

## Rheodynamics non-viscous medium in long (cylindrical) pipes: using the Ostwald-De Ville model

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*Abstract.* While heating the soil in greenhouses, ground heat exchangers are used, as well as when accumulating heat in soil massifs for further use of heat by heat pumps, or when accumulating heat from solar collectors in heat accumulators. In such cases, a coolant moves in the heat exchangers, which will not freeze or boil, most often these are solutions polyethylene/ethylene glycol, which belong to nonlinear viscous fluids. In this work, the non-isothermal motion of a nonlinear-viscous fluid (within the framework of the Ostwald - de Ville model) in a flat channel with a given heat flow on the wall is investigated. A characteristic feature of the flow of such media is their high thermal sensitivity due to phase and structural transformations. Therefore, with a change in temperature, there is a sharp change in rheological properties. For a number of materials, for example, for polyethylene glycol (ethylene glycol) mixtures, for glycerol solutions, these changes can occur in a fairly narrow temperature range. The flow in a channel is considered, the length of which is much greater than the length of the hydrodynamic and thermal initial sections. The case is investigated when the temperature changes along the channel. Channels of arbitrary cross-section (and, in particular, rectangular and circular) are considered. The situation is analyzed when the temperature changes along the channel, and this change is rather weak in relation to the temperature change along the channel walls (in the transverse direction, that is, in the plane of symmetry). To describe the state of the system, the so-called state diagram (dependence of the temperature on the axis of symmetry of the channel on the heat flux).

*Keywords:* rheodynamics, nonlinear viscous medium, long channels, rectangular and round channel sections, Ostwald - de Ville model, heat transfer, nonisothermal flow, heat exchangers, polyethylene glycol, ethylene glycol.

### FORMULATION OF THE PROBLEM. LITERATURE REVIEW

When heating the soil in greenhouses, ground heat exchangers are used, as well as when accumulating heat in soil massifs for further use of heat by heat pumps, or when accumulating heat from solar collectors in heat accumulators of heat [1]. In such cases, a coolant moves in the heat exchangers, which will not freeze or boil, most often these are polyethylene / ethylene glycol solutions, which belong to nonlinear viscous liquids. Studies of non-isothermal flow of nonlinear viscoplastic and nonlinear viscous fluids in flat channels with a given heat flux on the wall is of great interest to researchers. A characteristic feature of the flow of these media is their high thermal sensitivity due to phase and structural transformations. Therefore, with a change in temperature, there is a sharp change in the rheological properties of the liquid itself. For a number of materials, for example, for concrete mixtures, for polyethylene glycol, glycerin solution, these changes can occur in a narrow temperature range.

The flow in a channel (rectangular / circular section) is considered, the length of which is significantly greater than the length of the hydrodynamic and thermal section of the channel (pipe section). For a stable flow, rheodynamics and heat transfer under various boundary conditions, taking into account the dependence of properties on temperature, were studied in a number of works [2-4]. Here, we investigated the case when the temperature changes along the channel. The situation is analyzed when this change is weak enough:

$$\left| \frac{\partial(T(x,y)-T_c(x))}{\partial T_c(x)} \right| \ll 1,$$

where,  $\{T(x, y) - T_c(x)\}$  is a distribution of the relative temperature in the transverse direction to the channel axis;

$T_c(x)$ - temperature on the plane of symmetry (it can be the channel wall).

The purpose of this study is in the substantiation and analysis of rheodynamic features of nonlinear viscous media in long pipes (cylindrical and rectangular sections) within the framework of the Ostwald - de Ville model and on this basis the study of the temperature profile in the channel along its axis in the processes of heat and mass transfer with the environment.

#### PRESENTATION OF THE MAIN CONTENT OF THE RESEARCH

The mathematical formulation of the problem, which includes equations, the boundary condition in the initial section, and the condition for the constancy of the heat flux on the channel wall, in this formulation has the form:

$$\begin{aligned} p c u \times \frac{dT_c}{dx} &= \lambda \times \frac{\partial^2}{\partial y^2} + \tau \times \frac{\partial u}{\partial y}; -\frac{dp}{dx} + \frac{\partial \tau}{\partial y} = 0; \\ \tau &= k(T) \times \left| \frac{\partial u}{\partial y} \right|^{n-1} \times \frac{\partial u}{\partial y}; \\ T(0, y) &= T_0; \frac{\partial T(x, 0)}{\partial y} = 0; \frac{\partial T(x, h)}{\partial y} = \frac{q}{h} \end{aligned} \quad (1)$$

where  $y$  is a transverse coordinate;

$h$  is a channel half-width;

$\tau$  is a shear stress;  $p$  is a pressure;

$T$  is a temperature,  $u$  is a flow rate;

$T_0$  is a temperature on the plane  $x = 0$ ;

$q$  is a heat flow;

$k(T)$  is a temperature-dependent consistency factor  $T$ ;

$n$  is a nonlinearity parameter in the Ostwald - de Ville model [5];

$\lambda$  is a coefficient of thermal conductivity;

$\rho$  is a the density of the substance;

$c$  is a specific heat capacity of the substance.

Transition to dimensionless variables and parameters by the formulas:

$$\left\{ \begin{aligned} \zeta &= \frac{x}{h}; \eta = \frac{y}{h}; \theta = \frac{T}{T_*}; \\ \dot{u} &= \frac{u}{u_0}; v = \frac{v}{v_0}; \\ q &= \frac{q \times h}{\lambda \times T_*}; f(\theta) = \frac{k(T)}{k(T_*)}; \\ Pe &= u_0 \times \frac{h}{a}; \beta = k(T_*) \times \frac{u_0^{n+1}}{\lambda T_* \times h^{n-1}}, \end{aligned} \right. \quad (2)$$

where  $u_0 = Q/(2h)$ ;  $T_*$  is a characteristic speed and temperature, respectively;

$Q$  is a costs per unit channel width;

$a = \frac{\lambda}{(\rho c)}$  is a coefficient of thermal conductivity;

$Pe$  is a Peclet number.

Peclet number reduces these equations (1) to the form:

$$\begin{aligned} Pe \times \dot{u} \times \frac{d\theta_c}{d\xi} &= \frac{\partial^2 \theta}{\partial \eta^2} + \beta \times \left(-\frac{\partial \dot{u}}{\partial \eta}\right)^{n+1} \times \\ f(\theta), \quad \theta_c &= \frac{T_c}{T_*} \end{aligned} \quad (3)$$

Integration of equation (3) over the channel cross section, taking into account the boundary conditions, gives the relation for the reduced longitudinal temperature gradient:

$$\frac{d\theta_c}{Pe^{-1} \times d\xi} = q + \beta \int_0^1 \left(-\frac{\partial \dot{u}}{\partial \eta}\right)^{n+1} \times f(\theta) d\eta \quad (4)$$

Equations (3) and (4) yield the following integral-differential equation:

$$\begin{aligned} u \times \left[ q + \beta \times \int_0^1 \left(-\frac{\partial \dot{u}}{\partial \eta}\right)^{n+1} \times f(\theta) d\eta \right] &= \\ = \frac{\partial^2 \theta}{\partial \eta^2} + \beta \times \left(-\frac{\partial \dot{u}}{\partial \eta}\right)^{n+1} \times f(\theta) \end{aligned} \quad (5)$$

together with the boundary conditions  $\theta(\xi, 0) = \theta_c$ ;  $\frac{\partial \theta(\xi, 0)}{\partial \eta} = 0$  makes it possible to determine the temperature profile accurate to the value  $\theta_c$ . This value

is found from the solution of equation (4). Thus, the two-dimensional problem was reduced to two one-dimensional ones. As a result of studying equation (3), the obtained relationship between the temperature gradient in the transverse direction and the value of the reduced longitudinal gradient.

$$\Phi = \frac{d\theta_c}{d\left(\frac{\xi}{Pe}\right)}$$

$$\frac{\partial\theta}{\partial\eta} =$$

$$= \frac{1}{\tau_w} \times \left\{ \Phi \times \tau \times \dot{u}(\tau) + \dot{q} \times \tau_w \times \int_0^\tau \tau \times \frac{f_1(\tau) d\tau}{\int_0^{\tau_w} \tau \times (\tau) d\tau} \right\}, \quad (6)$$

where  $\tau_w$  is s shear stress on the channel wall ( $\tau_w < 0$ ),

$f_1(\tau)$  is a shear rate

$$\{f_1(\tau) = \frac{\partial\dot{u}}{\partial\eta}\}.$$

From relation (6), it can be concluded that only three cases (situations) exist:

a) if the total heat inflow (value  $\Phi$ ) positive and heat flux heats the liquid ( $q, \dot{q} > 0$ ) Then the temperature over the channel cross-section increases monotonically and the maximum temperatures are reached on the channel wall (the process of heating the liquid heat-accumulating material in the channel, the process of heating the discharge heat exchanger during the AO discharging process and the process of heating the heat exchanger in the solar collector from solar radiation [1], heating the soil in which there is a heat exchanger due to solar radiation).

b) if the total inflow of heat is negative and the liquid gives off heat to the environment, then the temperature decreases monotonically with distance from the plane of symmetry and, accordingly, the maximum temperature is at  $\eta = 0$  (The process of heating solid heat storage material from the charging heat exchanger and the processes of heating water in the DHW tank and buffer tank from a heat accumulator or solar collectors through heat exchangers in these tanks [1], heating the soil from a heat mine located in it);

c) if the total inflow of heat is positive, and the liquid gives off heat to the environment, then the temperature increases with increasing coordinate "y" of the channel and then falls, that is, the maximum temperature is shifted from the plane of symmetry to the wall (the process of heating solid heat storage material from liquid heat storage material, which is located in channels located in a solid TAM and heats

up faster due to the higher heat capacity of the material [1]).

To analyze the problem, the case of a step function was investigated:

$$\begin{cases} f(\theta) = a, a = \frac{k_x}{k_r}, & 0 < \theta \leq 1, \\ f(\theta) = 1, & \theta > 1, \end{cases} \quad (7)$$

where  $(k_x, k_r)$  is a mix ratios for ("cold") / ("hot") liquid, respectively.

A state diagram is used to describe the state of the system: dependence  $q(\theta_c)$  - that is, the dependence of the heat flow on temperature on the axis of symmetry of the channel (fig. 1).

In the half-plane  $\theta_c < 1$  near the plane of symmetry, there is always a layer of "cold" water. For intensive cooling ( $-q > -q_{AB}$ ) the temperature will be maximum, therefore, a "cold" liquid will flow over the entire section (on line AB, the total heat flow  $\Phi = 0$ ). With decreasing cooling intensity for  $0 < -q < -q_{AB}$ , the maximum temperature begins to vary from the axis. For sufficiently low temperatures,  $\theta_c$  maximum temperature  $\theta_{max} < 1$  and throughout the cross section we have a "cold" liquid (region ABRP). The boundary of this region is the RB curve  $d\theta_{max} = 1$ . At a temperature on the axis greater than  $\theta_{CRB}$  that inside the "cold" liquid, a zone of a "hot" medium appears and a three-ball flow is realized: "cold" + "hot" + "cold" liquid (BRE region). On the RE curve (for sufficiently small cooling flows), the wall temperature reaches the transition temperature, and in the ERL region a two-ball flow is realized: a "cold" liquid near the symmetry plane and a "hot" liquid near the walls. In the part of the half-plane that remains, the external medium heats the liquid and the maximum is the wall temperature.

For a low temperature on the pipe axis, this temperature remains less than the transition temperature (PRS region) and a "cold" liquid flows over the entire section. In the other part of the square, there is a two-spherical flow: "cold" + "hot" liquid.

For the right half-plane, the temperature on the axis is greater than the transition temperature, and there will always be a "hot" liquid near the plane of symmetry. It is shown that in the NBCF region there is a two-bead flow: "hot" + "cold" liquid, and in the FCD region, a one-bead flow with a maximum temperature on the plane of symmetry. On the CD line, the total heat inflow is zero, and on the line, the wall temperature becomes equal to the transition temperature. In the BEC and MLECD regions, the maximum is displaced toward the wall; moreover, in the BEC region, there is a two-ball flow, and in the MLECD, a single-ball flow. On the line EC, the wall temperature is equal to the transition temperature. In

the NLM region, the cross-section temperature monotonically increases; therefore, there is a "hot" liquid throughout the cross-section. If a point in the initial state is set above the line ABCD, then later it moves from right to left (liquid is cooled). When  $-q < -q_{ABCD}$  then the liquid heats up during the flow in the channel, and the given point moves from left to right.

## CONCLUSIONS

The non-isothermal motion of a nonlinear viscous fluid (within the framework of the Ostwald - de Vilya model) in a flat channel with a given heat flux on the

wall has been investigated, while the case is investigated when the temperature changes along the channel and three cases of heat exchange with the environment are described.

To analyze the problem of changing the rheological properties, the case of a step function with consistency coefficients was investigated.

To describe the state of the system when the temperature changes along the channel, a state diagram is shown (the dependence of the temperature on the axis of symmetry of the channel on the heat flux).

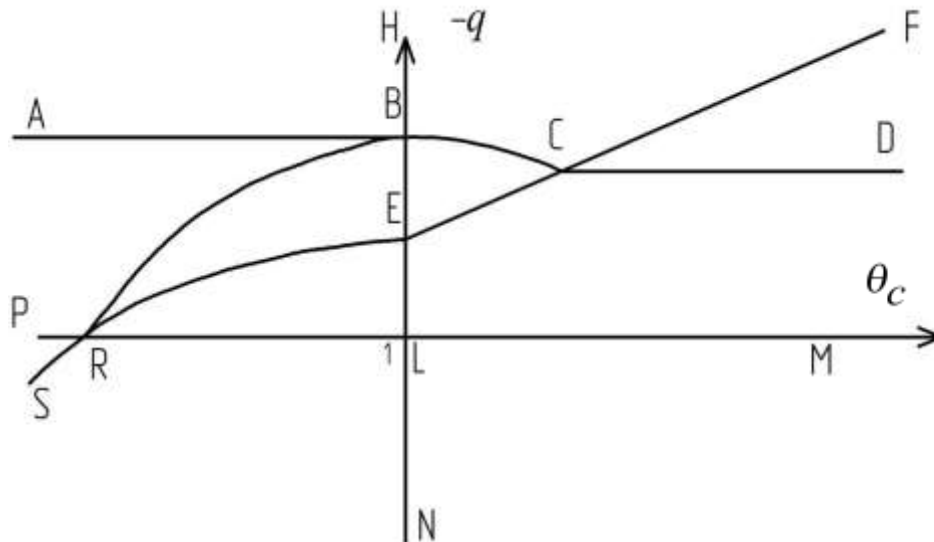


Fig. 1. State diagram

## REFERENCES

1. **Moskvitina A. S. 2020.** Analytical model of heat supply system with solar collectors and heat accumulator, *Young scientist*, №3, pp. 193-198.
2. **Bostanjiyan S. A., Merzhanov A. G., Khudyaev S. I. 1965.** Some problems on nonisothermal steady flow of a viscous fluid. *Applied Mechanics and Technical Physics*, No. 5, pp. 45-50.
3. **Trowbridge E.A., Karran I.H. 1973.** A discussion of critical parameters which can occur in frictionally heated non-newtonian fluid flows. *International Journal Heat and Mass Transfer*, Vol.16, №10, pp. 1833-1848.
4. **Benderskaya S.L., Khusid B.M., Shulman Z.P. 1980.** Non-isothermal flow of non-Newtonian fluids in a channel, *Mechanics of liquids and gases*, № 3, pp. 3-10.
5. **Shulman Z.P. 1975.** Convective Heat and Mass Transfer of Rheologically Complex Liquids, Energiya, Moscow.