Computer Applications in Electrical Engineering

Accuracy of the intelligent dynamic models of relational fuzzy cognitive maps

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Applying fuzzy relational cognitive maps in dynamic modelling work of the systems involves restrictions deriving from the assumed model parameters. The selection of these parameters depends mostly on abilities of calculating equipment used for the simulation and on the modelling purposes. In most cases it is necessary to balance between increasing the mapping accuracy (which is connected with the calculation time lengthening) and shortening the calculation time (which, in consequence, worsens the accuracy). Additionally, aiming at the accuracy maximization not always can really improve it, but is always connected with the growing of the computational load. In this chapter the analysis of the intelligent cognitive maps work accuracy in the realization dynamic models is elaborated. As a result of the numerical analysis there was shown the existence of certain optimal parameters of analyzed signals fuzzyfication and connected with them sampling parameters in fuzzy arithmetical operations performed during the modelling processes.

1. Introduction

In works [1-5] there were introduced and analyzed applying static and dynamic models of fuzzy relational cognitive maps in decisional monitoring low-structural objects. The research results showed the existence of certain dependencies between accuracy of such models work, selected parameters of fuzzy cognitive maps and the length of the sampling step chosen for numerical calculation. It is specially demonstrated in dynamic models, where the stabilization of the system after stimulating by external signals needs certain number of the cycles of the signals flow through feedbacks.

In the chapter, the results of the simulation analysis of the relationship between actions accuracy, membership functions parameters and fuzziness measure of the applied signals, will be presented on the example of chosen fuzzy relational cognitive map. The results will be shown in the form of appropriate diagrams, derive from which the existence of certain optimal parameters of the models fuzzyfication.

2. Model of the analyzed cognitive map

Generally speaking, a cognitive map can be presented as following pair of sets:

$$\langle \mathbf{X}, \mathbf{R} \rangle \tag{1}$$

where: $\mathbf{X} = [X_1, ..., X_N]^T$ – the set of values of the map concepts (state vector), $R = \{R_{ij}\}$ – matrix of relations between variables X_i and X_j ($i,j = 1, ..., N$).

Figure 1 presents graphical model fuzzy relational cognitive map chosen for numerical analysis.

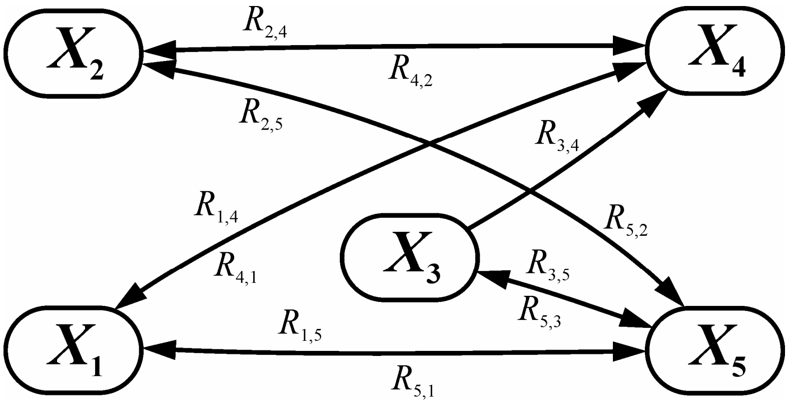

Fig. 1. The explored cognitive map graphical model ($N = 5$)

Dynamic model of fuzzy relational cognitive maps for the object from Fig. 1 can be presented in the form (2) [4]:

$$X_k(t+1) = X_k(t) \oplus \bigoplus_{i=1}^{5} [X_i(t) \ominus X_i(t-1)] \circ R_{i,k} \tag{2}$$

where: X_k – the k-th concept value ($k = 1, ..., 5$), t – discrete time, $\oplus$ – operation of fuzzy addition, $\ominus$ – operation of fuzzy subtraction, $R_{i,k}$ – individual fuzzy relation between fuzzy concepts numbered i and k, $\circ$ – operation of max-min fuzzy composition.

Matching fuzzy parameters, it should be considered: the type of membership function, according to which individual concepts will be fuzzyfied (Fig. 2), universum domain (which depends on the expected concept values) and the number of the universum sampling points.

The essential question is also determining the method of normalization of concepts, which is necessary owing to their distinct physical characteristics. For the needs of this analysis there was chosen non-dimensional normalization to the domain [-1, 1].

For fuzzyfication, according to the algorithm presented in [3, 4], Gauss type function (3) whose graphical representation (for selected parameters) is presented in Fig. 3, was chosen.

$$\mu(x) = e^{-\left(\frac{(x-c)}{\sigma}\right)^2} \tag{3}$$

where: $\mu(x)$ – membership function, x – argument, c – $\mu(x)$ function centre, σ – $\mu(x)$ function width coefficient.

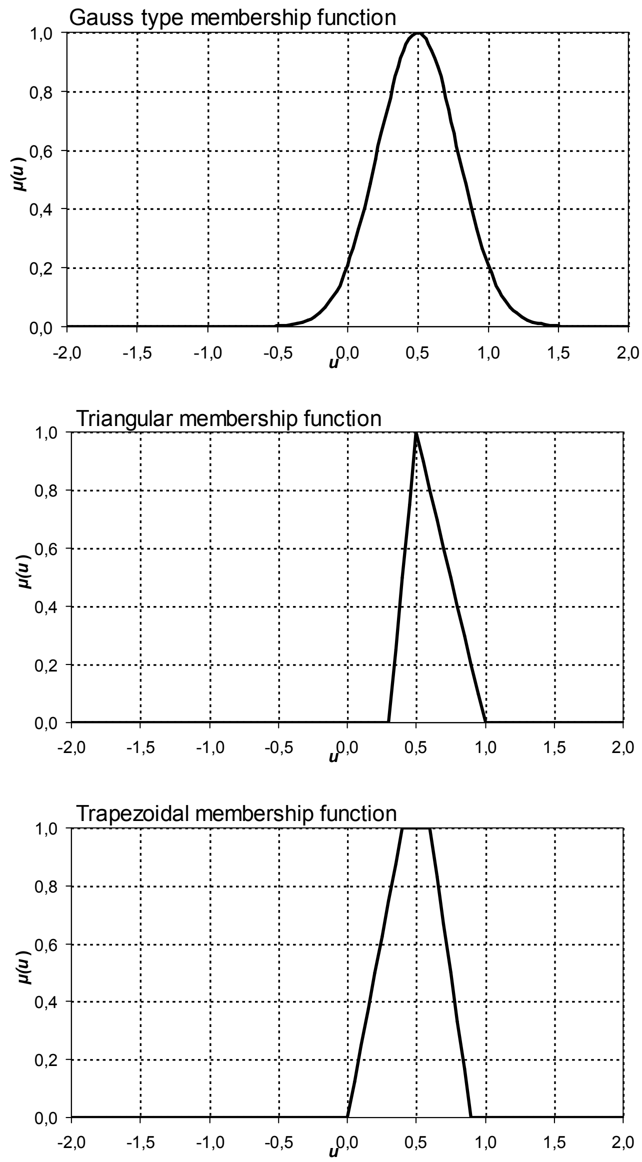

Fig. 2. Examples of membership functions, which can be used for fuzzyfication of cognitive map concepts, u – universum, $\mu(u)$ – membership function

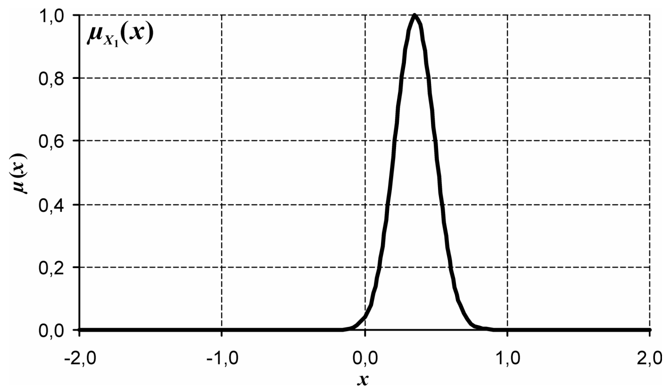

Fig. 3. Hypothetical course of one of input signals, marked as X_1 (after Gauss type fuzzyfication with $\sigma = 0.2$), with momentary value (centre) equals to 0.35 (after non-dimensional normalization to domain [-1, 1])

Similarly to fuzzyfication of concepts, also at the choosing of fuzzy relation characteristics one can use different membership functions, which can be the base for creating such the relations (Fig. 4).

For the needs of present simulation there were chosen the Gauss type relations, strengths of which were corresponding with values of the crisp relations presented in (2).

During the process of determining the universum domain, two concepts should be taken into consideration: normalization domain boundaries and width coefficient of the membership function – σ. Normalization domain boundaries should be chosen in the way which secures maximal symmetry of fuzzy concepts shapes in full range of normalized values. It means e.g. that if $\sigma < 0.8$ for normalization range [0, 1], universum domain can amount from -1 to 2, but for normalization range [-1, 1], universum domain should be wider – from -2 to 2. Generally, for higher values of σ the universum should be wider owing to necessity of keeping the above mentioned symmetry. Finally the universum with domain [-2, 2] was chosen.

In the further part of the chapter there will be presented results of the research on dependency of the accuracy of fuzzy relational cognitive map activities on membership function parameter σ (fuzziness degree *FUZ*) of the model presented in (4)-(5) [6] and on sampling step Δx of the fuzzy sets universum $X = [-2, 2]$ ($x_k = -2 + \Delta x \cdot k$, $k = 0, ..., K$).

$$FUZ(X_i) = 1 - \frac{1}{K^{1/2}} D_2(\mu_{X_i}(x)) \tag{4}$$

$$D_2(\mu_{X_i}(x)) = \sqrt{\sum_{k=0}^{K} \left(2\mu_{X_i}(x_k) - 1\right)^2} \tag{5}$$

where: $\mu_{X_i}(x)$ – membership function of fuzzy set X_i type (3), $i = 1, ..., 5$.

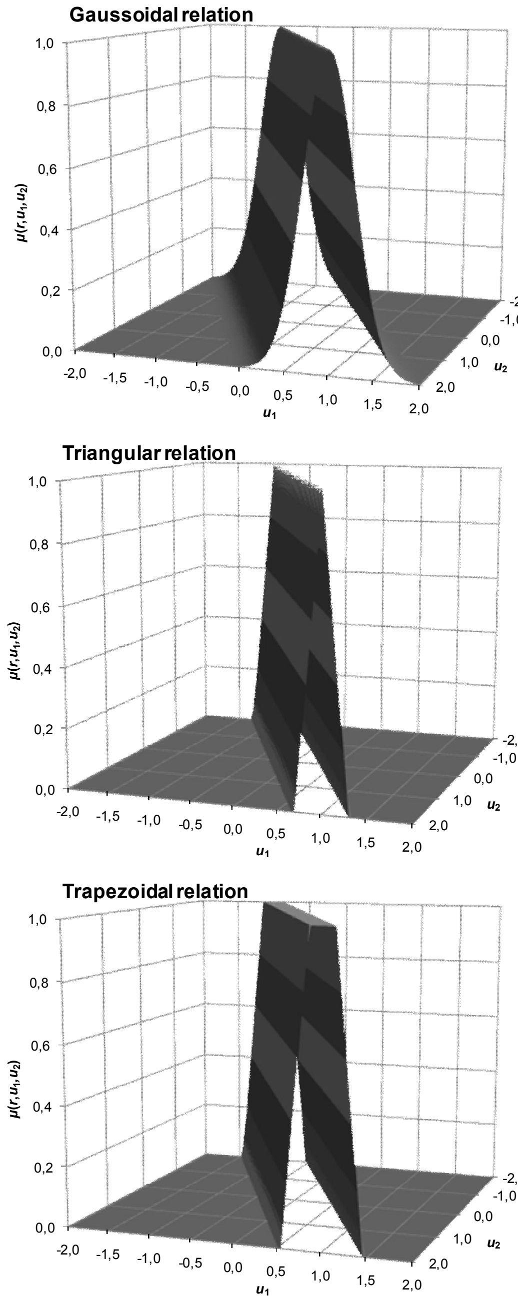

Fig. 4. Examples of fuzzy relations built on the basis of different kinds of membership functions; u_1, u_2 – universum variables, $\mu(r,u_1,u_2)$ – membership function of the relation

3. Selected results of simulation analysis

Matrix of relations $\mathbf{r} = \{r_{i,k}\}$ (i, k = 1, ..., 5) was determined as follow (according to Fig. 1):

$$\mathbf{r} = \begin{bmatrix} 0 & 0 & 0 & 0{,}6 & 0{,}4 \\ 0 & 0 & 0 & 0{,}3 & 0{,}4 \\ 0 & 0 & 0 & 0{,}1 & 0{,}1 \\ -0{,}2 & -0{,}1 & 0 & 0 & 0 \\ -0{,}3 & -0{,}4 & -0{,}5 & 0 & 0 \end{bmatrix} \tag{6}$$

Elements of matrix **r** are values of reference for the constructing individual fuzzy relations [3, 4], which can be designed by experts or during the learning process. These relations are elements of the fuzzy relations matrix **R**, which is the basis of the operating of the relational fuzzy cognitive map used in the tested model.

It was assumed that the system will be acting under influence of one-shot forcing selected concepts to certain values. These values are shown in Table 1.

Table 1. Values of stimulating signals

Concept number	1	2	3	4	5
Stimulating value (normalized)	0.5	0.4	0	0	0

In consecutive calculation steps the system obtains a certain state of equilibrium, which is the basis for the conclusion. The simulation was carried out for 200 steps of discrete time.

A. *Comparative results of analysis for crisp and defuzzyfied courses of concepts presented in Fig. 1, according to matrix **R** from (6) – for $\sigma = 0.2$ and different values of K*

In Fig. 4 there is presented comparison of time courses of tested system concepts in dynamic crisp model and fuzzy model for different number of sampling points of the universum (after defuzzyfication with weighted average method).

From Fig. 4 results that the lower number of the universum sampling points the larger differences between time courses of the fuzzy model concepts. Further research also shows dependency of this difference on the value of σ coefficient (Fig. 5).

Therefore it can be stated that accuracy of fuzzy models depends on the number of the universum sampling points K (which is identical with the number of linguistic functions selected for fuzzyfication) and the coefficient of the membership function width σ. It is quite intuitive assessment, based on visual comparison of time courses, but the conclusions objectification bears the presentation numerical criterion, which would allow to appraise the accuracy of the mapping more accurately and to point the sufficient level of this accuracy.

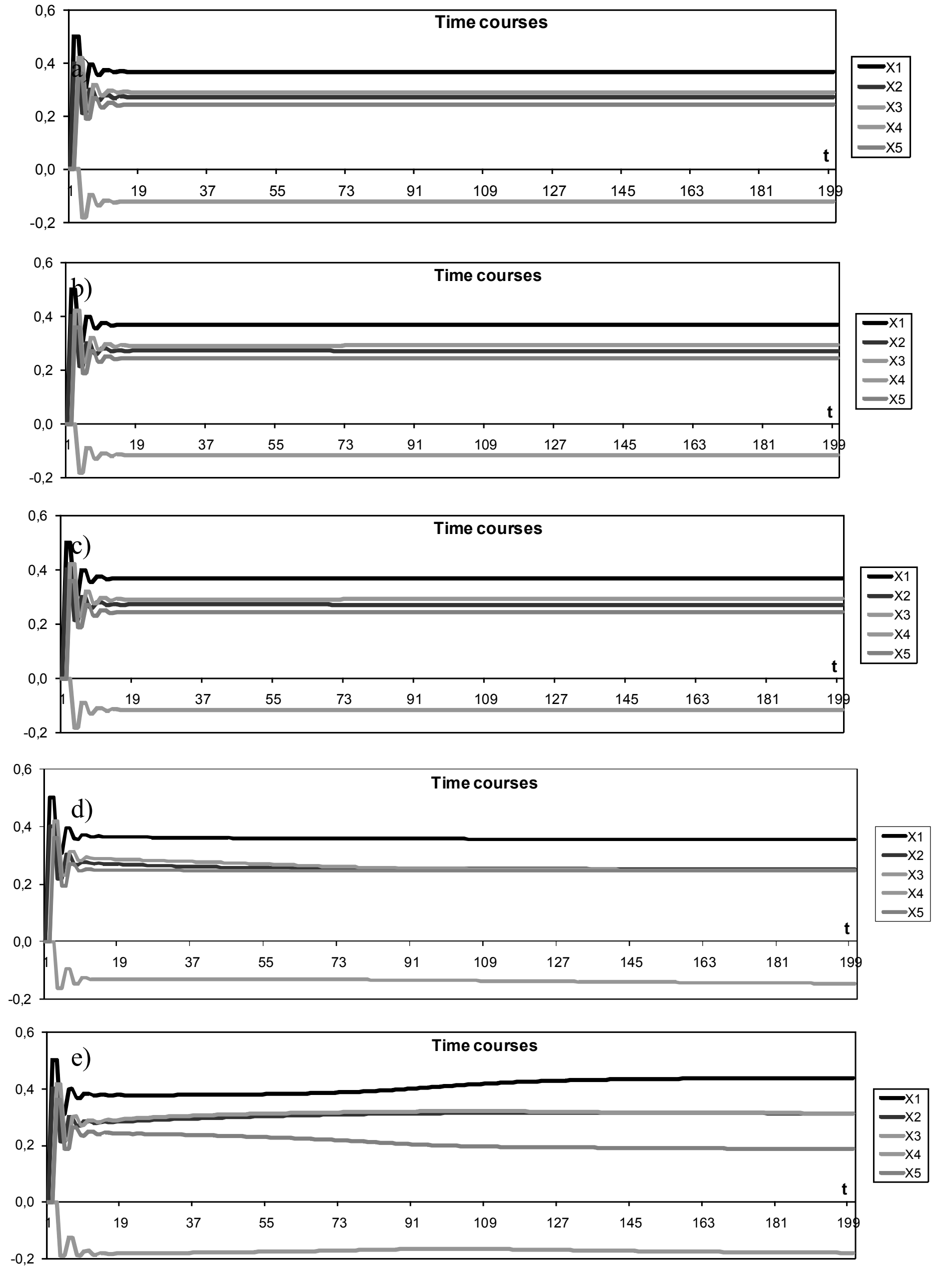

Fig. 4. Time courses of concepts for different models. a) crisp system, b) – e) fuzzy systems with different numbers of linguistic functions (K) on the universum domain (with constant value of the coefficient $\sigma = 0.2$): b) $K = 101$, c) $K = 65$, d) $K = 41$, e) $K = 33$

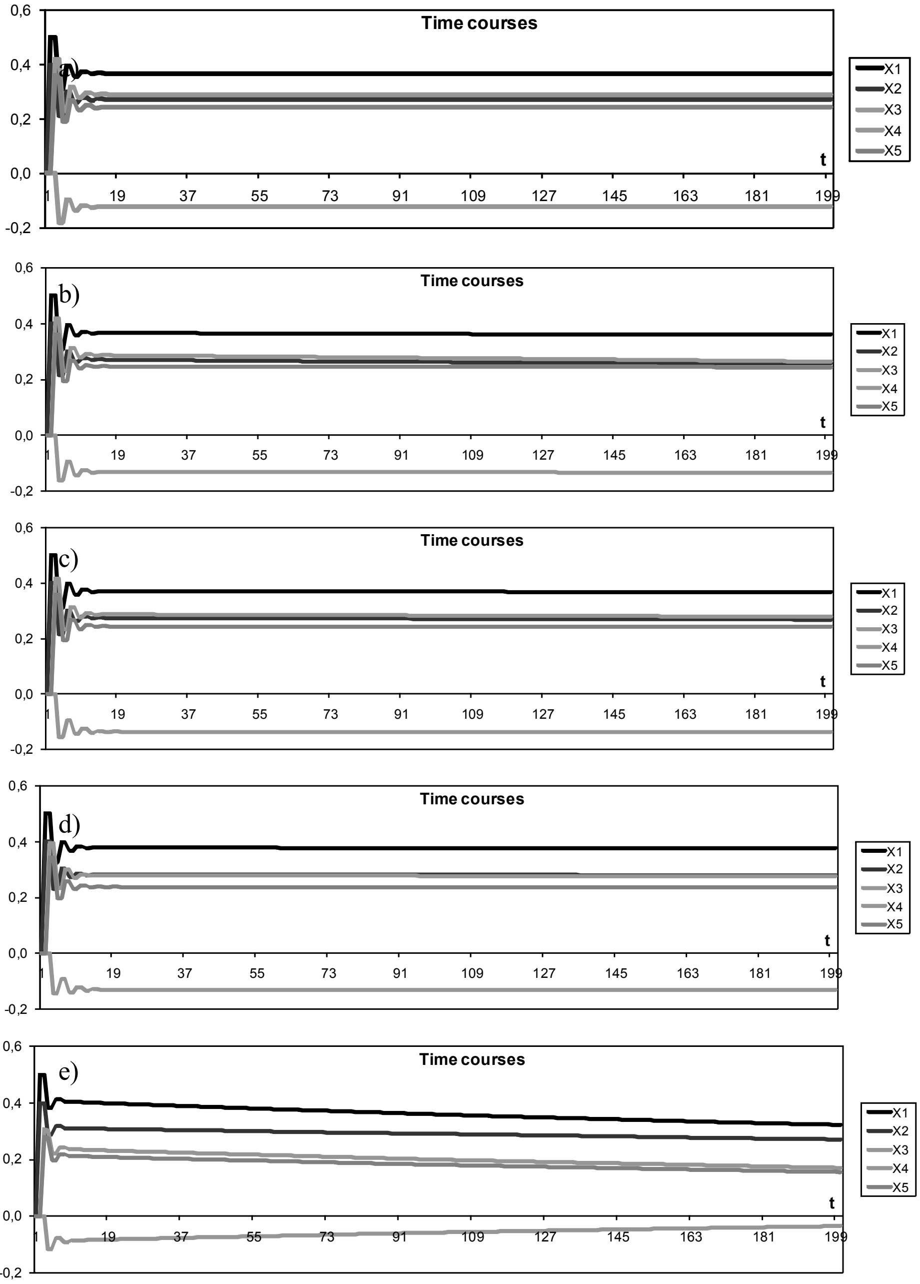

Fig. 5. Time courses of concepts for different models. a) crisp system, b) – e) fuzzy systems with different values of σ coefficient (with constant number of the linguistic functions on the universum domain K = 41): b) $\sigma = 0.3$, c) $\sigma = 0.4$, d) $\sigma = 0.5$, e) $\sigma = 0.7$

***B.** Results of numerical appraisal of the criterion of nearness between crisp and defuzzyfied values*

Appraisal of the accuracy level of the mapping of courses by fuzzy model was performed using nearness criterion (7) considering deviation between fuzzy course and crisp course that was taken as the comparison base. The aim of the studying of the above mentioned criterion is an attempt at finding the number of the universum K sampling points and the membership function width coefficient σ that secure the minimal value of the criterion (7) in specific circumstances.

$$J(FUZ) = \sqrt{\frac{1}{200}\sum_{n=0}^{200}\left(X_i^w(n) - X_i^o(n)\right)^2} \rightarrow \min_{FUZ} \qquad (7)$$

where: $X_i^w(n)$ – defuzzyfied course of i-th concept of the cognitive map (1), $X_i^o(n)$ – crisp course of i-th concept of the cognitive map (in equation (1) fuzzy operators was replaced with arithmetical operators), $i = 1, ..., 5$.

Determining the course of function $J(FUZ)$ allows to discover its minimum for given value of K. This minimum can take different values for different values of K, moreover it also depends on earlier assumed limitations of the calculating system (e.g. on the universum u domain boundaries). It should be also considered that such research is carried out independently for each concept and its results can be different for different concepts.

Figs. 6 and 7 present diagrams of function $J(FUZ)$ for concept X_1 for two different values of K. It should be noticed that "de facto" they present dependency on the membership function with coefficient σ because FUZ is function of σ.

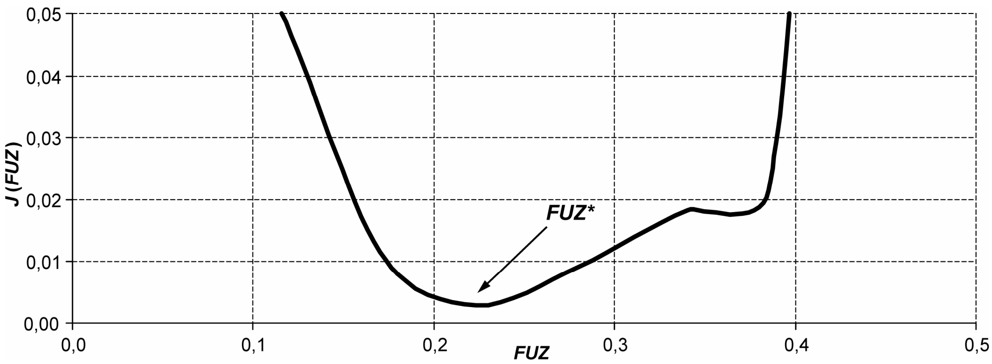

Fig. 6. The course of function $J(FUZ)$ values of concept X_1 for $K = 33$

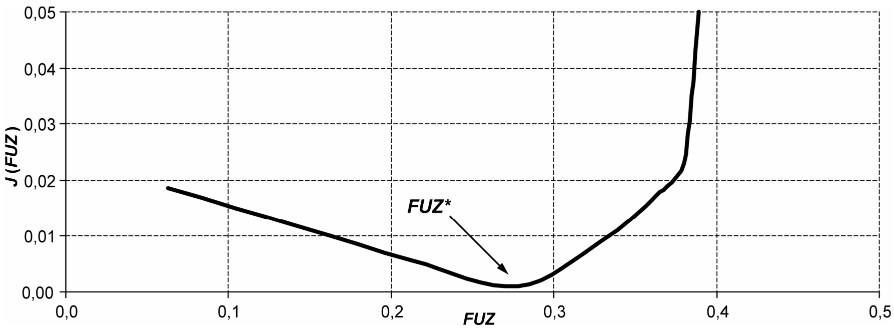

Fig. 7. The course of function $J(FUZ)$ values of concept X_1 for $K = 41$

The comparison of courses from figs. 6 and 7 leads to the observation that minimal value of the nearness criterion $J(FUZ)$ can occur for different values of the fuzziness degree FUZ of given concept and its location depends on the assumed technical parameters of calculating system (number of the universum K sampling steps and, indirectly, the universum u domain width). Therefore it can be stated that for a constant value of K (or Δx) there is optimal value FUZ^* dependent on the parameter σ. According to this there can be formulated the problem of finding up the optimal value of σ.

Generally, analyzing the results of A. and B. it can be stated that there is a problem of the optimization of the selecting parameters σ, Δx and **R**, which can be solved by using different optimization algorithms (e.g. gradient or genetic) [7].

4. Conclusions

Results of the partial numerical analysis of the accuracy of intelligent dynamic models of fuzzy cognitive maps presented in the work, lead to existence of certain optimal parameters of the fuzzyfication with using max-min composition between fuzzy concepts and appropriate fuzzy relations.

The problem of seeking optimal parameters of the model is connected not only with the modelled system parameters themselves. It also requires consideration of the calculating equipment technical abilities and expected calculation time.

For finding optimal parameters there is proposed using certain optimization methods based on gradient or genetic algorithms that will be presented in further works.

References

[1] Borisov V. V., Kruglov V. V., Fiedulov A. C. Fuzzy models and networks. Telekom, Moscow 2007 (in Russian).

[2] Jastriebow A., Gad S., Słoń G., Analysis of the fuzzy cognitive maps dynamics in diagnostic monitoring of systems. Proc. of XIV Scientific Conference Computer Applications in Electrical Engineering ZKwE'2009, Poznan 2009, pp. 287-288 (in Polish).

[3] Jastriebow A., Słoń G., Fuzzy cognitive maps in relational modelling of low-structural systems. In: Jastriebow A. (red.) Computer Science in XXI Century. Information Technologies in Science, Technology and Education. Science Publishing House of Institute of Exploitation Technology – National Research Institute, Radom 2009, pp. 35-38 (in Polish).
[4] Jastriebow A., Słoń G., Fuzzy cognitive maps in relational modelling of monitoring systems. In: Kowalczuk Z. (red.) Systems of faults detection, analysis and toleration. PWNT, Gdansk 2009, str. 217-224 (in Polish).
[5] Kosko B., Fuzzy cognitive maps. Int. Journal of Man-Machine Studies, Vol. 24. pp. 65-75, 1986.
[6] Osowski S., Neural networks for the information processing. Printing house of Warsaw University of Technology, Warszawa 2000 (in Polish).
[7] Stach W., Kurgan L., Pedrycz W., Reformat M., Genetic Learning of Fuzzy Cognitive Maps, Fuzzy Sets and Systems, Vol. 153, August 2005, pp. 371-401.