Journal of Machine Engineering, 2024, Vol. 24. No. 1, 60–73
ISSN 1895-7595 (Print)ISSN 2391-8071 (Online)

Received: *30. January 2024.* / Accepted: *15 March 2924* / Published online: *19 March 2024*

profiled rail guides,
rolling contact related lifetime calculation,
multiparametric polynomial regression

Danny STAROSZYK[1*],
Jens MÜLLER[1],
Steffen IHLENFELDT[1,2]

SIMPLIFICATION OF THE ROLLING CONTACT-RELATED LIFETIME CALCULATION OF PROFILED RAIL GUIDES WITH A POLYNOMIAL REGRESSION

The calculation of the lifespan of profile rail guides is an essential part in the design process of machines. Conventional lifespan models yield good results when calculating lifespan values under a homogeneous distribution of individual rolling contact forces on the raceways. In the case of an uneven load distribution, significantly too low lifespan values are calculated, resulting in a considerable loss of lifetime potential. The novel and experimentally validated rolling contact-based lifespan calculation (RCBL) takes the transferred force on each rolling element into account, resulting in more realistic lifespan values that can be up to 4 times higher than those obtained through the classical method. The disadvantage lies in the complex calculation of the necessary individual rolling contact forces, which until now has been done by using extensive finite element models, along with the computationally intensive optimization problem of the RCBL. To overcome these disadvantages, a method is introduced that efficiently calculates the individual rolling contact forces, taking into account all relevant system elasticities, and pre-solves the RCBL for a variety of potential superimposed load combinations. The results are subsequently approximated through an analytical multiparametric polynomial function and can be utilized with the conventional lifespan formula for rolling bearings.

1. INTRODUCTION AND STATE OF THE ART

Profiled rail guides with rolling bearing supports are standard elements in mechanical and plant engineering that enable linear guided relative movements between components. They fulfil the constantly increasing requirements for precision, load capability and cost-effectiveness. Profiled rail guides are force-transmitting elements that are subjected to wear, which means that the machine accuracy depends largely on the state of wear. Machine downtimes caused by wear are a major drawback for cost-effective production, which is why it is very important to calculate the expected lifetime of the profiled rail guides as precisely as possible. Many approaches have been developed to estimate the expected lifetime of pro-

[1] Faculty of Mechanical Engineering, Institute of Mechatronic Engineering, TU Dresden, Germany
[2] Fraunhofer-Institut für Werkzeugmaschinen und Umformtechnik IWU, Fraunhofer Gesellschaft, Germany
[*] E-mail: danny.staroszyk@tu-dresden.de
https://doi.org/10.36897/jme/186130

filed rail guides. The conventional calculation methods have deficits in terms of prediction accuracy, which can be overcome with a rolling contact-related lifetime calculation (RCLE) [1]. For loads that cause a homogeneous or almost equal load distribution on all rolling elements, as is the case with vertical and horizontal forces F_z and F_y as well as rolling moments M_x (Fig. 2), good and often experimentally validated lifetime values can be calculated using the standard lifetime formula (1). The superimposed loads can be combined with (2) to an equivalent force F_{comb}, taking into account the described restrictions [2].

$$L = a_1 \cdot \left(\frac{C_{dyn}}{F_{comb}}\right)^p \cdot L_{Ref} \text{ with } a_1=1 \text{ (survival probability 90\%)} \tag{1}$$

$$F_{comb} = |F_y| + |F_z| + C_{dyn} \cdot \frac{|M_x|}{M_T} + C_{dyn} \cdot \frac{|M_y|}{M_L} + C_{dyn} \cdot \frac{|M_z|}{M_L} \tag{2}$$

where: a_1 – lifetime coefficient, C_{dyn} – dynamic load rating in N, F_{comb} – equivalent force in N, p – lifetime exponent, L_{Ref} – reference lifetime in km, F_y – horizontal force in y-direction in N, F_z – vertical force in z-direction in N, M_x – rolling moment in Nm, M_y – pitching moment in Nm, M_z – yawing moment in Nm, M_T – torsional load moment in Nm, M_L – longitudinal load moment in Nm.

Too low lifetime values are calculated for pitching moments M_y and yawing moments M_z, as these loads cause an inhomogeneous load distribution on the rolling elements. With the rolling contact-related lifetime calculation, the probability of survival in each rolling contact can be considered and an overall probability of survival of the profiled rail guide can be calculated (3)–(8) [1]. Based on the rolling contact-related dynamic load rating C_{SC} and the force in each rolling element $F_{SC\,i}$, the lifetime (4) is adjusted iteratively until equation (3) is fulfilled. The calculation of the rolling contact-related dynamic load rating is carried out with a homogeneous load case, for which (1) is fully valid and the calculated lifetime is known accordingly, and known individual rolling contact forces $F_{SC\,i}$ iteratively with (7) and the associated equations. The individual rolling contact forces are determined by calculation according to the state of the art, as no suitable measurement methods are known yet. A summary of the known calculation methods used here is presented in section 3.

$$P_W = \prod_{i=1}^{N_{RE\,max}} P_{SC\,i} = S \tag{3}$$

$$L_{SC\,i} = L_{RCLE} \tag{4}$$

$$P_{SC\,i}(a_{1\,SC\,i}) = \left(\left(\frac{1}{0.9}\right)^{a_{1\,SC\,i}^{1.5}}\right)^{-1} \tag{5}$$

$$\mathrm{N_{RET\,max}} = \mathrm{int}\left(\frac{\mathrm{l_{carriage}}}{\mathrm{D_{RE\,nom}}}\right) + 1 \tag{6}$$

$$a_{1\,SC\,i} = \frac{L_{SC\,i}}{\left({C_{SC}}/{F_{SC\,i}}\right)^p \cdot L_{Ref}} \tag{7}$$

$$N_{RE\,max} = N_{RET\,max} \cdot N_T \tag{8}$$

where: P_W – survival probability of the linear guideway, S – desired survival probability, P_{SC} – survival probability for one contact, L_{RCLE} – RCLE lifetime of the whole guideway, $a_{1\,SC}$ – lifetime coefficient for one contact, L_{SC} – lifetime for one contact, C_{SC} – dynamic load rating for one contact, F_{SC} – force on one contact, $N_{RET\,max}$ – maximum number of rolling elements on one track, $N_{RE\,max}$ – maximum number of rolling elements on all tracks, N_T – number of tracks, $l_{carriage}$ – length of the carriage, $D_{RE\,nom}$ – nominal diameter of rolling elements.

2. EXPERIMENTAL VALIDATION OF THE RCLE

The individual rolling contact-related lifetime calculation was examined for validity in experimental lifetime tests. Two series of tests were carried out with size 25 ball profiled rail guides with 8% preload. The first was carried out with a centric vertical force F_z = – 14300 N in accordance with DIN 631 [3] (homogeneous load distribution) in order to determine the real dynamic load rating $C_{dyn\,exp}$ of the examined profiled rail guide charge. The second series of tests was carried out with an eccentric vertical load F_z = –5250 N (35 mm lever arm), resulting in a superimposed pitching moment M_y (inhomogeneous load distribution), based on DIN 631, in order to verify the RCLE. This load was chosen in order to achieve a comparable calculated lifetime as for test series 1 and to maximize the inhomogeneity of the load distribution on the rolling element raceways.

The lifetime values achieved in each of the tests are evaluated using a Weibull analysis in accordance with the specifications of DIN 631. The test parameters and results can be found in Fig. 1 and Table 1. The dynamic load rating specified by the manufacturer could not be confirmed in the first series of tests, as the experimental lifetime on the Weibull equilibrium line for a failure probability of 10% with L_{exp} = 435 km is less than the calculated lifetime of L_{calc} = 800 km. The achieved experimental lifetime L_{exp} results in a real dynamic load rating $C_{dyn\,exp}$ = 23360 N. If this dynamic load rating is used for the calculation of the rolling contact-related load rating $C_{dyn\,exp\,RC}$ (see Section 1), a lifetime of L_{calc} = 339 km is calculated with the RCLE for the load case of the 2nd test series (inhomogeneous load distribution). The lifetime achieved experimentally for this 2nd load case is L_{exp} = 446 km and is greater in comparison to the RCLE, which means that the RCLE can be considered validated. Compared to the conventional catalogue method, a 3.4-fold higher lifetime was calculated with the RCLE and a 4.5-fold higher lifetime was achieved in the tests.

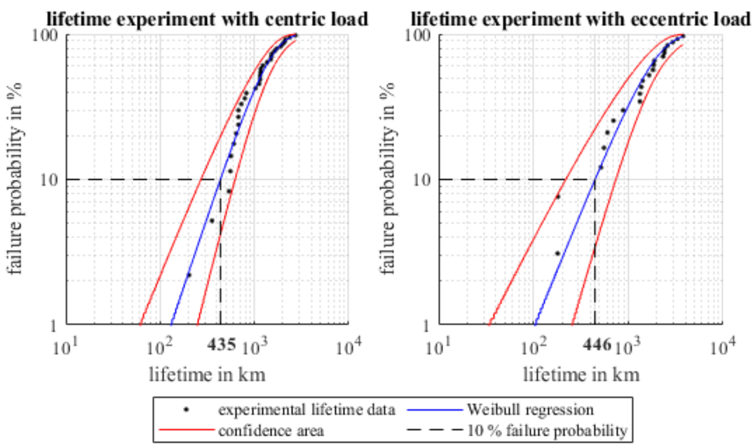

Fig. 1. left: Weibull-analysis for a centric load, right: Weibull-analysis for an eccentric load

Table 1. Experimental parameters and results

	Catalogue centric	RCLE centric	Catalogue eccentric	RCLE eccentric
C_{dyn} ($C_{dyn\ RC}$)	28600 N (3450 N)			
Load	F_z = –14300 N		F_z = –5250 N; M_y = 183.75 Nm	
Calculated lifetime L_{calc}	800 km	800 km	183 km	793 km
	Experimental results			
Exp. lifetime (Weibull) L_{exp}	435 km		446 km	
$C_{dyn\ exp.}$ ($C_{dyn\ exp.\ RC}$)	23360 N (2139 N)		/	
Calculated lifetime with $C_{dyn\ exp.}$	435 km	435 km	100 km	339 km

The calculation of the RCLE is associated with complex calculations, as both the RCLE itself must be calculated iteratively using a suitable optimizer and the required individual rolling contact forces must be calculated using a comprehensive calculation procedure. As a result, the RCLE can only be calculated with specially programmed software tools, whereby it is neither computationally efficient nor user-friendly and therefore not suitable for a more general application. This deficit can only be solved with an analytical and therefore easy-to-use calculation method. In order to achieve this goal, the RCLE, including the calculation of the individual rolling contact forces, can be pre-calculated for a large number of complex superimposed load combinations. With these results, the RCLE can then be converted into an analytical form using a multi-parametric polynomial regression. This simplifies the calculation of the RCLE, as only a few input parameters are required and only a few calculation operations need to be performed. By saving computational resources, the RCLE could be implemented directly in machine control systems, for example, in order to consider real operating conditions in the RCLE (see section 5 for more details).
To illustrate the necessary simplification of the RCLE, the extensive steps for calculating the individual rolling contact forces are briefly summarized in Section 3.

3. CALCULATION OF THE SINGLE ROLLING CONTACT FORCES

Many approaches can be found in the literature as to how the single rolling contact forces of profiled rail guides can be calculated. Most works use FE models [1, 4–7] or develop analytical calculation methods, whereby the elastic conditions in the Hertzian rolling contact are considered, but the elasticities of the rail and carriage bodies are not taken into account [1, 8–17]. Although these elasticities are considered in other works, the calculation approaches are unsuitable, as they either show very large deviations from experimental measurements [18–22] or are very computationally time-consuming [6–7]. Only Sarfert et. al [23] describe an analytical method for considering all system elasticities with the help of influence numbers and Castigliano's theorem, which provides good results and a starting point for the calculation steps summarized below. None of the works listed describe a holistic calculation approach that is suitable for the RCLE, which is why the most important steps for calculating the single rolling contact forces are described in this section.

As it is not possible to measure the single rolling contact forces directly, numerically calculated FE models provide the most precise calculation results. The profiled rail guide to be calculated must be modelled in a suitable manner, which requires a lot of expertise and experience, as the quality of the calculation results depends heavily on the selected modelling strategy. The modelling and calculation of the FE model is both time-consuming and resource-intensive and requires special software, which is usually not available to the user [24].

For the analytical regression model presented in this paper, many load combinations must be calculated in order to train the regression model with a sufficient number of load cases. The computational effort of the FE model is too high for this, which makes a more analytical and thus more computationally efficient way of calculation necessary.

The most basic model for the analytical calculation of the single rolling contact forces considers the geometric and elastic conditions in the rolling contact areas according to Hertz. The elasticities of the carriage and rail body are initially neglected. In this model, the flattening of each rolling element is calculated according to (10), based on a relative displacement of the carriage with reference to the rail, and from this the single rolling contact force is determined according to (9) [25]. The flattening in each rolling element is determined by the difference in displacement of the centres of the raceway radii, which in turn results from the rolling element displacements z_{RE} and y_{RE} and the geometric conditions in the rolling contact (Fig. 2). The rolling element displacements result from the relative displacements of the carriage in relation to the rail. The contact angles α_{RER} that occur under load for ball profiled rail guides are thus calculated correctly and considered in the transformation matrix **T** ($5\times N_{RE\,max}$) in the calculation of the resulting loads **B** (1×5). The preload of the profiled rail guide can be considered by an initial flattening or preload of the "Hertz spring".

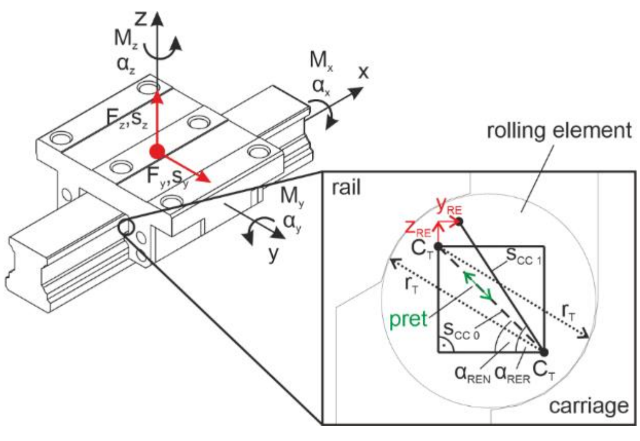

Fig. 2. Geometric conditions in the rolling contact

$$F_{SC} = {\delta_{RE\,rigid}}^{1.5} \cdot C_\delta \tag{9}$$

$$\delta_{RE\,rigid} = s_{CC\,1} - s_{cc\,0} \tag{10}$$

$$b_i = \sum_{k=1}^{N_{RE\,max}} (t_{i,k} \cdot F_{SC\,k}) \tag{11}$$

$$\boldsymbol{B} = (F_z \quad F_y \quad M_y \quad M_z \quad M_x)^{\boldsymbol{T}} \tag{12}$$

where: δ_{RE} – rolling element flattening, C_δ – geometric factor, $s_{CC\,0}$ – distance between the centres (C_T) of the track radii (r_T) in the initial state, $s_{CC\,1}$ – distance between the centres (C_T) of the track radii (r_T) in the load state for the contact, **B** – load vector, **T** – transformation matrix.

Based on this, the carriage and rail elasticities can be considered using the influence numbers approach and Castigliano's theorem [23]. For this purpose, the rolling element flattening of the "rigid solution" at rolling element i (10) is reduced by an elastic component (13), which is caused by all rolling contact forces of the rolling elements *k* on the point *i* (14).

$$\delta_{RE\ elastic\ i} = \delta_{RE\ rigid\ i} - \delta_{influence\ i}(\delta_{RE\ elastic\ k}) \tag{13}$$

$$\delta_{influence\ i}(\delta_{RE\ elastic\ k}) = \sum_{k=1}^{N_{RE\ sum}} \left(g_{k,i} \cdot \delta_{RE\ elastic\ k}^{\frac{3}{2}} \cdot C_\delta \right) \tag{14}$$

where: **G** – influence number matrix, $\delta_{influence}$ – elastic influence of all rolling element forces.

The non-linear system of equations is solved using the Newton method and the influence number matrix **G** can be calculated once using an FE model. The single rolling contact forces can then be calculated with (13) analogous to (9). The calculation time of the extended elastic analytical model is approx. 1.5 s in Matlab and calculates similarly precise results as an FE model (see Fig. 3).

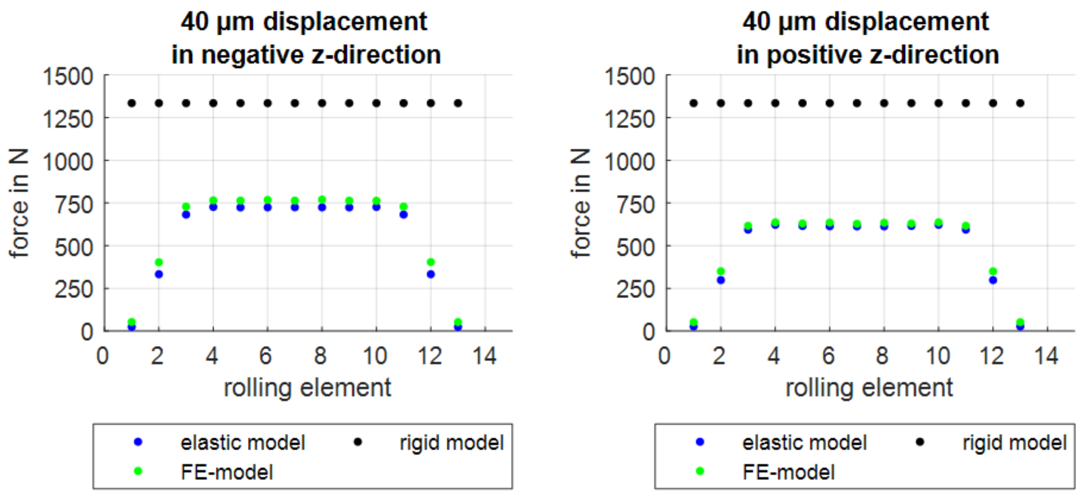

Fig. 3. Distribution and comparison of the calculated rolling contact forces

4. ANALYTICAL CALCULATION OF THE RCLE

The workflow described for calculating the RCLE involves a lot of complex calculation steps, some of which have to be performed iteratively. A programming implementation is therefore very complex and requires special software and a lot of specialist knowledge. This can be rectified by an analytical implementation of the RCLE, which is able to model all relevant influencing variables, but greatly simplifies the mathematical complexity and thus the use of the RCLE. The aim is to calculate the RCLE lifetime L_{RCLE} analytically from the specified load or displacement parameters. For this purpose, a multi-parametric polynomial regression is performed in the hyperplane of the 5th degree, which has as input parameters either the 5 loads or the 5 displacements in the freedoms constrained for a profiled rail guide. In order to train the regression model, a large number of possible load or displacement combinations must be calculated in advance using the methods described for calculating the RCLE, including the calculation of the single rolling contact forces ((3)–(8)). Since the load distribution model (Fig. 2) and ((9)–(12)) has the relative displacements of the carriage in relation to the rail as input parameters and the resulting loads B as output parameters, it is practical to specify ranges of carriage displacements for the training data, within which discrete displacement combinations are calculated. The defined ranges are shown in Table 2. The range limits were defined in such a way that the profiled rail guide is not yet overloaded in the most unfavourable displacement combination (e.g. due to edge running or excessive contact pressures in the Hertz contacts). The range limits shown and the specified step widths result in a total of 16807 (m^5 calculations with m=7 displacement values per displacement direction) necessary calculations of the RCLE.

If the loads were specified directly with corresponding range limits for the calculation of the training data, the carriage displacements would first have to be calculated iteratively with the load distribution model of the profiled rail guide, which would significantly increase the calculation time. With equation (11), however, the resulting loads are also available as information for the regression model with the specification of the carriage displacements.

Table 2. Ranges for the displacement parameters

Displacement	Range
S_z	$-22.5\ \mu m \cdot 0.5^n \ldots 0 \ldots 5.625\ \mu m \cdot 2^n$
S_y	$-22.5\ \mu m \cdot 0.5^n \ldots 0 \ldots 5.625\ \mu m \cdot 2^n$
α_{pitch}	$-750\ \mu rad \cdot 0.5^n \ldots 0 \ldots 187.5\ \mu rad \cdot 2^n$
α_{yaw}	$-750\ \mu rad \cdot 0.5^n \ldots 0 \ldots 187.5\ \mu rad \cdot 2^n$
α_{roll}	$-750\ \mu rad \cdot 0.5^n \ldots 0 \ldots 187.5\ \mu rad \cdot 2^n$
	$with\ n = 0 \ldots 2$

The information from the calculated training data can be used to set up various regression models. First of all, a distinction can be made as to whether either the relative displacements **D** (equation (20)) of the carriage or the loads **B** on the carriage are to be used as independent variables (input parameters). In the design process of a machine with profiled rail guides, it is primarily the load parameters that are available as information, whereas during operation of the profiled rail guides, the relative displacements of the carriage in relation to the rail can be measured using suitable sensors. In a first regression approach, a regression model is trained in for each of these cases, which directly calculates the RCLE lifetime L_{RCLE} as a dependent variable (DL-model and BL-model). Table 3 shows the coefficient of determination R^2, which describes the goodness of fit of the regression, for different polynomial degrees. For both the BL-regression model and the DL-regression model, the best regression quality is achieved with a polynomial degree of 4. For the BL-regression model, the regression quality is slightly better.

Table 3. Polynomial degree and coefficient of determination *R* for the BL and DL-regression models

Polynomial degree	R^2 BL-model	R^2 DL-model
2	0.3933	0.4464
3	0.5585	0.4613
4	**0.6720**	**0.6139**
5	0.5098	0.6144
6	-2.0829	0.6159

In order to validate the regression models, 16807 further calculations were carried out with randomly selected displacement combinations within the limits shown in Table 2. For each of these calculations, the exact RCLE-value and the RCLE-value estimated with the regression models were calculated. In order to be able to quantify each of the loads on the profiled rail guide in a parameter, the equivalent force was also determined, which would have to be used in the standard lifetime formula according to (1) in order to calculate the correct RCLE lifetime L_{RCLE} (15).

$$F_{eq} = \frac{C_{dyn\ exp}}{\sqrt[p]{\frac{L_{RCLE}}{a_1 \cdot L_{ref}}}} \tag{15}$$

The test data with the randomly superimposed displacements were sorted in ascending order based on this equivalent force and displayed in the figures, whereby the exact RCLE lifetime, the RCLE lifetime estimated with the corresponding regression model, the lifetime according to the catalogue method and the confidence interval around the exact RCLE are compared (Fig. 4). The confidence interval was determined on the basis of the Weibull analysis of the experimental lifetime tests carried out, whereby the RCLE was calculated for this with rolling contact-related dynamic load ratings, which result according to the B_{10} running values of the experimental confidence range (see Fig. 1).

For the BL- and DL-regression models, it is seen that many lifetime values lie outside the confidence intervals, which are considered here as limit values for an accuracy assessment. The relatively low coefficients of determination of approx. 0.67 and 0.61 also indicate an inadequate prediction quality. The problem is due to the fact that the calculated lifetime values depend in principle on the load via an inverse cubic function (see equation (1)), which can only be inadequately approximated by polynomials.

To improve the prediction quality, the inverse cubic relationship must be eliminated from the regression model. One way to achieve this is to use the regression model to estimate only the equivalent force F_{eq} according to equation (15), which must be inserted into the standard lifetime formula (1) to calculate the RCLE lifetime L_{RCLE} (DF-model and BF-model). Although this requires an additional calculation step, the additional calculation effort of the standard lifetime formula is only small. As can be seen in Table 4, the coefficients of determination R^2 for the BF- and DF-regression models are significantly higher than for the BL- and DL-regression models (Table 3). For both the BF- and DF-regression models, the optimal coefficient of determination is reached at a polynomial degree of 4 and is close to the optimum of 1.0. With a coefficient of determination of 0.99, the BF-regression model is more precise than the DF-regression model.

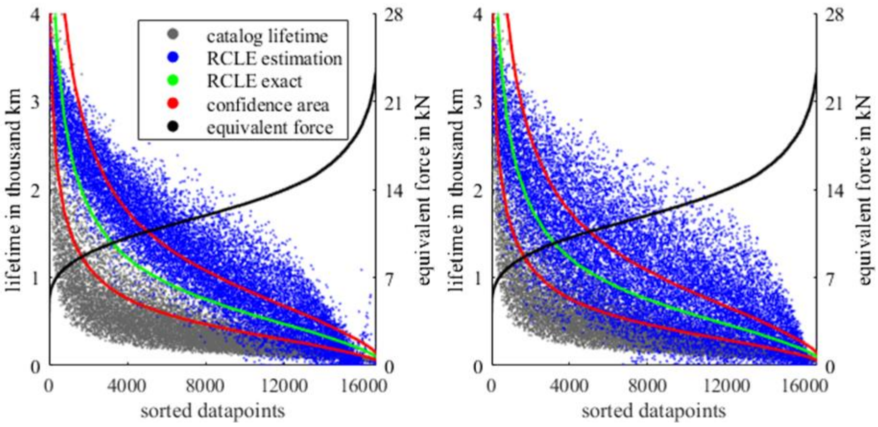

Fig. 4. L_{RCLE}-regression results left: BL-model right: DL-model

Table 4. Polynomial degree and coefficient of determination R for the BF- and DF-regression models

Polynomial degree	R^2 BF-model	R^2 DF-model
2	0.9496	0.8152
3	0.9748	0.9570
4	**0.9907**	**0.9685**
5	0.8894	0.9694
6	0.2596	0.9694

For both regression models, the calculated lifetime values with the estimated equivalent forces are almost completely within the confidence interval (see Fig. 5). In accordance with the slightly better coefficient of determination, the BF-regression model can be used to calculate lifetime values that are closer to the exact RCLE.

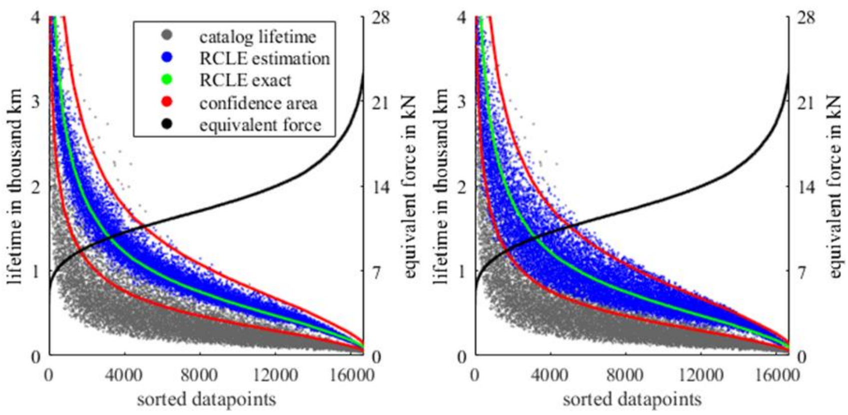

Fig. 5. F_{eq}-regression results left: BF-model right: DF-model

For the BF- and DF- regression models in particular, it is clear that although the lifetime values estimated using these models scatter around the exact RCLE lifetime values, they are still largely within the confidence interval of the lifetime to be expected in reality. Compared to the lifetime values calculated using the conventional catalogue method, which are clearly too low, a more realistic lifetime can be determined using the regression models, whereby the calculation effort is many times less than when calculating the exact RCLE.

Figure 6 shows the percentage deviations of the 4 regression models presented for the exact individual rolling contact-related lifetime calculation as a function of the lifetime. The percentage deviation values of the data points were calculated in blocks of 200 km for the illustration. The first data point in each of Fig. 6 represents the percentage deviation for the range 0 km to 200 km, the second the deviations from 200 km to 400 km and in such a way. The course of the average error and the median error values have a similar quantitative and qualitative course, which is characteristic of approximately uniformly distributed data and a regression that minimizes the error squares. The regression models that calculate the equivalent force for the standard lifetime formula show significantly lower percentage deviations from the exact RCLE, especially in the lifetime ranges below 2000 km.

The regression was performed with the Python library scikit-learn [26]. For each regression model, scikit-learn outputs the power matrix pow (n×5), the coefficient vector coeff (n×1) and the intercept inter. This allows the regression results to be calculated for any combination of parameters when the loads on the profiled rail guide are specified using (16) and (18) and when the relative carriage displacements are specified according to (17) and (18). For the calculation of the estimated RCLE, it is recommended to choose the BF- or DF-regression model, with which the equivalent force is calculated, which can then be used in the standard lifetime formula (1), as the percentage deviations from the exact RCLE are lower compared to the other regression approaches (BL- and DL-model).

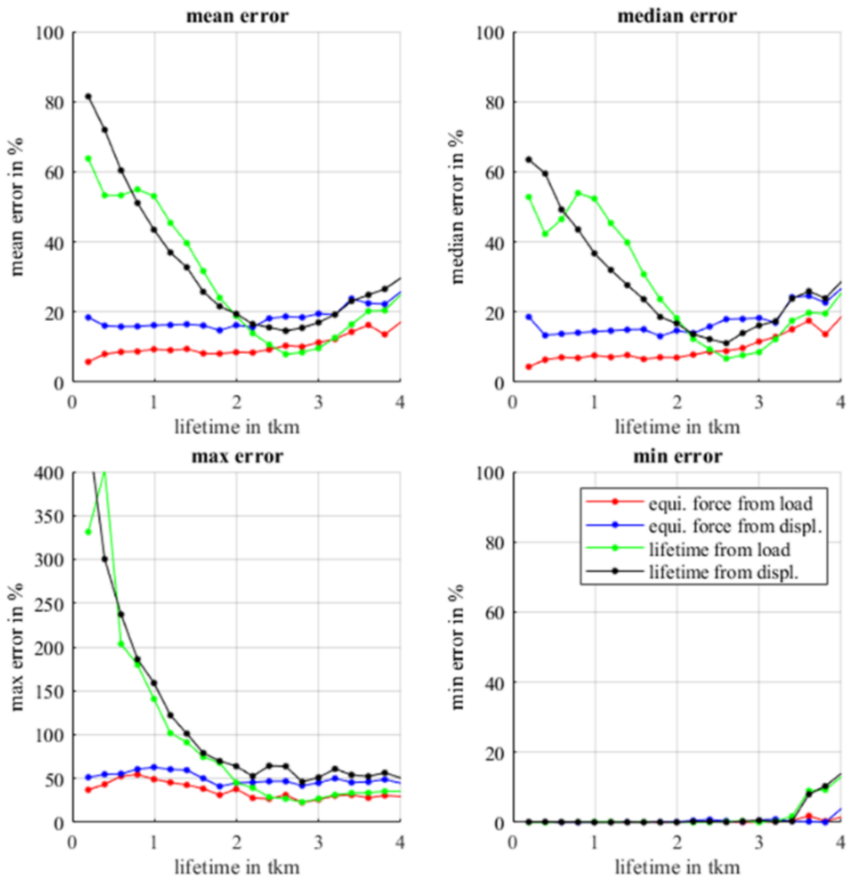

Fig. 6. Errors of the predicted lifetime values compared to the exact RCLE top left: mean error top right: median error bottom left: max error bottom right: min error

$$F_{eq}\ or\ L_{RCLE} = \sum_{i=1}^{n} \left(coeff_i \cdot F_z^{pow_{i,1}} \cdot F_y^{pow_{i,2}} \cdot M_y^{pow_{i,3}} \cdot M_z^{pow_{i,4}} \cdot M_x^{pow_{i,5}} \cdot \right) + inter \tag{16}$$

$$F_{eq}\ or\ L_{RCLE} = \sum_{i=1}^{n} \left(coeff_i \cdot s_z^{pow_{i,1}} \cdot s_y^{pow_{i,2}} \cdot a_y^{pow_{i,3}} \cdot a_z^{pow_{i,4}} \cdot a_x^{pow_{i,5}} \cdot \right) + inter \tag{17}$$

$$n = \frac{(m+5)!}{m! \cdot 5!} - 1 \tag{18}$$

where: n – number of coefficients, m – polynomial degree (in this example m = 4), coeff – vector of regression coefficients, inter – independent term in the regression model, s_z – displacement in z in m, s_y – displacement in y in m, α_y – pitching angle of the carriage in rad, α_z – yawing angle of the carriage in rad, α_x – rolling angle of the carriage in rad.

5. FURTHER POSSIBLE APPLICATIONS OF THE REGRESSION MODEL

The calculated data can also be used to train other regression models that do not have the objective of a lifetime calculation. For example, the relative displacements of the carriage with specified loads on the profiled rail guide can be calculated according to equation (19) in order to determine the stiffnesses in all freedoms in the machine design process. The result of the regression model is a displacement vector **D** according to equation (20) (m = 3; R^2 = 0.9900).

$$d_k = \sum_{i=1}^{n} \left(coeff_{k,i} \cdot F_z^{pow_{i,1}} \cdot F_y^{pow_{i,2}} \cdot M_y^{pow_{i,3}} \cdot M_z^{pow_{i,4}} \cdot M_x^{pow_{i,5}} \cdot \right) + inter_k \tag{19}$$

$$\boldsymbol{D} = (s_z \quad s_y \quad \alpha_y \quad \alpha_z \quad \alpha_x)^{\boldsymbol{T}} \tag{20}$$

Similarly, a regression model can also be trained which calculates the load vector **B** (see equation (12)) as a function of the relative carriage displacements **D** (equation (21); m = 4; R^2 = 0.9971).

$$b_k = \sum_{i=1}^{n} \left(coeff_{k,i} \cdot s_z^{pow_{i,1}} \cdot s_y^{pow_{i,2}} \cdot \alpha_y^{pow_{i,3}} \cdot \alpha_z^{pow_{i,4}} \cdot \alpha_x^{pow_{i,5}} \cdot \right) + inter_k \tag{21}$$

The model can be implemented very easily and memory-efficiently in a machine control system, which allows indirect force measurement if the displacements are measured in a suitable manner, for example using capacitive or inductive distance sensors. To validate this regression model, the carriage displacements for the size 25 ball profiled rail guide with 8% preload discussed here were recorded with capacitive distance sensors under a vertical tensile and compressive load (Fig. 7. top) and under a pitching moment load that was superimposed with a vertical load (Fig. 7 bottom). The measurement results were compared with the calculation results of the exact load distribution model and the regression model. As the absolute carriage displacements for the measurement and model calculations are very close to each other, the difference between the exact load distribution model and the regression model and the measured values was formed for each of them in Fig. 7 on the right for a better assessment of the results. The absolute carriage displacements (Fig. 7 left) are nevertheless shown for a better assessment of the results.

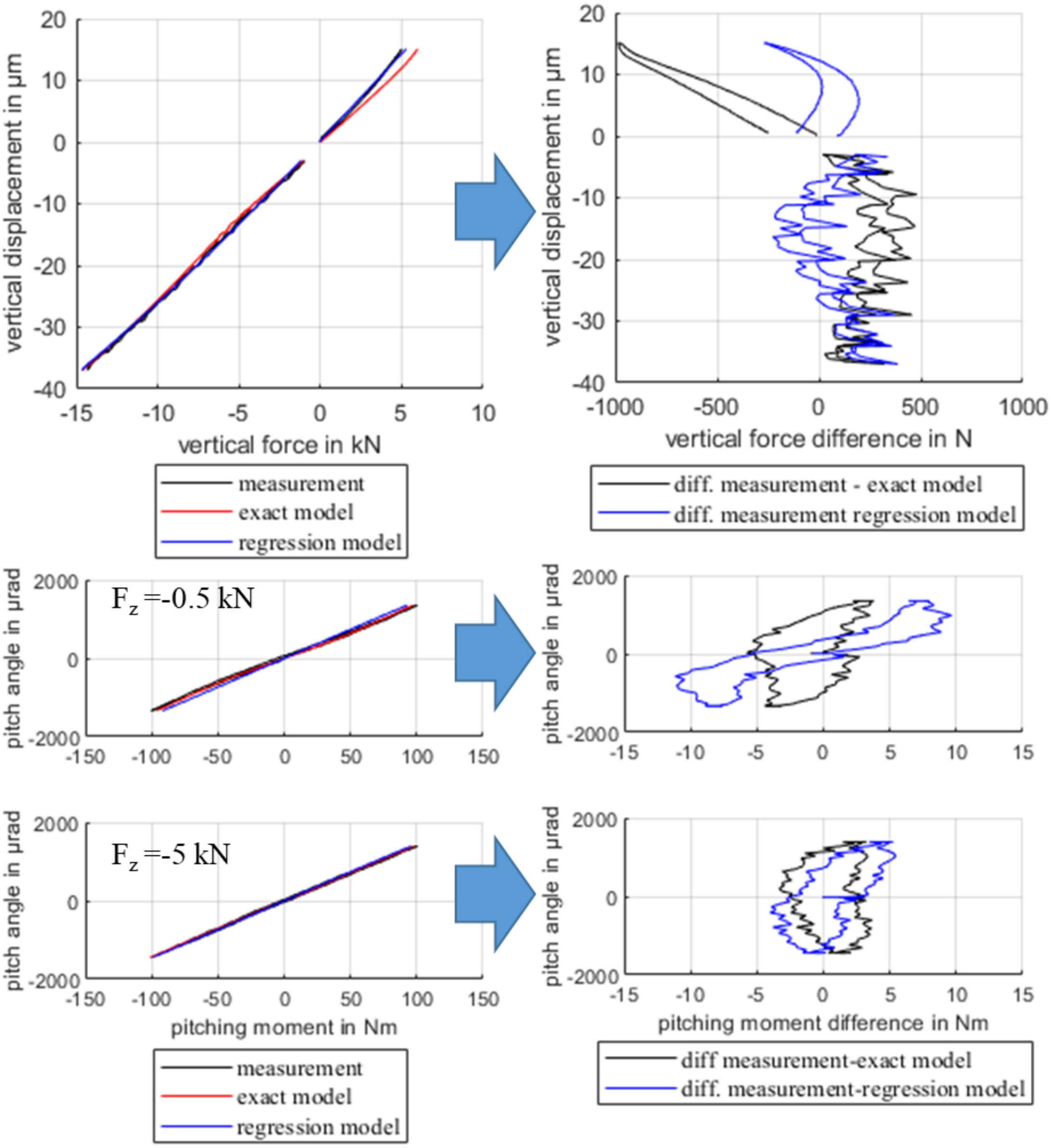

Fig. 7. Comparison of the experimentally measured and calculated carriage displacements

The results of the regression model are similarly precise as the results of the exact load distribution model. The deviations from the measured values are less than 500 N for a tensile and compressive load and around 10 Nm for a pitching moment load for the regression model and 5 Nm for the exact load distribution model. The stiffness of profiled rail guides exhibits a hysteresis for loading and unloading. This is not modelled in the exact load distribution model and therefore also not in the regression model, which results in the hysteresis-related course of the differences in Fig. 7 on the right.

6. CONCLUSIONS

This paper describes a method for simplifying the complex calculations of the single rolling contact-related lifetime calculation of profiled rail guides (RCLE). For this purpose, the validity of the RCLE is first demonstrated by experimental lifetime tests and the steps for calculating the RCLE are presented. To reduce the calculation effort, various multi-parametric polynomial regression models are trained, which estimate the lifetime values of the RCLE based on an analytical equation. To generate the training data, a large number of possible load combinations are systematically calculated in advance using the exact RCLE. The regression models are then validated with random load combinations (test data). The regression models can be used to calculate sufficiently precise lifetime values that lie within the experimentally determined confidence intervals of the exact RCLE. Compared to the conventional lifetime formula, as specified by many manufacturers in their catalogues, more accurate lifetime values for pitch and yaw moments, which cause an inhomogeneous load distribution in the rolling element rows, can be calculated quickly and in a computationally efficient way.

REFERENCES

[1] NEIDHARDT L., 2013, *Wälzkontaktbezogene Lebensdauer von Profilschienenführungen – Bewertung der experimentellen Ermittlung des Lebensdauerkennwerts*, PhD thesis, TU Dresden.

[2] Bosch Rexroth AG, 2017, *Linear Motion Handbook*.

[3] DIN 631, 2020, *Rolling Bearings – Testing Conditions for the Experimental Verification of the Dynamic Load Rating of Ball Carriage or Roller Carriage Profiled Rail Guides*, Deutsches Institut für Normung, March.

[4] DADALAU A., GROH K., REUSS M., VERL A., 2012, *Modeling Linear Guide Systems with CoFEM*: *Equivalent Models for Rolling Contact*, Production Engineering, 6/1, 39–46, https://doi.org/10.1007/s11740-011-0349-3.

[5] GROSSMANN K., NEIDHARDT L., 2006, *FEM-Gestützte Analyse von Profilschienen-Führungssystemen, Teil 1: Technologien und Beispiele zur Untersuchung von Profilschienenführungen und geführten Baugruppen*, Zeitschrift für wirtschaftlichen Fabrikbetrieb, 101/9, 517–521, https://doi.org/10.3139/104.101067.

[6] TONG V.-C., KHIM G., HONG S.-W., PARK C.-H., 2019, *Construction and Validation of a Theoretical Model of the Stiffness Matrix of a Linear Ball Guide with Consideration of Carriage Flexibility*, Mechanism and Machine Theory, 140, 123–143, https://doi.org/10.1016/j.mechmachtheory.2019.05.021.

[7] OHTA H., TANAKA K., 2010, *Vertical Stiffnesses of Preloaded Linear Guideway Type Ball Bearings Incorporating the Flexibility of the Carriage and Rail*, Journal of Tribology, 132/1, https://doi.org/10.1115/1.4000277.

[8] SHAW.D., SU W.L., 2013, *Stiffness Analysis of Linear Guideways Without Preload*, Journal of Mechanics, 29/2, 281–286, https://doi.org/10.1017/jmech.2012.136.

[9] TAO W., ZHONG Y., FENG H., WANG Y., 2013, *Model for Wear Prediction of Roller Linear Guides*, Wear, 305/1–2, 260–266, https://doi.org/10.1016/j.wear.2013.01.047.

[10] KONG W., SUN B., WANG B., WEN X., 2015, *Dynamic and Stability Analysis of the Linear Guide with Time-Varying, Piecewise-Nonlinear Stiffness by Multi-Term Incremental Harmonic Balance Method*, Journal of Sound and Vibration, 346, 265–283, https://doi.org/10.1016/j.jsv.2015.02.021.

[11] ZOU H., WANG B., 2015, *Investigation of the Contact Stiffness Variation of Linear Rolling Guides Due to the Effects of Friction and Wear During Operation*, Tribology International, 92, 472–484, https://doi.org/10.1016/j.triboint.2015.07.005.

[12] WANG W., ZHANG Y., LI C., 2017, *Dynamic Reliability Analysis of Linear Guides in Positioning Precision*, Mechanism and Machine Theory, 116, 451–464, https://doi.org/10.1016/j.mechmachtheory.2017.06.011.

[13] WANG W., ZHOU Y., WANG H., LI C., ZHANG Y., 2019, *Vibration Analysis of a Coupled Feed System with Nonlinear Kinematic Joints*, Mechanism and Machine Theory, 134, 562–581, https://doi.org/10.1016/j.mechmachtheory. 2019.01.014.

[14] LI C., XU M., HE G., ZHANG H., LIU Z., HE D., ZHANG Y., 2020, *Time-Dependent Nonlinear Dynamic Model for Linear Guideway with Crowning,* Tribology International, 151, https://doi.org/10.1016/j.triboint.2020.106413.

[15] LI C., XU M., SONG W., ZHANG H., 2023, *A Review of Static and Dynamic Analysis of Ball Screw Feed Drives, Recirculating Linear Guideway, and Ball Screw*, International Journal of Machine Tools and Manufacture, 188, https://doi.org/10.1016/j.ijmachtools.2023.104021.

[16] CHRISTOV S., 1997, *Einbaugenauigkeit und Querschnittsberechnung von Profilschienenführungen*, PhD thesis, TU Dresden,

[17] OHTA H., KATO S., MATSUMOTO J., NAKANO K., 2005, *A Design of Crowning to Reduce Ball Passage Vibrations of a Linear Guideway Type Recirculating Linear Ball Bearing*, J. Tribol., Apr. 2005, 127/2, 257–262, https://doi.org/10.1115/1.1828436.

[18] IHLENFELDT S., MÜLLER J., STAROSZYK D., 2019, *Lebensdauer von Profilschienenführungen unter Momentenbelastung – Entwicklung Einer Vereinfachten Wälzkontakt-Bezogenen Lebensdauerberechnung für Profilschienenführungen Unter Nick- und Giermomentenbelastung*, Gleit- und Wälzlagerungen, 379–384, https://doi.org/10.51202/9783181023488-379.

[19] IHLENFELDT S., MÜLLER J., STAROSZYK D., 2020, *Lifespan of Profile Rail Guides – Developing of a Simplified Rolling Contact Related Life Calculation for profile rail Guides – Part 2*, WT Werkstattstechnik, 110/3, 151–158.

[20] ISPAYLAR M.H., 1997, *Betriebseigenschaften von Profilschienen-Wälzführungen*. PhD thesis, Würzburg.

[21] SCHEUERMANN M., 2010, *Dynamiksimulation zur Virtuellen Produktentwicklung von Rollenschienen-führungen. Maschinenelemente- und Getriebetechnik-Berichte*, Techn. Univ., ISBN 3941438492.

[22] SCHNEIDER M., 1991, *Statistisches und Dynamisches Verhalten Beim Einsatz Linearer Schienenführungen Auf Wälzlagerbasis Im Werkzeugmaschinenbau*, Phd Thesis.

[23] SARFERT J., LENSSEN S., 1994. *Berechnung Wälzgelagerter Linearführungen*, Konstruktion, 46/6, 209–214.

[24] IHLENFELDT S., MÜLLER, J. STAROSZYK D., 2021, *Lifespan Investigations of Linear Profiled Rail Guides at Pitch and Yaw Moments*, 294–303, Springer Berlin Heidelberg.

[25] BRÄNDLEIN J., ESCHMANN P., HASBARGEN L., WEIGAND K., 2002, *Die Wälzlagerpraxis*, Vereinigte Fachverlage GmbH, Mainz.

[26] PEDREGOSA F., VAROQUAUX G., GRAMFORT A., et al., 2011, *Scikit-Learn: Machine Learning in Python*, Journal of Machine Learning Research, 12, 2825–2830.