \end{bmatrix},$$

$$B(t) = \begin{bmatrix} \dfrac{1}{L_1(t)} & 0 \\ 0 & \dfrac{1}{L_2(t)} \end{bmatrix},$$

(26b)

where

$$a^* = \frac{dL_1(t)}{dt} \qquad \text{and} \qquad b^* = \frac{dL_2(t)}{dt}.$$

From (26b) it follows that for $R_1(t) > 0$, $R_2(t) > 0$, $R_3(t) > 0$, $L_1(t) > 0$, $L_2(t) > 0$ and $e_1(t) \geq 0$, $e_2(t) \geq 0$ for $t \in [0, +\infty)$ the matrix $A(t) \in M_2$ and $B(t) \in \Re_+^{2 \times 2}$ for $t \in [0, +\infty)$. Therefore, the electrical circuit is a positive time-varying system.

**Example 3.** Consider the time-varying electrical circuit shown in Fig. 2 with given nonzero resistances $R_1(t)$, $R_2(t)$, $R_3(t)$, inductance $L(t) > 0$, capacitance $C(t) > 0$ and source voltage $e(t) \geq 0$ for $t \in [0, +\infty)$.
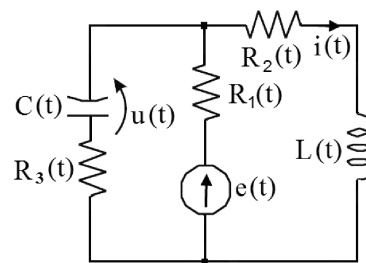


Fig. 2. Electrical circuit

It is shown that the electrical circuit is a positive time-varying linear system if and only if $R_1(t) = 0$ for $t \in [0, +\infty)$.

Using Kirchhoff's laws, we can write the equation

$$e(t) = R_1(t)\left[i(t) + C(t)\frac{du(t)}{dt} + \frac{dC(t)}{dt}u(t)\right]$$
$$+ R_3(t)\left[C(t)\frac{du(t)}{dt} + \frac{dC(t)}{dt}u(t)\right] + u(t),$$

(27)

$$e(t) = R_1(t)\left[i(t) + C(t)\frac{du(t)}{dt} + \frac{dC(t)}{dt}u(t)\right]$$
$$+ L(t)\frac{di(t)}{dt} + \frac{dL(t)}{dt}i(t) + R_2(t)i(t),$$

which can be written in the form

$$\frac{d}{dt}\begin{bmatrix} i(t) \\ u(t) \end{bmatrix} = A(t)\begin{bmatrix} i(t) \\ u(t) \end{bmatrix} + B(t)e(t), \qquad \text{(28a)}$$

where

$$A(t) = \begin{bmatrix} 0 & c^*C(t) \\ L(t) & R_1(t)C(t) \end{bmatrix}^{-1}$$

$$\cdot \begin{bmatrix} -R_1(t) & -c^*\dfrac{dC(t)}{dt} - 1 \\ -R_1(t) - R_2(t) - \dfrac{dL(t)}{dt} & -R_1(t)\dfrac{dC(t)}{dt} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{R_1^2(t)}{c^*L(t)} - \dfrac{R_1(t) + R_2(t) + \dfrac{dL(t)}{dt}}{L(t)} & -\dfrac{R_1(t)}{c^*L(t)} \\ -\dfrac{R_1(t)}{c^*C(t)} & -\dfrac{c^*\dfrac{dC(t)}{dt} + 1}{c^*C(t)} \end{bmatrix},$$

$$B(t) = \begin{bmatrix} 0 & c^*C(t) \\ L(t) & R_1(t)C(t) \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \dfrac{R_3(t)}{c^*L(t)} \\ \dfrac{1}{c^*C(t)} \end{bmatrix},$$

(28b)

where

$$c^* = [R_1(t) + R_3(t)].$$

From (28b) it follows that $A(t) \in M_2$ if and only if $R_1(t) = 0$ for $t \in [0, +\infty)$. Therefore, the electrical circuit is a positive time-varying system if and only if $R_1(t) = 0$ for $t \in [0, +\infty)$.

Now let us consider the electrical circuit shown in Fig. 3 with given positive resistances $R_k(t)$, $k = 0, 1, ..., n$, inductances $L_i(t)$, $i = 2, 4, ..., n_2$, capacitances $C_j(t)$, $j = 1, 3, ..., n_1$ depending on time $t$ and source voltages $e_1(t), e_2(t), ..., e_n(t)$. We shall show that this electrical circuit is a positive and asymptotically stable time-varying linear system.

Using Kirchhoff's law we can write the equations

$$e_1(t) = R_k(t)C_k(t)\frac{du_k(t)}{dt}$$
$$+ \left[R_k(t)\frac{dC_k(t)}{dt} + 1\right]u_k(t) \qquad \text{(29a)}$$
$$\text{for} \qquad k = 1, 3, \ldots, n_1,$$

$$e_1(t) + e_k(t) = L_k(t)\frac{di_k(t)}{dt}$$
$$+ \left[R_k(t) + \frac{dL_k(t)}{dt}\right]i_k(t) + u_k(t) \qquad \text{(29b)}$$
$$\text{for} \qquad k = 2, 4, \ldots, n_2,$$

which can be written in the form

$$\frac{d}{dt}\begin{bmatrix} u(t) \\ i(t) \end{bmatrix} = A(t)\begin{bmatrix} u(t) \\ i(t) \end{bmatrix} + B(t)e(t), \qquad \text{(30a)}$$

where

$$u(t) = \begin{bmatrix} u_1(t) \\ u_3(t) \\ \vdots \\ u_{n_1}(t) \end{bmatrix}, \qquad i(t) = \begin{bmatrix} i_2(t) \\ i_4(t) \\ \vdots \\ i_{n_2}(t) \end{bmatrix},$$

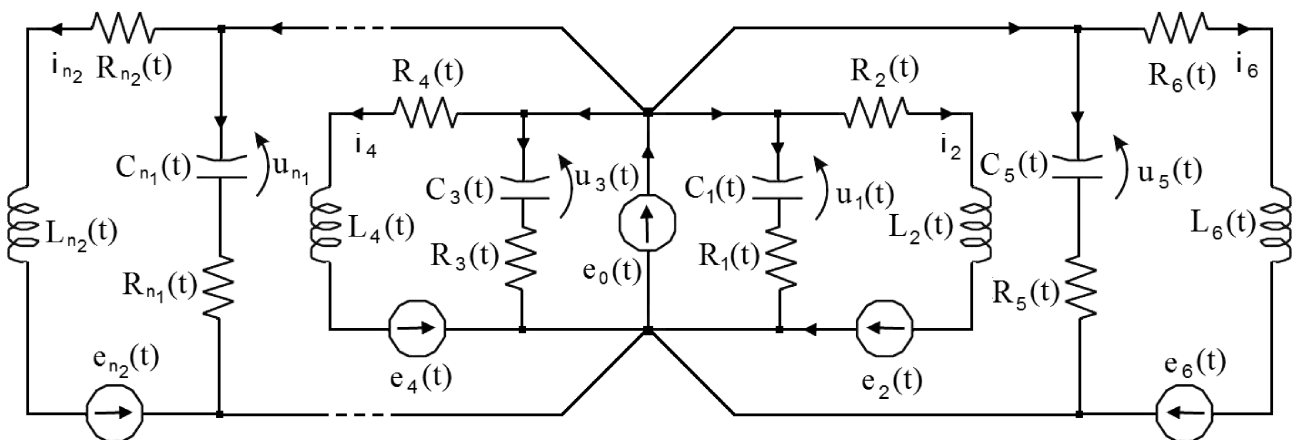$$e(t) = \begin{bmatrix} e_1(t) \\ e_3(t) \\ \vdots \\ e_n(t) \end{bmatrix}, \qquad (n = n_1 + n_2) \qquad \text{(30b)}$$



Fig. 3. Positive and stable electrical circuit

and

$$A(t) = \text{diag}[-a_1(t), -a_3(t), ..., -a_{n_1}(t),$$

$$-a_2(t), -a_4(t), ..., -a_{n_2}(t)],$$

$$a_k(t) = \frac{R_k(t)\dfrac{dC_k(t)}{dt} + 1}{R_k(t)C_k(t)} \quad \text{for} \quad k = 1, 3, ..., n_1,$$

$$a_k(t) = \frac{R_k(t) + \dfrac{dL_k(t)}{dt}}{L_k(t)} \quad \text{for} \quad k = 2, 4, ..., n_2,$$

$$B(t) = \begin{bmatrix} B_1(t) \\ B_2(t) \end{bmatrix},$$

$$B_1(t) = \begin{bmatrix} \dfrac{1}{R_1(t)C_1(t)} & 0 & 0 & ... & 0 \\ \dfrac{1}{R_3(t)C_3(t)} & 0 & 0 & ... & 0 \\ \vdots & & \vdots & \vdots & ... & \vdots \\ \dfrac{1}{R_{n_1}(t)C_{n_1}(t)} & 0 & 0 & ... & 0 \end{bmatrix},$$

$$B_2(t) = \begin{bmatrix} \dfrac{1}{L_2(t)} & \dfrac{1}{L_2(t)} & 0 & ... & 0 \\ \dfrac{1}{L_4(t)} & 0 & \dfrac{1}{L_4(t)} & ... & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \dfrac{1}{L_{n_2}(t)} & 0 & 0 & ... & \dfrac{1}{L_{n_2}(t)} \end{bmatrix}.$$

(30c)

The electrical circuit is positive time-varying linear system since all diagonal entries of the matrix $A(t)$ are negative functions of $t \in [0, +\infty)$ and the matrix $B(t)$ has nonnegative entries for $t \in [0, +\infty)$ if $\dfrac{dL_k(t)}{dt} \geq 0$ and $\dfrac{dC_k(t)}{dt} \geq 0$. The solution of Eq. (29a) can be found using Lemma 1.

## 5. Concluding remarks

The positivity of time-varying continuous-time linear systems and electrical circuits have been addressed. Necessary and sufficient conditions for the positivity of the system and electrical circuits have been established. It has been shown that there exists a large class of positive electrical circuits with time-varying parameters. The considerations have been illustrated by positive electrical circuits. The consideration can be extended to fractional time-varying electrical circuits.

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