The role of properties of the membership function in construction of fuzzy sets ranking

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The paper presents a several new definitions of concepts regarding the properties of fuzzy sets in the aspect of their use in decision support processes. These are concepts such as the image and counter – image of the fuzzy set, the proper fuzzy set, the fuzzy support and the ranking of fuzzy set. These concepts can be important in construction decision support algorithms. Particularly a lot of space was devoted to the study of the properties of membership function of the fuzzy set as a result of operations on fuzzy sets. Two additional postulates were formulated that should be fulfilled by the membership function product of fuzzy sets in decision making.

Keywords: fuzzy set, fuzzy set image, fuzzy set ranking, discrimination postulate, no internal contradiction postulate

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1. Introduction

The work concerns the use of the concept of fuzzy sets in mathematical modeling of optimization decision support processes [3, 4, 5, 7, 8, 13, 14, 15, 16, 17, 29]. Very often sets of decisions which are a proposal of a decision maker support are presented (due to on the specifics of the circumstances) in the form of a fuzzy set. A classic example is a fuzzy medical diagnosis [1, 2, 6, 9, 10, 11]. In such a situation, the natural solution is to use the fuzzy set ranking as a tool for defuzzification the fuzzy decisions [12, 18, 19, 20, 21, 26, 27]. In simple cases this problem is usually trivial. The situation becomes more complicated if the resulting fuzzy set is the result of additional operations on fuzzy sets, such as the product of sets, sum or difference [1, 2, 6, 22, 23, 24, 29]. The problem of constructing the global membership function, which has an obvious impact on the form of the final ranking of fuzzy set elements, and thus on the final decision in defuzzification process. Classical definitions related to fuzzy sets in many situations of mathematical modeling are not sufficient. The problem seems to be particularly important when defining the membership function of fuzzy sets being the result of operations on fuzzy sets (product of fuzzy sets, sum of fuzzy sets, etc.) [10, 25, 29].

2. Fuzzy sets – basic characteristics

Let X be a finite set of elements, called space.

Definition 1 [29]
The fuzzy set A in space X is the set of ordered pairs

$$A = \{(x, \mu_A(x)) | x \in X\} \tag{1}$$

where $\mu_A(x),\ x \in X$ denotes the degree of belonging of the element x to the set A. Function μ_A is called the membership function and takes values from the range [0,1].

Definition 1a
Fuzzy set $A = \{(x, \mu_A(x)) | x \in X\}$ we call the proper set if it doesn't exist $x \in X$, that $\mu_A(x) = 0$.

Definition 2
The *image* of fuzzy set A in space X will be called the set

$$O_A(X) = \{\mu_A(x) \neq 0 | x \in X\} \tag{2}$$

Counterimage of the set $C \subset [0,1]$ there is a set

$$\mu_A^{-1}(C) = \{x \in X | \mu_A(x) \in C\} \tag{2a}$$

Definition 3 [29]
The support of fuzzy set A will be called the classic (sharp, crisp) set:

$$supp(A) = \{x \in X \mid \mu_A(x) > 0\} \quad (3)$$

In practice, often the classic (Definition 1) of a fuzzy set is not convenient to use. Especially in situations where space X is "very extensive" and the support of the fuzzy set A is very small. This is the case, for example, when defining medical diagnosis as a fuzzy set. Space X (set of potential disease entities) formally contains about 20,000 diseases. Systems supporting medical diagnostics define a potential diagnosis in the form of a few or at most a dozen or so potential, most likely disease entities in the "fuzzy variant". The value of the membership function for other disease entities is "epsilon" or equal to zero. Therefore, in practical terms it is better to use a a subset of a particular character of the fuzzy set A which is its *fuzzy support*.

Definition 4
The fuzzy *support* of set A is the set with the following form:

$$\overline{A} = \{(x, \mu_A(x)) \in A \mid x \in \text{supp}(A)\} \subset A \quad (4)$$

This set is also called *reduced fuzzy set A* (*truncation* to the *support*). It can also be defined as follows:

$$\overline{A} = \{(x, \mu_A(x)) \in A \mid \mu_A(x) > 0\}$$

In decision-making practice, the *support* of set A itself can also be modified [10] as follows (by entering a small number $\varepsilon > 0$):

$$supp_\varepsilon(A) = \{x \in X \mid \mu_A(x) \geq \varepsilon\}, \quad \text{where} \quad \varepsilon > 0$$

certain threshold value of membership function (this set is also called ε – section [19, 29]).

Definition 5
The *ceiling* (*upper pole*) of fuzzy set A is the set:

$$roof(A) = \left\{ \begin{array}{l} x \in \text{supp}(A) \mid \textit{no exists } y \in \text{supp}(A), \\ \text{that } \mu_A(y) > \mu_A(x) \end{array} \right\} \quad (5)$$

Definition 6
The *floor* (*bottom pole*) of fuzzy set A is the set:

$$floor(A) = \left\{ \begin{array}{l} x \in \text{supp}(A) \mid \text{no exists } y \in \text{supp}(A), \\ \text{that } \mu_A(y) < \mu_A(x) \end{array} \right\} \quad (6)$$

The concept of ranking of its elements is naturally associated with the fuzzy set [3, 5, 11, 25].

Definition 7
The ranking of elements of fuzzy set A will be called the ranking of its support elements.
Membership functions play an important role in situations where the fuzzy sets are used in decision support algorithms (for example, diagnostic decision support). Their typical use is to create rankings of fuzzy set elements [5, 13]. Ranking of fuzzy set elements is most often identified with ranking elements of $supp(A)$ set.

Definition 8
Ranking elements of the set $supp(A)$ we will call any finite sequence $r(A)$ subsets $A_k \subset \text{supp}(A)$, forming its *division* [3, 5].

$$r(A) = (A_1, \dots, A_k, \dots, A_K)$$

It is therefore a sequence of subsets A_k for $k \in \mathcal{K} = \{1, \dots, k, \dots, K\}$, that

1) $A_k \cap A_m = \varnothing$ for $k \neq m$
2) $\bigcup_{k \in \mathcal{K}} A_k = supp(A)$ (7)

The number of possible *divisions* of set is determined by the so-called *Bell number* [5].

Definition 9
The set A_k is called the *k-th* element of the ranking (*k*-th cluster or *k*-th rank) of fuzzy set A. Each fuzzy set is naturally accompanied by a specific ranking resulting from its membership function μ_A [5]. Sets A_k for $k \in \mathcal{K} = \{1, \dots, k, \dots, K\}$ in this case, it defines the following recursive formula:

$$A_k = \arg \max_{x \in X - \bigcup_{l=0}^{k-1} A_l} \mu_A(x) \quad (8)$$

for $k \in \mathcal{K} = \{1, \dots, k, \dots, K\}$, $A_0 = \varnothing$

The ranking of elements of the fuzzy set determined according to the formula (8) is a very effective decision support tool in the case of solving optimization problems using fuzzy sets of permissible decisions. A numerical example will be presented below illustrating the importance (impact) of formulas determining global membership functions on the result of decision optimization problem. The results obtained using various membership functions of multiaspect fuzzy sets as well as the sets being the product of the classic fuzzy sets will be analyzed. The following example illustrates the previously defined characteristics of a typical fuzzy set. Some of the characteristics presented below were defined in [10].

Example 1

Let $X = \{a,b,c,d,e,f,g,h\}$ – finite space of elements, while A is a fuzzy set:

$$A = \left\{ \begin{matrix} (a,0),(b,0),(c,0.6),(d,0.5), \\ (e,0.5),(f,1),(g,0),(h,1) \end{matrix} \right\}$$

Let $C = \{0.5\}$ someone-element subset of the set [0,1].

About the fuzzy set A we can say that the elements f and h certainly belong to it, while the elements*:* a, b, g certainly do not belong to it. The other elements belong to with the level of 0.5 or 0.6. We will obtain the following characteristics of the fuzzy set A:*A image* of fuzzy set A is:

$$O_A(X) = \{\mu_A(x) \neq 0 | x \in X\} = \{0.5,\ 0.6,\ 1\}$$

A counterimage of a one-element set $C = \{0.5\} \subset [0,1]$ there is

$$\mu_A^{-1}(\{0.5\}) = \{x \in X | \mu_A(x) \in \{0.5\}\} = \{d,e\}$$

A support of fuzzy set A is:

$$supp(A) = \{c,d,e,f,h\}$$

A fuzzy support of the set (*truncation* of the fuzzy set) A is the set:

$$\overline{A} = \{(x,\mu_A(x)) \in A | x \in \mathrm{supp}(A)\} =$$
$$\{(c,0.6),(d,0.5),(e,0.5),(f,1),(h,1)\}$$

A core [10] of the fuzzy set A: $core(A) = \{f,h\}$

A height of fuzzy set $hgt(A)$=1 (this is the so--called normal set) [29, 30, 31].

A threshold of the set is:

$$thres(A) = \min\{\mu_A(x)|\ \ x \in \mathrm{supp}(A)\} = 0.5$$

A extension of the set [10] is:

$$exten(A) = hgt(A) - thres(A) = 1 - 0.5 = 0.5$$

A sharpness of the set is [10]:

$$sharp(A) = \frac{\sum_{x \in X} \mu_A(x)}{|supp(A)|} = 0.72 \tag{9}$$

A fuzzyness of the set is [10]:

$$fuzze(A) = 1 - \frac{\sum_{x \in X} \mu_A(x)}{|supp(A)|} = 1 - 0.72 = 0.28 \tag{10}$$

A ceiling (*upper pole*) of the set A is:

$$roof(A) = \left\{ \begin{matrix} x \in \mathrm{supp}(A) | \textit{no exists } y \in \mathrm{supp}(A) \\ \text{that } \mu_A(y) > \mu_A(x) \end{matrix} \right\} =$$
$$= \{f,h\} \tag{11}$$

A floor (*bottom pole*) of fuzzy set A is:

$$floor(A) = \left\{ \begin{matrix} x \in \mathrm{supp}(A) | \textit{no exists } y \in \mathrm{supp}(A) \\ \textit{that } \mu_A(y) < \mu_A(x) \end{matrix} \right\} =$$
$$= \{d,e\} \tag{11a}$$

The ranking of elements of fuzzy set A, resulting from its belonging function, is as follows $r(A) = (\{f,h\},\{c\},\{d,e\})$.

The elements of this ranking (clusters) are the following sets:

$A_1 = \{f,h\}, A_2 = \{c\}, A_3 = \{d,e\}$. Note that the set $A_1 = \{f,h\}$ is the *roof* of the fuzzy set and the set $A_3 = \{d,e\}$ is the *floor* of a fuzzy set.

3. Analysis of properties of the membership function in operations on fuzzy sets

On fuzzy sets, similarly to classical sets, one can perform a number of operations [29, 30, 31]. For the purposes of this work, we will use only some of them, mainly the operation of the product and sum of the sets.

Definition 10 [29]

The sum of the two sets A and B will be called the set C of the form: $C = \{(x,\mu_C(x)) | x \in X\}$, where

$$\mu_C(x) = \max\{\mu_A(x),\mu_B(x)\}, x \in X \tag{12}$$

Other formulas for constructing the membership function a set being the sum of sets [19, 29, 31] are also used. These are, for example:

$$\mu_{prod}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x),\ x \in X$$
$$\mu_{Luk}(x) = \min(\mu_A(x) + \mu_B(x),1),\ x \in X$$

Definition 11 [29]

The product of two sets A and B will be called the set C of the form:

$C = \{(x,\mu_C(x)) | x \in X\}$, where

$$\mu_C(x) = \min\{\mu_A(x),\mu_B(x)\},\ x \in X \tag{13}$$

In some situations, other formulas are used to construct the membership function to a set being the product of sets [19, 29, 31]. These are, for example:

$$\mu_{prod}(x) = \mu_A(x)\mu_B(x),\ x \in X \qquad (13a)$$

$$\mu_{Luk}(x) = \max\left(0, \mu_A(x) + \mu_B(x) - 1\right) \qquad (13b)$$

In practical applications of the operations of the product of fuzzy sets, the so-called assumption of non-triviality in the form of the condition is often adopted:

$$\mu_A(x)\mu_B(x) \neq 0,\ x \in X$$

It means that every element belongs to both sets A and B even in the least degree. Failure to fulfill this condition by any $x \in X$ makes that $\mu_C(x) = 0$ regardless of the formula – see (13), (13a), (13b). Different definitions of the above membership functions μ_C in practical applications can sometimes raise some doubts. The use of different formulas most often leads to different results. A particularly important problem is the proper selection of the appropriate membership function (in the case of the product of fuzzy sets). This takes place in the construction of algorithms for supporting diagnostic decisions using similarity models and pattern recognition [9, 11, 28]. The following considerations will be limited to examining the operations of the product of fuzzy sets and the consequences of adopting various formulas of belonging to the fuzzy set. In the further part of the work, decision making consequences (resulting from defuzzification process) based on rankings obtained on the basis of various literature concepts of the membership function (see formulas (13), (13a), (13b)) will be examined. In the process of supporting medical diagnostics, the proposal of the resulting (final) diagnosis can be understood as the intersection (product of sets) of two fuzzy diagnoses: diagnosis based on disease symptoms and diagnosis based on risk factors [6, 8, 9, 10]. A dilemma arises which formula of the membership function $\mu_C(x)$ to the product of fuzzy sets take: (13) (most often cited [10, 19, 20, 21, 23, 29, 30, 31]) or formula (13a) or may be (13b)? or any new?

Example 2

Let $X = \{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10\}$ set of people (people numbers). Let A is a set of tall people (height in centimeters – an overview).

$$A = \begin{Bmatrix} (1,160),(2,200),(3,170),(4,180),(5,160), \\ (6,190),(7,170),(8,180),(9,150)(10,200) \end{Bmatrix}$$

And B, a set of fat people (body weight in kilograms – illustrative record).

$$B = \begin{Bmatrix} (1,180),(2,190),(3,200),(4,100),(5,150), \\ (6,190),(7,170),(8,180),(9,200)(10,150) \end{Bmatrix}$$

After normalizing the features, these sets will take the form of typical fuzzy sets:

$$A = \begin{Bmatrix} (1,0.80),(2,1.00),(3,0.85),(4,0.90),(5,0.80), \\ (6,0.95),(7,0.85),(8,0.90),(9,0.75)(10,1.00) \end{Bmatrix}$$

$$B = \begin{Bmatrix} (1,0.90),(2,0.95),(3,1.00),(4,0.50),(5,0.75), \\ (6,0.95),(7,0.85),(8,0.90),(9,1.00)(10,0.75) \end{Bmatrix}$$

Let's assume that we want to define a set of "big fat people" – that is, a set of people who are both tall and fat. This operation gives the product of the sets A and B (see (13). We will get a fuzzy set C:

$$C = \begin{Bmatrix} (1,0.80),(2,0.95),(3,0.85),(4,0.50),(5,0.75), \\ (6,0.95),(7,0.85),(8,0.90),(9,0.75),(10,0.75) \end{Bmatrix}$$

Based on this set, we can create a ranking of belonging elements from set X to the set C of tall fat people. It will be the following ranking (see (6), (7):

$$r(C) = r(A \cap B) = (\{2,6\},\{8\},\{3,7\},\{1\},\{5,9,10\},\{4\})$$

The position of the element in the ranking (see Definition 8) results from the value of the membership function (13) to the fuzzy set of large fat people. Ranking is not linear. It is ambiguous and "out of focus". The first place ex aequo are people with the number 2 and 6 (the value of the membership function 0.95) etc. However, the analysis of the ranking can lead to some doubts. Generally, without any reservations we can say about a person $a \in X$, that it more closely ("more belongs") to the set of C large fat people than a person $b \in P$, if the condition is fulfilled $\mu_A(a) \geq \mu_A(b)$ i $\mu_B(a) \geq \mu_B(b)$.

We have such a situation for example $a = 2$, $b = 8$ and $a = 6$, $b = 1$ etc. However, steam raises doubts $a = 2$, $b = 6$. The ranking according to the value of the membership function shows that their degree of belonging is identical (0.95), while person no. 2 has the same weight as person no. 6 but is much higher. Therefore, they should be ranked before the person no. 6 (should have a bit higher value of belonging function). Similarly, pair $a = 3$ and $b = 7$. Also, person no. 10 is more suited to the set of large fat people than person no. 5 and they have the same value belonging to the set belonging function (0.75).

Similar doubts are more. The ranking thus obtained does not present "reliable information" about the degree of belonging to set *C* which is the product of sets *A* and *B*. The considered membership function of the fuzzy set in the context of taking into account two aspects (height and weight:

$\mu_C(x) = \min\{\mu_A(x), \mu_B(x)\},\ x \in X$

we can interpret as a certain aggregation (function) of two membership functions. This function, however, did not give a precise possibility to distinguish some people despite obvious premises. Other probably aggregation rules are presented in formulas (13a) and (13b). Defining, for example, the function of belonging to a set being the product of the formula (13a), we will obtain a fuzzy set in the form:

$$C_1 = \left\{ \begin{matrix} (1,0.72),(2,0.95),(3,0.85),(4,0.45),(5,0.60), \\ (6,0.90),(7,0.723),(8,0.81),(9,0.75),(10,0.75) \end{matrix} \right\}$$

In turn, using the formula (13b) we get the set:

$$C_2 = \left\{ \begin{matrix} (1,0.70),(2,0.95),(3,0.85),(4,0.40),(5,0.55), \\ (6,0.90),(7,0.70),(8,0.80),(9,0.75),(10,0.75) \end{matrix} \right\}$$

Both sets are very similar [18, 26, 31]. All characteristics of these two sets (see definitions 2–5 and [4]) are almost identical. The rankings obtained are also very similar [26, 31]. However, they differ significantly from the base product set ranking

$r(C) = (\{2,6\},\{8\},\{3,7\},\{1\},\{5,9,\{4\},10\})$.

$$r(C_1) = \begin{pmatrix} \{2\},\{6\},\{3\},\{8\},\{9,10\}, \\ \{7\},\{1\},\{5\},\{4\} \end{pmatrix}$$

$$r(C_2) = \begin{pmatrix} \{2\},\{6\},\{3\},\{8\},\{9,10\}, \\ \{1,7\},\{5\},\{4\} \end{pmatrix}$$

The received rankings, in contrast to the first case (formula (13)) are "almost linear" – they are "sharpened". *Ex aequo case* studies do not lead to dilemmas in this situation as if using formula (13). From a decision support point of view two recent cases have a significant advantage *over the first (classic) case. Expectations in the context of the possibility of more adequate* modeling of decision-making situations using fuzzy sets lead to the formulation of certain postulates as to the properties of defined membership functions of (especially in the case of performing operations on fuzzy sets). They can be entered and described as follows:

Let *X* finite set (space), and *A* and *B* two fuzzy sets in the form:

$A = \{(x, \mu_A(x)) | x \in X\}$,

$B = \{(x, \mu_B(x)) | x \in X\}$

and the fuzzy set *C* which is their product:

$C = \{(x, \mu_C(x)) | x \in X\}$

Suppose sets *A* and *B* are *proper*. Let it simplify the notation: $\mu(x) = (\mu_A(x),\ \mu_B(x)),\ x \in X$

Definition 12 (demand for discrimination postulate)

Membership function $\mu_C(x),\ x \in X$ fulfills the demand for *discrimination postulate* if for everyone $x, y \in X$ such that $\mu(x) \geq \mu(y)$ and $\mu(x) \neq \mu(y)$ there is $\mu_C(x) > \mu_C(y)$.

This postulate can be interpreted as if an element $x \in X$ "Belongs more" to both sets *A* and *B* than an element $y \in X$ (in the sense of membership function, respectively) μ_A i μ_B, so the degree of his belonging $\mu_C(x)$ to the product of sets should be "slightly" greater than the degree of belonging $\mu_C(x)$ of the element *y* to this set.

Definition 13 (postulate of no internal contradiction)

A membership function $\mu_C(x),\ x \in X$ fulfills the postulate of *no internal contradiction*, if for everyone $x, y \in X$, that $\mu_C(x) = \mu_C(y)$ and $\mu(x) \neq \mu(y)$ there is neither $\mu(x) \geq \mu(y)$ neither $\mu(y) \geq \mu(x)$.

The fulfillment of this postulate means that if two different elements they have the same degree of belonging to the product of sets, no one can "belong" more to both sets at the same time than the other element. It is easy to see that the membership function $\mu_C(x),\ x \in X$ from Example 2 determined according to the classic formula (13) does not fulfill both of these postulates. In turn, membership functions constructed according to formulas (13a) and (13b) in Example 2, these postulates fulfill.

The postulates formulated above regarding the properties of the membership function to a set being the product of two sets have very important practical significance. The requirement to fulfill these postulates in practical terms should therefore be considered very obvious. It is surprising that the typical, classical [29, 31] function

$\mu_C(x) = \min\{\mu_A(x), \mu_B(x)\}$, $x \in X$

not fulfill them. From the analysis of the above example, the significance of the construction of the membership function is clearly visible from the point of view of obtaining final results, allowing to indicate decision proposals in models based on fuzzy sets. The problem, of course, requires a more thorough and formal analysis (including the formulation of appropriate postulates and theorems) of the membership function, being the aggregation of partial membership functions in the case of the product of sets. Examples of theorems in this area can be the following two theorems regarding the function of belonging to the product of sets given in the form (13a). Let *X* finite set (space), and *A* and *B* two fuzzy sets in the form:

$A = \{(x, \mu_A(x)) | x \in X\}$,

$B = \{(x, \mu_B(x)) | x \in X\}$

and the fuzzy set *C* which is their product:

$C = \{(x, \mu_C(x)) | x \in X\}$

Let's assume for the sake of sets *A* and *B* that they are *proper*.

Theorem 1

Function $\mu_C(x) = \mu_A(x)\mu_B(x)$ fulfills the demand for discrimination postulate.

Proof:

To fulfill the demand for discrimination postulate, for each pair of elements $x, y \in X$ such that $\mu(x) \geq \mu(y)$ and $\mu(x) \neq \mu(y)$ must happen $\mu_C(x) > \mu_C(y)$,

so

$\mu_A(x)\mu_B(x) > \mu_A(y)\mu_B(y)$

(*)

Formula $\mu(x) \geq \mu(y)$ means

$\mu_A(x) \geq \mu_A(y)$ and $\mu_B(x) \geq \mu_B(y)$

There are three possible cases:

1) $\mu_A(x) > \mu_A(y)$ and $\mu_B(x) > \mu_B(y)$
2) $\mu_A(x) = \mu_A(y)$ and $\mu_B(x) > \mu_B(y)$
3) $\mu_B(x) = \mu_B(y)$ and $\mu_A(x) > \mu_A(y)$

Ad 1) condition (*) is obvious

Ad 2) in this situation it is enough to divide the inequality (*) by pages $\mu_A(x)$ to receive

$\mu_B(x) > \mu_B(y)$

and that means that $\mu_C(x) > \mu_C(y)$

Ad 3) by analogy.

So the thesis is true. □

Theorem 2

Function $\mu_C(x) = \mu_A(x)\mu_B(x)$ fulfills the postulate of *no internal contradiction*.

Proof:

To fulfill the postulate of no internal contradiction, it should be shown that for each pair of elements $x, y \in X$ such that $\mu_C(x) \geq \mu_C(y)$ and $\mu(x) \neq \mu(y)$

a) does not occur $\mu(x) \geq \mu(y)$

b) does not occur $\mu(y) \geq \mu(x)$

Ad a) (ad absurdum)

Suppose it happens $\mu(x) \geq \mu(y)$. There are two possible cases:

1) $\mu_A(x) > \mu_A(y)$ and $\mu_B(x) \geq \mu_B(y)$
2) $\mu_B(x) > \mu_B(y)$ and $\mu_A(x) \geq \mu_A(y)$

Ad 1) multiplying inequalities 1) by sides we get

$\mu_A(x)\mu_B(x) > \mu_A(y)\mu_B(y)$

However, this contradicts the assumption that $\mu_C(x) = \mu_C(y)$

Ad 2) by analogy.

Therefore, the thesis of the theorem is true. □

4. Final conclusions

The paper shows how important the modeling of the membership functions of sets resulting from operations on fuzzy sets in the context of decision making (including the construction of rankings) can be. Two important postulates were defined that should be met by membership functions of fuzzy sets used in decision support systems.

Determining the properties of constructed membership functions in the context of specific, practical applications can be very important in practice. This problem can be very important, for example in building diagnostic classifiers. Similar analyzes can, of course, be carried out for other operations on fuzzy sets in the context of the meaning and role of the defined membership function in modeling. If this type of approach is used in modeling and optimization, for example, diagnostic decision support processes, the fulfillment of the above postulates by belonging functions should be a prerequisite for their practical supporting application

5. Bibliography

[1] M. Albin, *Fuzzy sets and their applications to medical diagnosis*, Berkeley, 1975.

[2] M. Anvari and G.F Rose, "Fuzzy relational databases", in: *Proceedings of the 1st International Conference on Fuzzy Information Processing*, J.C. Bezdek (Ed.), CRC Press, 1984.

[3] A. Ameljańczyk, *Optymalizacja wielokryterialna w problemach sterowania i zarządzania*, Ossolineum, 1984.

[4] A. Ameljańczyk, "Multicriteria similarity models in medical diagnostics support algorithms", *Bio-Algorithms and Med--Systems*, Vol. 21, No. 1 (2013).

[5] A. Ameljańczyk, „Metryki Minkowskiego w tworzeniu uniwersalnych algorytmów rankingowych", *Biuletyn WAT*, Vol. LXIII, Nr 2, 324–336 (2014).

[6] A. Ameljańczyk, "Property analysis of multi-label classifiers in the example of determining the initial medical diagnosis", *Computer Science and Mathematical Modelling*, No. 1, 11–16 (2015).

[7] A. Ameljańczyk, „Analiza wpływu przyjętej koncepcji modelowania systemu wspomagania decyzji medycznych na sposób generowania ścieżek klinicznych", *Bliuletyn Instytutu Systemów Informatycznych*, Nr 4, 1–10 (2009).

[8] A. Ameljańczyk, „Rozpoznawanie wzorców w systemie komputerowego wspomagania diagnostyki medycznej", in: *Systemy informatyczne na potrzeby bezpieczeństwa państwa oraz gospodarki narodowej: modele i metody*, 29–54, WAT, Warszawa 2019.

[9] A. Ameljańczyk, "Pareto filter in the process of multi-label classifier synthesis in medical diagnostics support algorithms", *Computer Science and Mathematical Modelling*, No. 1, 3–10 (2015).

[10] A. Ameljańczyk, "Fuzzy sets in modelling of patient's disease states", *Computer Science and Mathematical Modelling*, No. 9, 5–11 (2019).

[11] A. Ameljańczyk, „Wielokryterialne mechanizmy wspomagania podejmowania decyzji klinicznych w modelu repozytorium w oparciu o wzorce", POIG.01.03.01-00-145/08/2009, WAT, Warszawa, 2009.

[12] A. Ameljańczyk, T. Ameljańczyk, "Determination of thresholds ranking functions applied in medical diagnostic support systems", *Journal of Health Policy and Outcomes Research*, No. 2, 4–12 (2015).

[13] M. Baczyński and B. Jayaram, *An Introduction to Fuzzy Implications*, Springer, 2008.

[14] J.F. Baldwin and B.W. Pilsworth, "Axiomatic approach to implication for approximate reasoning with fuzzy logic", *Fuzzy sets and systems*, 3(2), 193–219 (1980).

[15] W. Bandler and L.J. Kohout, "The use of checklist paradigm in inference systems", *Fuzzy Logic in Knowledge Engineering*, 95–111, 1986.

[16] J.I. Barone de Medeiros, M. Machado, "A fuzzy Inference System to Support Medical Diagnosis in Real Time", *Procedia Computer Science*, 122,167–173 (2017).

[17] C.M. Bishop, *Pattern recognition and machine learning*, Springer, 2007.

[18] D.V. Cross and T.A. Sudkamp, *Similarity and compatibility in fuzzy set theory: assessment and applications*, Studies in Fuzziness and Soft Computing, 93, Springer, 2002.

[19] J. Kacprzyk, *Wieloetapowe sterowanie rozmyte*, Wydawnictwa Naukowo--Techniczne, Warszawa 2001.

[20] A. Kandel, *Fuzzy techniques in pattern recognition*, Cambridge University Press, 1982.

[21] A. Kauffman and M.M. Gupta, *Introduction to fuzzy arithmetic: Theory and application*. Van Nostrand Reinhold, New York 1991.

[22] J.B. Kiszka, M. Kochanska and D.S. Sliwinska, "The influence of some parameters on the accuracy of a fuzzy model", in: M. Sugeno, *Industrial Applications of Fuzzy Control*, 187–230, Elsevier, 1985.

[23] G. Klir and B. Yuan, *Fuzzy sets and fuzzy logic*: *Theory and Applications*, Prentice Hall, New Jersey 1995.

[24] E. Sanchez, "Medical diagnosis and composite fuzzy relations", in: *Advances in fuzzy sets theory and applications*, North--Holland, 1979.

[25] P. Smets, "Medical diagnosis fuzzy sets and degrees of belief", *Fuzzy sets and Systems*, Vol. 5, 259–266 (1981).

[26] Seung-Seok Choi, Sung-Hyuk Cha, C.C. Tappert, “A Survey of Binary Similarity and Distance Measures”, *Systemics, Cybernetics and Informatics*, Vol. 8, No. 1 (2010).
[27] H. Wechsler, “Applications of fuzzy logic to medical diagnosis”, in: *Proc. Symp. on Multiple-Valned Logic*, Logan 1975.
[28] A. Tversky, I. Gati, “Studies of similarity”, in: *Cognition and categorization*, E. Rosch, B. Lioyd (Eds.), 79–98, NJ, 1978.
[29] L. Zadeh, “Fuzzy sets”, *Information and Control*, Vol. 8, 338–353 (1965).
[30] R. Zwick, E. Carlstein, D.V. Budescu, “Measures of similarity among fuzzy concepts: A comparative analysis”, *International Journal of Approximate Reasoning*, Vol. 1(2), 221–242 (1987).
[31] M. Żywica, *Miary podobieństwa i zawierania zbiorów rozmytych*, Wydawnictwo UAM, Poznań 2014.

Rola własności funkcji przynależności w konstrukcji rankingów zbiorów rozmytych

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W artykule przedstawiono kilka nowych definicji pojęć dotyczących własności zbiorów rozmytych w aspekcie ich wykorzystania w procesach wspomagania decyzji. Są to pojęcia takie jak obraz i przeciwobraz zbioru rozmytego, właściwy zbiór rozmyty i ranking zbioru rozmytego. Pojęcia te mogą być ważne w konstruowaniu algorytmów wspomagania decyzji. Szczególnie dużo miejsca poświęcono badaniu własności funkcji przynależności zbioru rozmytego będącego wynikiem działań na zbiorach rozmytych. Sformułowano dwa dodatkowe postulaty, które powinny spełniać funkcje przynależności zbioru będącego iloczynem zbiorów rozmytych.

Słowa kluczowe: zbiór rozmyty, obraz zbioru rozmytego, ranking zbioru rozmytego, postulat rozróżnialności, postulat braku wewnętrznej sprzeczności