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A logical device for processing nautical data

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Abstract

Nautical measurements are randomly and systematically corrupted. There is a rich scope of knowledge regarding the randomness shown by results of observations. The distribution of stochastic distortions remains an estimate and is imprecise with respect to their parameters. Uncertainties can also occur through the subjective assessment of each piece of available data. The ability to model and process all of the aforementioned items through traditional approaches is rather limited. Moreover, the results of observations, the final outcome of a quality evaluation, can be estimated prior to measurements being taken. This *a posteriori* analysis is impaired and it is outside the scope of traditional, inaccurate data handling methods. To propose new solutions, one should start with an alternative approach towards modelling doubtfulness. The following article focusses on belief assignments that may benefit from the inclusion of uncertainty. It starts with a basic interval uncertainty model. Then, assignments engaging fuzzy locations around nautical indications are discussed. This fragment includes transformation from density functions to probability distributions of random errors. Diagrams of the obtained conversions are included. The presentation concludes with a short description of a computer application that implements the presented ideas.

Introduction

The Mathematical Theory of Evidence (MTE) operates on the principle of belief assignments or belief functions. It exploits evidence sets and hypothesis frames. It is widely believed that the evidence at hand supports each of the considered hypothesis items, although degrees of endorsement vary in real terms (Dempster, 1968; Shafer, 1976). Relationships between the involved universes/frames are encoded within belief assignments. The measures of included support are belief and plausibility values. The theory also offers combination mechanisms in order to increase the informative context of the initial evidence. The combination scheme delivers results that support hypothesis space elements. The evidence is considered as a collection of facts and knowledge related to the observations. In navigation, facts are results of observations such as taken bearings, distances or horizontal/vertical angles. A combination scheme is expected to enable one to obtain the best solution provided by the given observations/measurements and knowledge of their quality. This theory has already been successfully implemented in many fields, some of which are related to the discussed area of application (Srivastava, Dutta & Johns, 1996; Ayoun & Smets, 2001; Filipowicz, 2009; 2014; 2014a).

The results of observations can be affected by random and systematic errors. Randomness is assumed to be governed by various distributions. The quality of measurements taken with different navigational aids differs. One may notice that discrepancies in the estimation of the randomness of distribution parameters have a prevailing character. It is a popular statement, related to nautical observations, that the mean error of a bearing taken with a radar is falls in the range [±1°; ±2.5°]. This appears as a fuzzy figure with a core and support as the respective intervals: [−1°; +1°], [−2.5°; +2.5°]. Evaluation of the mean error requires the fuzziness to be accepted and taken into account during computations.

Random distortions are inevitable; however, seafarers are aware of this fact. This phenomenon is called aleatory uncertainty; it cannot be eliminated, but it might be evaluated to a certain extent. It is crucial to include this type of doubtfulness into a processing scheme. Most analyses exploit theoretical distributions but the empirical approach can be used instead. The most important thing is the ability to objectively evaluate the obtained output along with measures indicating its accuracy, i.e. the probability of achieving alternative results (Filipowicz, 2015). It is also necessary to emphasise how uncertainties included in input data propagate to give the obtained results.

There are publications devoted to the implementation of the MTE concept in nautical science (Filipowicz, 2010; 2016). Most of them discuss practical navigational aspects of the concept. This paper presents details regarding aleatory uncertainty modelling and processing. This paper presents a basic uncertainty model. Then, the model is used while analysing erroneous observations. Fuzzy probability sets are introduced in the wake of the discussion of measurement errors. MTE was exploited in order to convert probability density into probability distributions. It is proven that conversion from fuzzy values to crisp ones is straightforward in nautical science. A combination scheme was introduced in order to show that the input uncertainty can be transferred to the obtained results. Finally, simple belief assignment was presented and its advantages depicted. The paper concludes with a presentation of a computer application implementing the presented ideas, showing a block diagram of the software solution and some practical results.

Basic interval uncertainty model

The popular basic uncertainty model includes a proposition and an associated range of probabilities, also called a belief interval (Lee & Zhu, 1995). Given proposition z and the range of real values $[a, b]$, one can define the model in terms of the truth of the statement. It can take the form of Equation 1:

$$z:[a,b]; \quad a,b \in [0,1] \quad \text{and} \quad a \le b \qquad (1)$$

where: a – upper limit of probability that the proposition z is true; $b–a$ – range of uncertainty, possibility that the truth of z is defined by a descending function; and b – lower limit of probability that proposition z is false.

The proposition statement and its contradiction can be transferred into the belief assignment (see also Denoeux, 2000). The assignment shown in Table 1 engages two elements' hypothesis space, Θ, reflecting the truth (true or false) of the considered proposition. Thus, one of the items is marked with z and another with $\neg z$, meaning the negation of z. Within the probability assignment, all elements of the power set of the considered frame might appear, consequently a multiple items subset of the frame: $\{z; \neg z\}$ is present in the table. The mass attributed to such a set expresses doubtfulness; all the considered items are equally possible. For this reason, the last row refers to uncertainty, since it involves the whole frame of discernment.

Table 1. Basic probability assignment for the uncertainty model

Notation	Probability value
$m(z)$	a
$m(\neg z)$	$1 - b$
$m(z, \neg z) = m(\Theta)$	$b - a$

$m(z)$ probability mass that proposition z is true
$m(\neg z)$ probability mass that the negation of proposition z is true
$m(z, \neg z)$ range within which one can doubt that the proposition is true

Usually, membership functions show the possibility of x belonging to two fuzzy probability sets indicated with a tilde, for example $\widetilde{S}_1, \widetilde{S}_2$. In the considered case, the first set covers the range $[0; b]$ and the second $[a; 1]$. Within the range $[0; a]$, the possibility of a true proposition z is equal to one, then it descends and reaches zero at point b. Interval $[a; b]$ contains an amount of ignorance and doubtfulness in the truth of proposition z. Furthermore, the statement cannot be true, contrary to $\neg z$ for which the possibility of being true is one within the rightmost $[b; 1]$ range. The value of a can be treated as the belief that z is true, and the upper bound, b, refers to the plausibility that z is true (proof may be found in (Lee & Zhu, 1995)).

Uncertainty model for nautical science

Uncertainty, which is related to random and systematic measurement deflections, is present in all nautical measurements. An observation made with a navigational aid is randomly deflected and can be treated as an instance of a random variable, governed by some kind of distribution. The Gaussian bell function is often used, although discrepancies in the parameters of such distributions frequently occur. It is a common statement (Jurdziński, 2014) that the mean error of a distance taken with a radar is distorted and its true value falls within the range of [±1%; ±2.5%] of the measured distance. Seafarers know much about the unavoidable random nature of measurements. This sort of doubtfulness can be referred to as aleatory uncertainty.

A graphical interpretation of the accuracy evaluation statement is presented in Figure 1. The limits of the introduced approximate range are three standard deviations of the bell functions. Therefore, for the measured distance of 10 km: σ_{min} = 33.33 m, σ_{max} = 83.33 m.

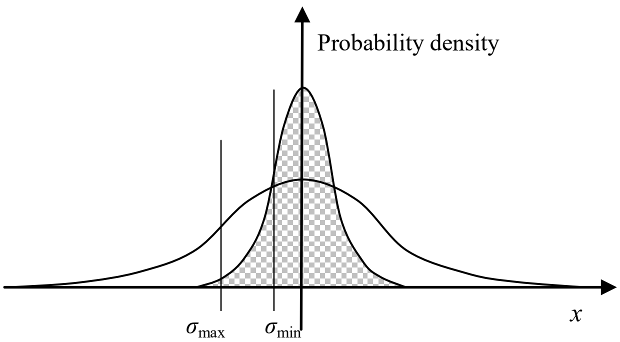

Figure 1. Interpretation of statement regarding popular accuracy evaluation

The range-valued evaluation of the mean error appears as a fuzzy figure; thus, fuzziness appears to be a helpful tool in constructing an adequate observation model.

The example of aleatory uncertainty related to the distance to a landmark observation is shown in Figure 2. Hereafter, it is assumed that randomness is governed by the Gaussian distribution. There are various estimates of the dispersion parameters available. Two of the dispersions, one of which is optimistic with a standard deviation σ_{min}, and the second assumed to be pessimistic with deviation σ_{max}, are presented in Figure 2. The distance measured is marked by the point on the *y*-axis and the abscissa intersection.

Given the above mentioned data, one can determine whether or not the proposition "the true distance to the landmark is represented by a point close to abscissa x_1" is true.. The measurement related proposition refers to the easily established interval [$C \cdot p_{1\ min}$; $C \cdot p_{1\ max}$], where *C* depends on the width of the considered neighbourhood of abscissa x_1, since the given diagram represents a probability density function.

Considering the presented information, the discussion of range-valued uncertainty is directly applicable to nautical science. Range-valued uncertainty is relevant while handling navigational observations. The possibility of various distances belonging to fuzzy probability sets is to be defined in a way that aims to be very much like the basic uncertainty model. Thus, as stated above, possibility and probability can be used jointly to include uncertainty in the defined mathematical model. In order to introduce the concept, appropriate fuzzy sets are to be defined regarding random distortions of nautical observations. Let us concentrate on errors in distance taking governed by the Gaussian distribution: an example is presented in Figure 3, which shows adjacent overlapping confidence intervals with fuzzy limits. Four fuzzy probability sets were established: $\widetilde{S}_1, \widetilde{S}_2, \widetilde{S}_3, \widetilde{S}_4$, which are also referred to using their corresponding cumulative probability values, for example: $\widetilde{0.673}, \widetilde{0.278}, \widetilde{0.047}, \widetilde{0.002}$.

Four confidence intervals with the rightmost mean borders at points: $i \cdot \sigma_m$ for $i \in \{1..4\}$ were introduced at first. The probabilities of each point within a given interval representing the true distance are constant. For the first one, it reaches 0.673

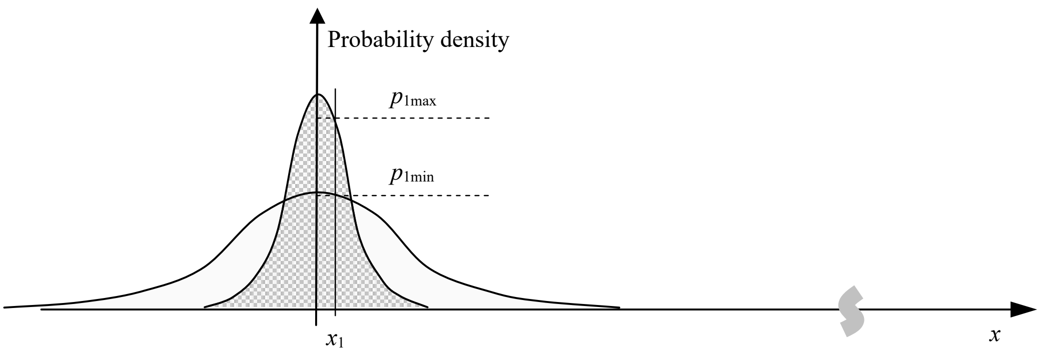

Figure 2. Aleatory uncertainty related to the distance to a landmark observation

assuming a single-sided analyses. Practical aleatory uncertainty enforces the interval-valued limits of the introduced ranges, these must be considered as: $[i\cdot\sigma_{\min};\ i\cdot\sigma_{\max}]$ for $i\in\{1..4\}$. Where σ_m is the mean of $\sigma_{\min}$ and $\sigma_{\max}$. Given fuzzy sets, one has to propose a membership function that enables the calculation of grades, i.e. the degree of belonging to each set.

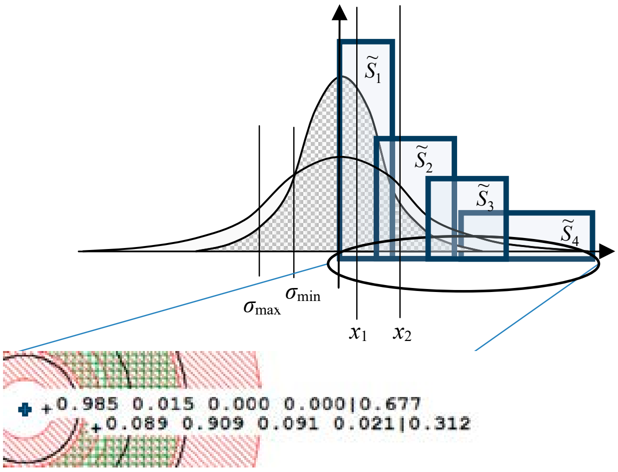

Figure 3. Adjacent overlapping confidence intervals with fuzzy limits and example membership grades for two selected points

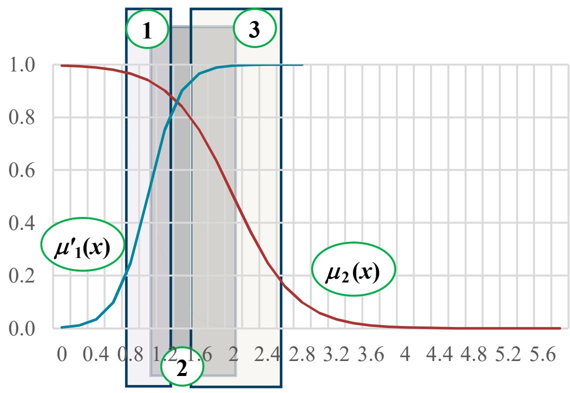

Figure 4. Second fuzzy set considered with its membership function consisting of two sigmoidal segments

The exploded insertion in Figure 3 shows two selected points' probabilities of representing the true measurement. The appropriate likelihood figures are the last of the presented data. They are preceded by membership grades calculated for each of the four considered fuzzy sets. The membership grades were calculated using sigmoidal functions. Their parameters were obtained with an algorithm which guarantees the highest descending rate of the sigmoidal function regarding the given standard deviation range (see Figure 4). Membership grades within introduced fuzzy probability sets and the probability of a point representing the true measurement are shown in Table 2. Note that in nautical science, the grade of belonging has geographical (geometrical) meaning.

As seen in Figure 1, the standard deviations of probability density dispersions can differ. Thus, instead of discussing confidence intervals with crisp limits, one should try to introduce their fuzzy borders. As it is widely assumed, standard deviations are usually interval-valued and separately defined based on a series of experiments. The second fuzzy set, $\tilde{S}_2$, considered in the paper, with its membership function consisting of two sigmoidal fragments, is illustrated in Figure 4. The function μ'_1 indicates not belonging to the first fuzzy set, $\tilde{S}_1$, at the same time, μ_2 indicates inclusion in the second set, $\tilde{S}_2$.

Table 2. Membership grades within introduced fuzzy probability sets and belief and plausibility measures of two points representing the true measurement

Fuzzy set name	Probability	Membership grades: $\mu(x_1)$	Membership grades: $\mu(x_2)$
$\tilde{S}_1$	0.673	0.985	0.089
$\tilde{S}_2$	0.278	0.015	0.909
$\tilde{S}_3$	0.047	0	0.091
$\tilde{S}_4$	0.002	0	0.021
bel(..)		0.604	0.254
pl(..)		0.677	0.317
PM		[0.604; 0.677]	[0.254; 0.317]

PM interval-valued probability mass that the point represents the true measurement

In discrete domains, fuzziness denotes membership within each item of the considered frame (Yen, 1990). Hereto inclusions into the introduced fuzzy sets are considered instead. From Table 2, it can be seen that point number 1 is located within the first and second set with degrees of 0.985 and 0.015 respectively. For point number 2, these values are 0.089 and 0.909. Additionally, the second point belongs to sets 3 and 4 with respective grades of 0.091 and 0.021. It should be stressed that degrees of belonging may sum to greater than one as the sum grows with increasing uncertainty of the data at hand.

The shaded part of Table 2 embraces belief assignment regarding the locations of points x_1 and x_2 as shown in Figure 3. The depicted assignment includes locations within consecutive fuzzy sets along with the probabilities assigned to these sets.

Given the assignment of this type, one can calculate the belief and plausibility supporting the truth of the considered statement. Formula (2) presents the general expressions for obtaining the required supporting data for normal fuzzy set, $\widetilde{A}$, embedded within the collection (Denoeux, 2000; Filipowicz, 2009a). Note that set $\widetilde{A}$ contains full membership of a single point from those within the hypothesis frame.

$$bel(x_l) = \sum_{k=1}^{n} m(\mu_k(x_i)) \cdot \mu_k(x_l) \cdot \min_{x_i \in \Omega;\, i \neq l} (\neg \mu_k(x_i))$$
$$pl(x_l) = \sum_{k=1}^{n} m(\mu_k(x_i)) \cdot \mu_k(x_l) \qquad (2)$$

where: x_i is i-th item of the considered frame Ω; $\widetilde{A}$ is a fuzzy set pattern of the singleton type {0, 1, 0, …}.

Applying Formula (2) with the $\widetilde{A}$ fuzzy set, one obtains the data presented in the last rows of Table 2. Due to the inequality of both measures, one arrives at the interval-valued probability that expresses the sought support for the considered proposition. Its belief and plausibility values for the location of x_1 representing the true measurement are 0.604 and 0.677 respectively.

The above discussion can be perceived from the perspective of propositions involving a set of points. In the example, a disjunctive kind of statement referring to atomic points x_1 or x_2, or even a molecule containing both points, as representatives of the true measurement is considered. Whether the proposition is true or not can be sought in the context of any single point or cluster of items within the considered space. The truth of the statement can be calculated using Formula (2), which is valid for a single point of interest. Support of the result, obtained for four fuzzy sets, is interval-valued. Calculating the belief measure, one has to reduce its value in order to take into account any other point located within the given fuzzy set. The general rule that "the greater the likelihood of something else located within a set, the smaller the belief measure" is valid for all considered fuzzy sets: note that this does not apply to the plausibility value.

Table 3. Membership grades within the introduced fuzzy probability sets and belief and plausibility measures of point x_1 representing the true measurement

Fuzzy set name	Probability	$\mu(x_1)$
$\widetilde{S}_1$	0.673	0.985
$\widetilde{S}_2$	0.278	0.015
$\widetilde{S}_3$	0.047	0
$\widetilde{S}_4$	0.002	0
$bel(x_1)$		0.677
$pl(x_1)$		0.677

Most interesting is the case where a single point, x_i, is considered. Data referring to example point x_1 is shown in Table 3. Note that belief and plausibility measures are the same when one attempts to find out the truth of whether x_1 represents the true measurement. Thus, the supporting probability is crisp-valued. This is a very practical finding since consequently logical formulae, indicating true measurement or fixed position, appear less complicated and the computation complexity is significantly reduced.

Based on probability density distributions, their interval-valued parameters, selected confidence intervals and membership functions, one can determine crisp probability of representing the true observation as defined by Formula (3). The formula is also valid for twodimensional distributions with the same discrepancies in density distribution estimates.

$$p_i = f(d(x, y), \sigma, \Delta\sigma) \qquad (3)$$

where: d – point (x, y) at a distance from the reference position (in the presented one-dimensional examples $y = 0$), σ – standard deviation of density distribution function, $\Delta\sigma$ – discrepancy in evaluation of the standard deviation of the density distribution function.

Using the formula, one can evaluate the probability that any point (x, y) represents the true measurement provided that bell-shaped density distribution functions are assumed. Figure 5 illustrates single-sided probabilities of representing the true measurement for two cases regarding discrepancies in probability density estimates. The injective function diagrams present the respective probabilities for single sided locations regarding the taken measurement. The diagrams were obtained for two discrepancies in the standard deviation estimation. It was assumed that for the lower diagram $\sigma_{max} - \sigma_{min} = 0.3\,\sigma_m$, and for the upper one $\sigma_{max} - \sigma_{min} = \sigma_m$. In the latest case, the range of more than zero probability is extended compared to the initial density function, which is very close to zero for relative distance equal to 3. Consequently, it can be suggested that $p(3)$ can be used as a measure indicating the quality of the observation or reliability of the indication, such that "the higher the $p(3)$ value the less credible the indication".

The general rule that "the higher the discrepancy in evaluation of the initial distribution characteristics, the wider the range of the probability diagram" appears to be fairly obvious. Paradoxically, it appears that less reliable observations have higher assigned probabilities. To reduce this phenomenon, one should consider $p(3)$ as a major factor in the subjective assessment of the measurement.

One statement/single observation contributes to the definition of a single belief assignment. It is practical to have more observations (pieces of evidence) and to find out the truth of a statement from the perspective of multiple inputs. A handful of measurements might deliver crucial results once all the items are combined. The association scheme is important from the uncertainty propagation point of view. In nautical science, the scope of doubtfulness regarding each of the observations should be transferred to the analysis of the accuracy of results. The statistical approach appears of limited value in this respect.

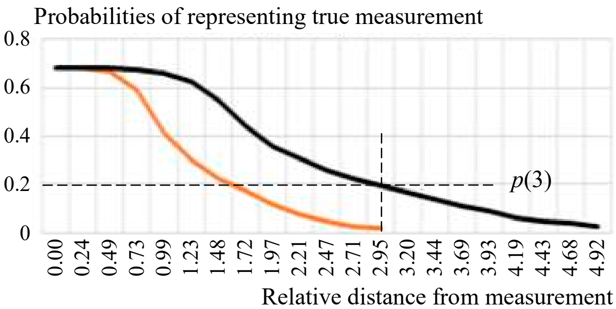

Figure 5. Single sided probabilities of representing the true measurement for two cases regarding discrepancies in density probability estimations

Considering the above transformation, belief functions can be perceived in different way as pairs of values defined by Formula 6. There are certain propositions set and levels of support embedded within the given piece of evidence e_i. In metrology and nautical science, measurements are perceived as evidence that is randomly and, potentially, systematically distorted. Evidence is also assumed to include nautical knowledge. All the above mentioned items appear as challenges to analysts once the traditional approach is exploited.

Belief assignments embrace the relationships between evidence and hypothesis frames. They tell how a single i-th piece of evidence supports each proposition, given their geographical location. In nautical science, an example hypothesis would be treating a given point as the fixed position of a ship (Filipowicz, 2012, 2014). Seeking support that a given location might represent the true observation (Filipowicz, 2014b) or looking for the distance between rescue units in a search and rescue (SAR) operation to guarantee success in detecting casualties are other practical issues.

$$\begin{aligned} &\mathrm{m}(e_i) = \\ &= \left\{ \left(z_{x,y},\, m(z_{x,y})_i\right), \left(\neg z_{x,y},\, m(\neg z_{x,y})_i\right),\, \left(\Theta, m(\Theta)_i\right) \right\} \\ &m(z_{x,y})_i = p_i \\ &m(\Theta)_i = f(\Delta\sigma, S_i) \end{aligned} \tag{4}$$

where: $z_{x,y}$ – proposition stating the truth of something located at point (x, y) (true observation, fixed position, etc.); $m(z_{x,y})_i$ – supporting mass of the proposition embedded within *the* i-th piece of evidence; S_i – observer's subjective evaluation of the i-th piece of evidence.

Belief functions in nautical applications represent evidence with encoded relationships with propositions and are subject to combination in order to increase their informative context. Representations of evidence and results of their combinations could include inconsistencies wherever null generating operations are involved. Conflicts appear when some probability is assigned to an empty set, refer to Table 4 for an example. Normalization procedures remove inconsistencies in order to avoid conflicting final results. Conflicts can cause belief to be greater than the plausibility measure. The most popular normalization procedures feature some disadvantages.

Two belief functions are presented in the shaded part of the first row and first column of Table 4. Conjunctive combination matrix cells for the above-mentioned belief assignments are not shaded. The summarised results of the association are presented in the column entitled $m_C(..)$. The data require conversion,

Table 4. Two probability assignments and their combinations with final results

	$m_2(z)$ =0.76	$m_2(\neg z)$ =0.05	$m_2(\Theta)$ =0.19	$m_C(..)$	$m_C^D(..)$	$m_C^Y(..)$
$m_1(z)$ =0.5	$m_{12}(z)$=0.38	$m_{12}(\varnothing)$=0.025	$m_{12}(z)$=0.095	0.741	0.861	0.741
$m_1(\neg z)$ =0.15	$m_{12}(\varnothing)$=0.114	$m_{12}(\neg z)$=0.008	$m_{12}(\neg z)$=0.029	0.054	0.062	0.054
$m(\Theta)$ =0.35	$m_{12}(z)$=0.266	$m_{12}(\neg z)$=0.018	$m_{12}(\Theta)$=0.067	0.067	0.077	0.206
$m(\varnothing)$				0.139	0.000	0.000

$m_C(..)$ – non-normalized combined mass values
$m_C^D(..)$ – combined mass values normalized through the Dempster method
$m_C^Y(..)$ – combined mass values normalized through the Yager method

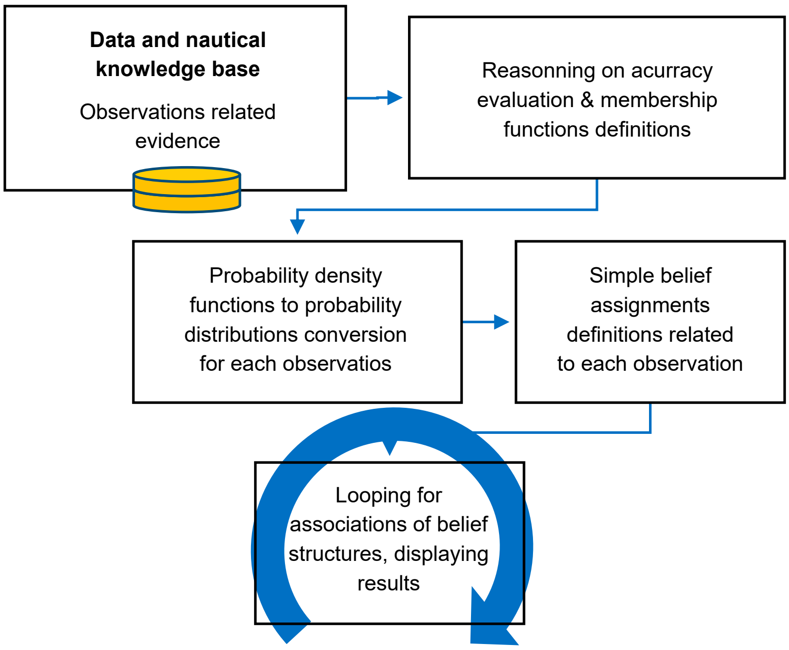

Figure 6. Block diagram of computer application implementing the presented approach

since there is some mass attributed to the empty set (which may be seen in the last row of the table). Such an assignment brings about an inconsistent, conflicting situation that should be avoided. When this occurs, mainly during null generating association, it must be eliminated. It is normalization that leads to belief structures and assignments without cases of unwanted inconsistency (Yager, 1996).

Application implementing the presented concept

Based on the above discussed ideas, a computer application was designed and implemented. It aims to calculate support for indicating a ship's position by a mouse follow-up procedure, given data and a nautical knowledge base extended by observation based evidence. Positions delivered by different navigational aids were used as input data. An example constellation of two-dimensional random variables is presented in Figure 7. The mean errors of each indication are illustrated by circles centered on the measured position. Discrepancies in dispersion evaluations were assumed high, equal to 90% for all cases ($\sigma_{\max} - \sigma_{\min} = 0.9\ \sigma_m$). At first the system is able to select the point with the highest belief/plausibility figures, it then loops through the mouse follow-up procedure.

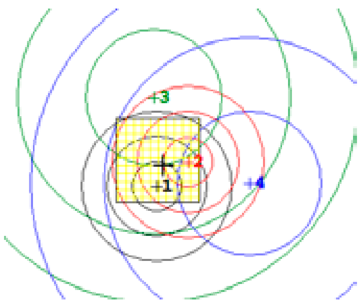

Figure 7. Example of four indications of position from various aids and the point for which the calculated belief/plausibility measures proved to be greatest

In order to indicate the required support, an evaluation of the accuracy is performed and membership functions are defined. Given those probability density functions, conversion to probability distributions for each observation is carried out. At this step, adjustment of the probability distributions is also performed considering the defined search space. Then, simple belief assignments (see Formula 4) related to each observation are developed. Further, on looping for and updating associations of belief structures, results are displayed. A block diagram of the computer application is presented in Figure 6.

In the mouse follow-up procedure, belief assignment for cursor coordinates was updated. Structures were further combined and support measures calculated and stored for future analysis. Displaying the results was the final step. An example of the application's outputs is shown in Figure 8. Plausibility values supporting the representation of the true position are shown for a coarse mesh of locations. In addition

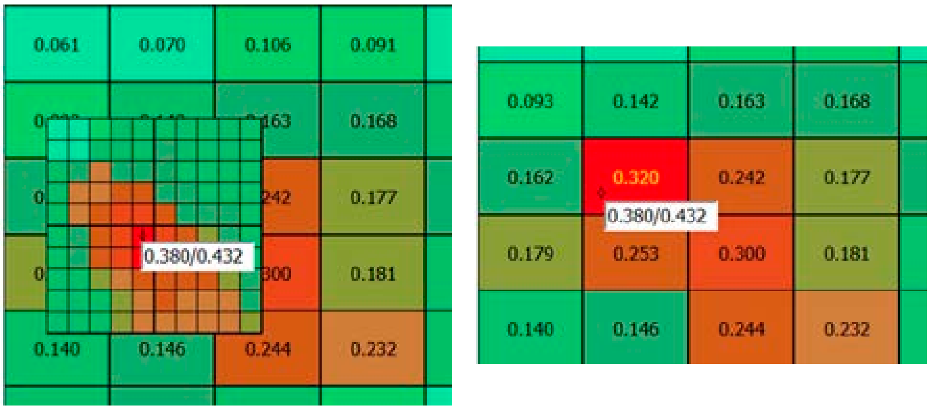

Figure 8. Screenshots from the application implementing the presented logical device

to this, belief and plausibility measures are displayed for the mouse cursor representing the true position.

Conclusions

Dealing with uncertain and imprecise evidence is a challenge in both nautical science and in practice. Formal descriptions of problems encountered in navigation involve models that accept imprecise, erroneous and, therefore, uncertain values. This gives rise to many practical problems that should be taken into consideration during navigation, such as true location measurement, position fixing and systematic error identification.

This paper presents an approach involving the application of belief assignments in nautical science. At first, the basic uncertainty model was introduced. Then aleatory doubtfulness encountered in metrology was presented. Knowledge related to random deflections of measurements fits into the model and therefore can be included into the processing scheme. The uncertainty model is intended for a fuzzy environment. Fuzziness can be interpreted in different ways although fuzzy probability sets are always involved. Membership functions are exploited to make decisions in dilemmas regarding the location of the true measurements of the given observations.

Fuzzy sets may be associated with cumulated probability calculated for specified intervals, once the bell function is considered. They can be related to bins when empirical distribution is involved. Fuzzy sets are defined by membership functions. Polyline and sigmoidal functions are used very often.

Aleatory doubtfulness arises once one makes nautical observations. They are systematically and randomly distorted. A seafarer knows a considerable amount about this fact and he or she has an in-depth knowledge concerning the diversity of probability density distributions. To include the knowledge into the computation scheme, one must invoke a fuzzy concept. Thanks to this concept, one can arrive at a crisp probability, which introduces the possibility of assessment of the truth of statements encountered in navigation, for example: "evaluate support that the true measurement is represented by a particular value". Apart from supporting the truth, the presented concept enables transformation from density functions to probability distributions. Further, simple crisp-valued belief assignments were introduced and discussed. This sort of function enables a straightforward combination scheme and a direct relationship between the calculated belief and plausibility measures to be observed. The difference between the two features is equal to the mass of uncertainty. This type of belief function is based on implementing a logical device to manage uncertain data. Therefore, processing uncertain nautical data can be split into two phases. At first, available evidence is analysed in order to identify probability distributions. This stage engages fuzzy approximate reasoning. Once achieved, simple crisp belief assignments are built in order to find solutions to the problem at hand. It should be noted that, in this way, the complexity of the computation is reduced, thereby complex iterative procedures can be executed more efficiently.

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