

# A new chaotic system with axe-shaped equilibrium, its circuit implementation and adaptive synchronization

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In the recent years, chaotic systems with uncountable equilibrium points such as chaotic systems with line equilibrium and curve equilibrium have been studied well in the literature. This reports a new 3-D chaotic system with an axe-shaped curve of equilibrium points. Dynamics of the chaotic system with the axe-shaped equilibrium has been studied by using phase plots, bifurcation diagram, Lyapunov exponents and Lyapunov dimension. Furthermore, an electronic circuit implementation of the new chaotic system with axe-shaped equilibrium has been designed to check its feasibility. As a control application, we report results for the synchronization of the new system possessing an axe-shaped curve of equilibrium points.

**Key words:** chaos, chaotic systems, curve equilibrium, Lyapunov exponents, circuit design, synchronization

## 1. Introduction

It is well-known that chaos theory has been applied to several areas such as such as lasers [1, 2], memristors [3, 4], chemical reactions [5, 6], finance [7], oscillators [8–15], neural networks [16], ecology [17], biology [18, 19], weather systems [20, 21], electrical circuits [22–27], sound encryption [28], image encryption [29], cryptosystems [30], robotics [31], secure communication devices [32, 33], etc.

An important area in chaos theory is the modelling of chaotic systems with infinite number of rest points (or equilibrium points) such as line rest points [34–36] or closed curve of rest points such as circle [37], square [38], rounded square [39], rectangle [40], cloud-shaped curve [41], etc.

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Recent research on chaos modelling has shown that the chaotic systems with infinite number of rest points can be classified as special chaotic systems with hidden attractors [42, 43]. Hence, discovery of chaotic systems with line equilibrium or closed curve equilibrium is an important research topic. The synchronization of the chaotic systems has useful applications in the control literature [44–47].

In this work, we report a new chaotic system with a closed curve of equilibrium points, which has the shape of an axe. We present the system dynamics, phase plots, and analysis of the chaotic system with axe-shaped equilibrium. Furthermore, we provide a circuital implementation of the chaotic system with axe-shaped equilibrium. As a control application, we report results for the synchronization of the new system possessing an axe-shaped curve of equilibrium points.

## 2. A new chaotic system with an axe-shaped curve of equilibrium points

Recently, Pham *et al.* [39] have reported a general model of dynamical systems given by

$$\begin{cases} \dot{x} = z, \\ \dot{y} = -zf(x, y, z), \\ \dot{z} = g(x, y), \end{cases} \quad (1)$$

where  $f(x, y, z)$  and  $g(x, y)$  are two arbitrary nonlinear functions.

We get the rest points of the general model (1) by solving the equations:

$$z = 0, \quad (2a)$$

$$-zf(x, y, z) = 0, \quad (2b)$$

$$g(x, y) = 0. \quad (2c)$$

It is clear from (2a) that  $z = 0$  for any rest point of the system (1).

Thus, we deduce from (2b) and (2c) that the rest points of the system (1) lie on the curve  $g(x, y) = 0$  on the plane  $z = 0$ .

Hence, by a suitable choice of the functions  $f(x, y, z)$  and  $g(x, y)$ , we can construct numerous dynamical systems with different types of open or closed curves of rest points.

In this paper, we choose the functions as

$$\begin{cases} f(x, y, z) = -x^2 - ay + by^2 + cxz, \\ g(x, y) = 4|x| + 4|y| + xy - 10, \end{cases} \quad (3)$$

which leads to the system

$$\begin{cases} \dot{x} = z, \\ \dot{y} = -z(-x^2 - ay + by^2 + cxz), \\ \dot{z} = 4|x| + 4|y| + xy - 10. \end{cases} \quad (4)$$

Thus, the rest points of the new system (4) are described by the closed curve

$$z = 0, \quad 4|x| + 4|y| + xy - 10 = 0, \quad (5)$$

which is an axe-shaped curve in the  $(x, y)$ -plane as shown in Figure 1. It is observed that the axe-shaped curve (Figure 1) is very different from the basic closed shapes like circle, square, rounded square and cloud [37–41].

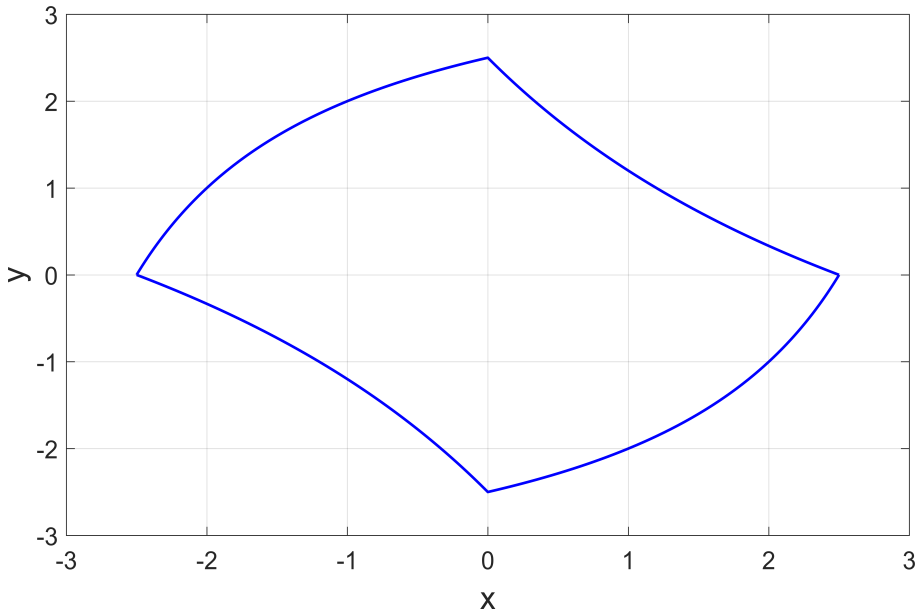


Figure 1: Axe-shaped curve of rest points of the system (4)

For the choice  $(a, b, c) = (6, 0.2, 0.1)$ , we show that the system (4) is chaotic.

For numerical simulations of phase plots and for the calculation of Lyapunov chaos exponents, we take  $X(0) = (0, 0, 0.2)$  and  $(a, b, c) = (6, 0.2, 0.1)$ . The chaos nature of (4) is guaranteed by the Lyapunov exponents calculated using Wolf’s algorithm [49] for  $T = 5000s$  as

$$L_1 = 0.11867 > 0, \quad L_2 = 0, \quad L_3 = -0.13175 < 0. \quad (6)$$

For the system (4),  $L_1 + L_2 + L_3 = -0.01308 < 0$ .

Thus, the system (4) is *dissipative*.

The local finite-time Lyapunov dimension [48] of the system (4) is estimated as

$$D_L = 2 + \frac{L_1 + L_2}{|L_3|} = 2.9007. \quad (7)$$

We observe high complexity of the system (4) with axe-shaped equilibrium by the high value of the Lyapunov dimension  $D_L$ .

It is observed that the system (4) is invariant under the change of coordinates  $(x, y, z) \mapsto (-x, -y, -z)$  for all values of the parameter set  $(a, b, c)$ . This shows that the system (4) has point reflection symmetry about the origin in  $\mathbf{R}^3$ . Hence, if  $(x(t), y(t), z(t))$  is a trajectory of the system (4), then  $(-x(t), -y(t), -z(t))$  will be also a trajectory of the system (4).

Figures 2–5 show the phase plots of the new chaotic attractor (4) for  $X(0) = (0, 0, 0.2)$  and  $(a, b, c) = (6, 0.2, 0.1)$ .

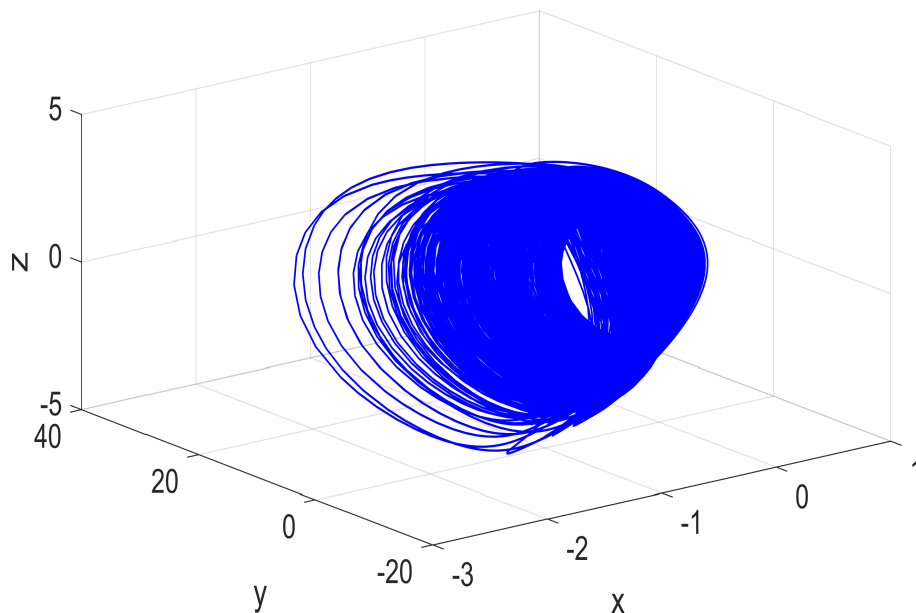


Figure 2: MATLAB simulation of (4) for  $X(0) = (0, 0, 0.2)$  and  $(a, b, c) = (6, 0.2, 0.1)$

The bifurcation diagram of the system (4) is illustrated in Figure 6. In addition, Poincaré map in Figure 7 also displays chaotic property of the system (4).

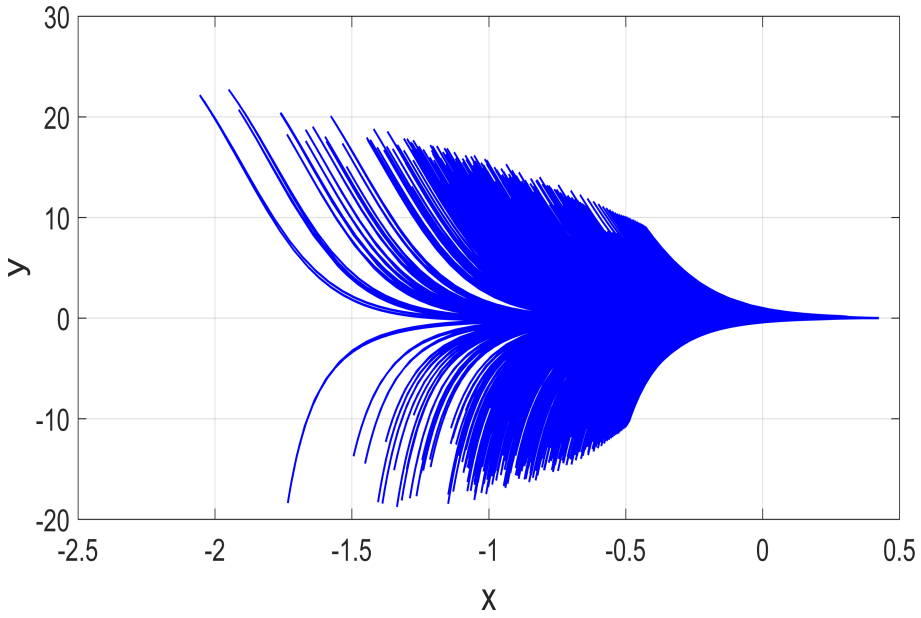


Figure 3: MATLAB simulation of (4) in  $(x, y)$ -plane for  $X(0) = (0, 0, 0.2)$  and  $(a, b, c) = (6, 0.2, 0.1)$

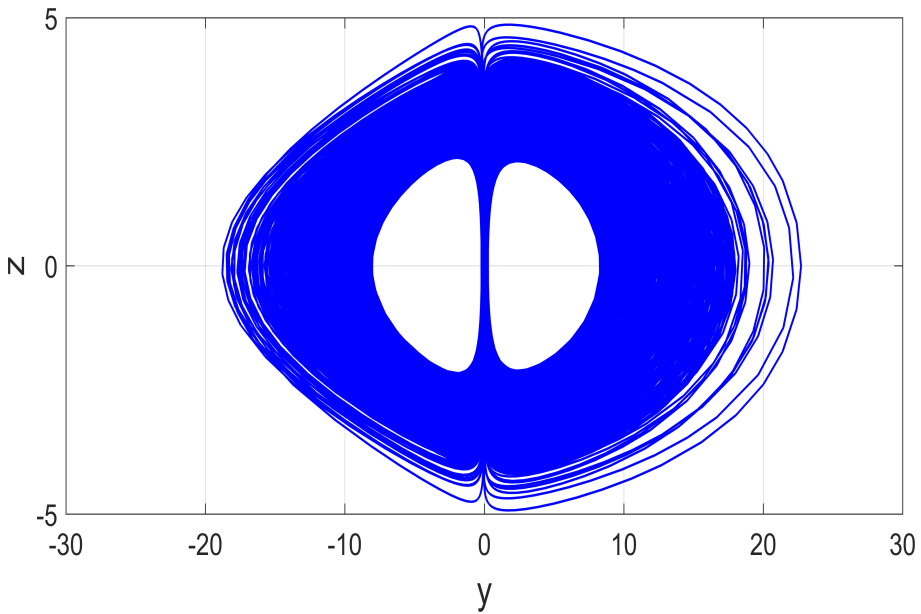


Figure 4: MATLAB simulation of (4) in  $(y, z)$ -plane for  $X(0) = (0, 0, 0.2)$  and  $(a, b, c) = (6, 0.2, 0.1)$

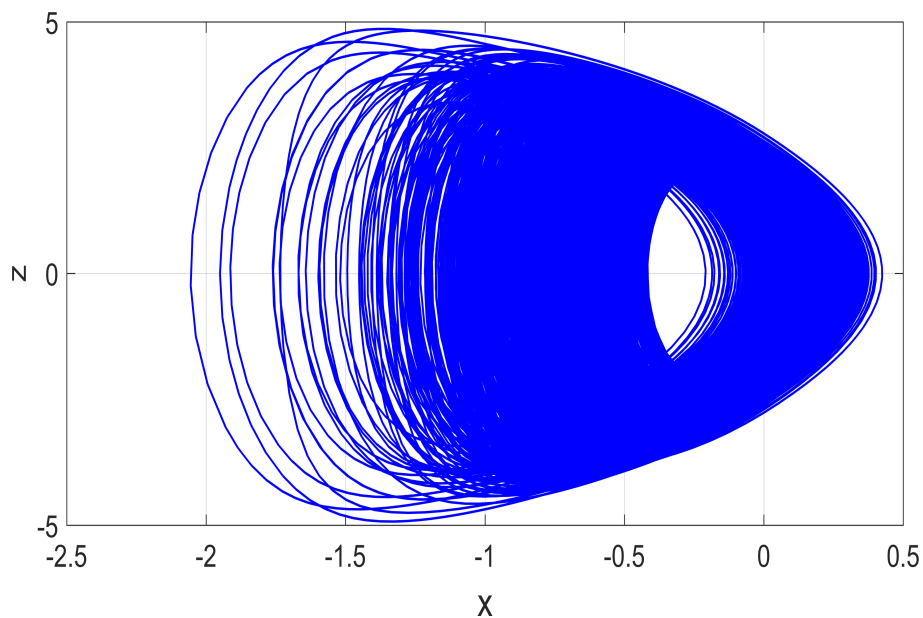


Figure 5: MATLAB simulation of (4) in  $(x, z)$ -plane for  $X(0) = (0, 0, 0.2)$  and  $(a, b, c) = (6, 0.2, 0.1)$

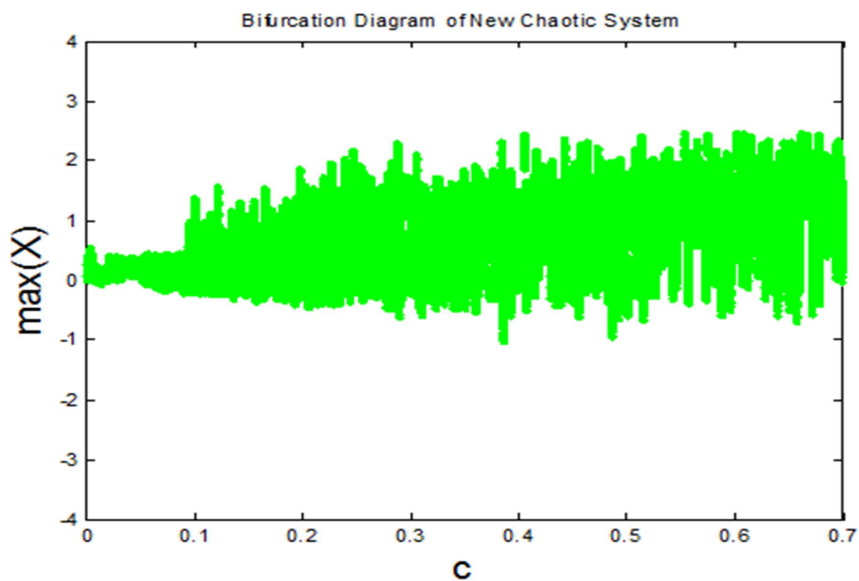


Figure 6: Bifurcation diagram of (4) with respect to the bifurcation parameter  $c$  when  $a = 6$  and  $b = 0.2$

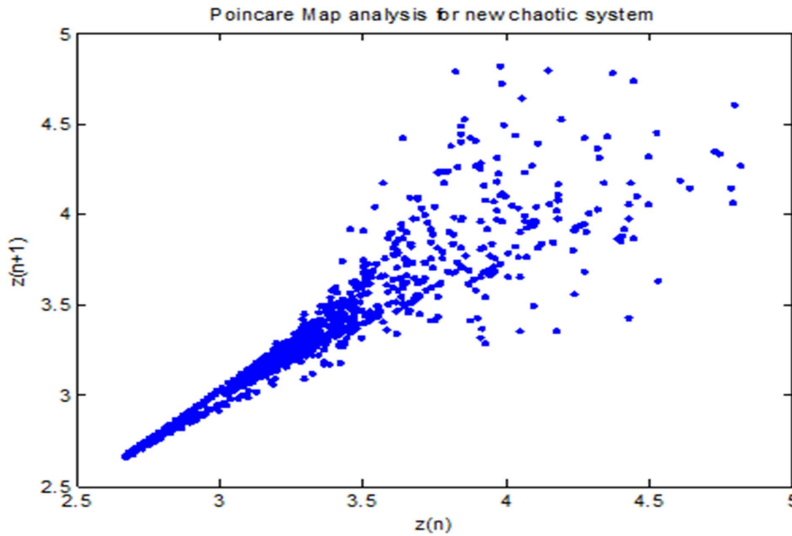


Figure 7: Poincaré map of (4) in the plane  $z(n + 1)$  versus  $z(n)$  for  $(a, b, c) = (6, 0.2, 0.1)$

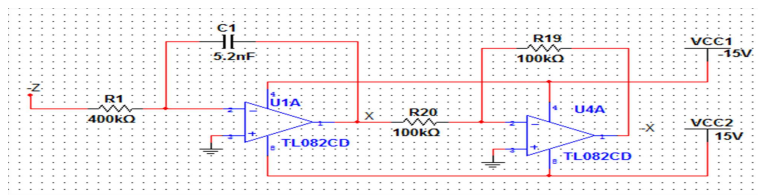
### 3. Circuit realization of the system with axe-shaped equilibrium points

In this section, the system (4) with axe-shaped equilibrium is realized by an electronic circuit shown in Figure 8. The main circuit that realizes the system (4), has three integrators (U1A, U3A, U5A), four inverting amplifiers (U2A, U4A, U6A, U10A), one absolute function by  $|x|$  signal (U8A, U9A) and one absolute function by  $|y|$  signal (U7A, U11A). The state  $X = (x, y, z)$  of the system (4) is characterized by the voltage across the capacitors  $(C_1, C_2, C_3)$ . For the scale setting on the circuit, we can use the amplitude control method [25–27]. Thus, the system (4) is changed into the following system.

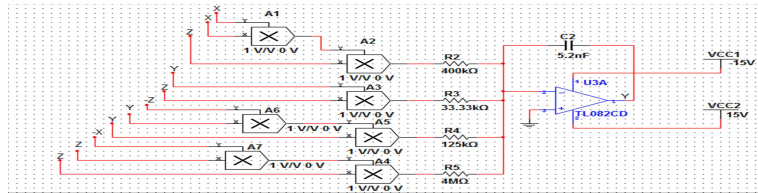
$$\begin{cases} \dot{x} = z, \\ \dot{y} = z(-x^2 - 2ay + 16by^2 + cxz), \\ \dot{z} = 4|x| + 16|y| + 8xy - 5. \end{cases} \quad (8)$$

Applying the Kirchoff's laws, the circuit of Figure 8 is described by the following equations:

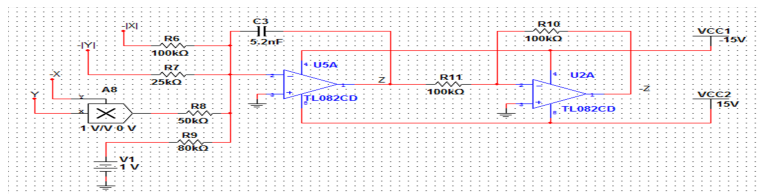
$$\begin{aligned} \dot{x} &= \frac{1}{C_1 R_1} z, \\ \dot{y} &= -\frac{1}{C_2 R_2} z x^2 - \frac{1}{C_2 R_3} z y + \frac{1}{C_2 R_4} z y^2 + \frac{1}{C_2 R_5} x z^2, \\ \dot{z} &= \frac{1}{C_3 R_6} |x| + \frac{1}{C_3 R_7} |y| + \frac{1}{C_3 R_8} xy - \frac{1}{C_3 R_9} V_1. \end{aligned} \quad (9)$$



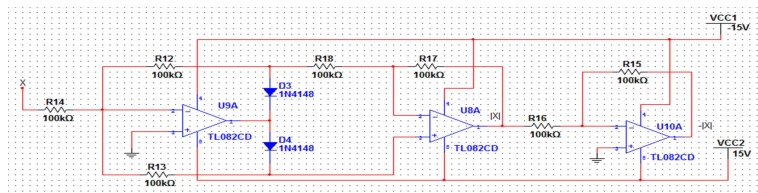
(a)



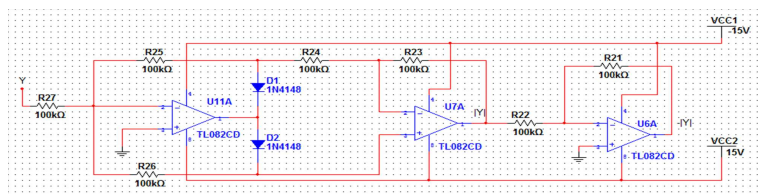
(b)



(c)



(d)



(e)

Figure 8: Circuit design of (4): (a) X signal, (b) Y signal, (c) Z signal, (d)  $|X|$  signal and (e)  $|Y|$  signal

The electronic components are chosen as:  $R_1 = R_2 = 400 \text{ K}\Omega$ ,  $R_3 = 33.33 \text{ K}\Omega$ ,  $R_4 = 125 \text{ K}\Omega$ ,  $R_5 = 4 \text{ M}\Omega$ ,  $R_7 = 25 \text{ K}\Omega$ ,  $R_8 = 50 \text{ K}\Omega$ ,  $R_9 = 80 \text{ K}\Omega$ ,  $R_6 = R_{10} = R_{11} = R_{12} = R_{13} = R_{14} = R_{15} = R_{16} = R_{17} = R_{18} = R_{19} = R_{20} =$



$R_{21} = R_{22} = R_{23} = R_{24} = R_{25} = R_{26} = R_{27} = 100 \text{ K}\Omega$ ,  $C_1 = C_2 = C_3 = 5.2 \text{ nF}$  and  $V_1 = 1V_{DC}$ . Oscilloscope phase portraits of the circuit are represented in Figures 9–11. It is easy to see the good agreement between MATLAB simulation results (Figures 3–5) and the circuit simulation results (Figures 9–11).

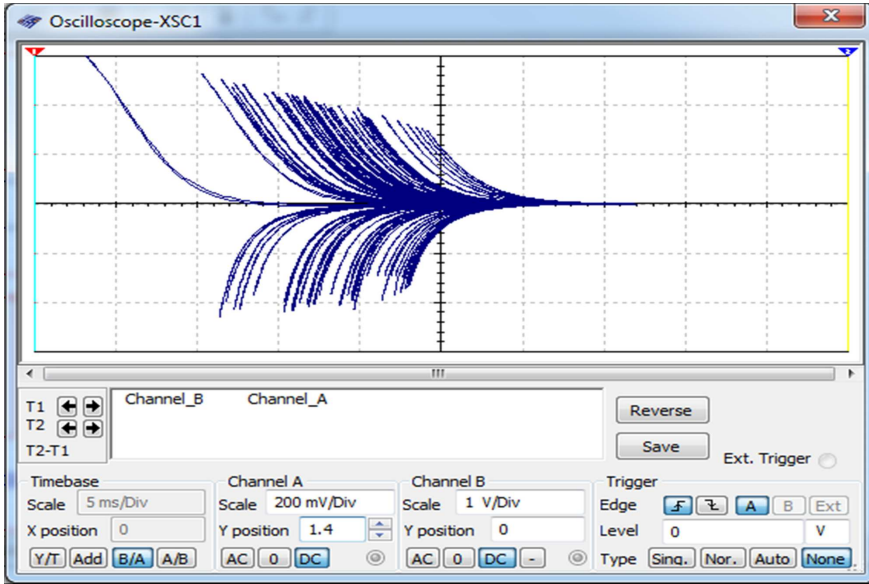


Figure 9: MultiSIM 10 simulation of the system in  $(x, y)$ -plane

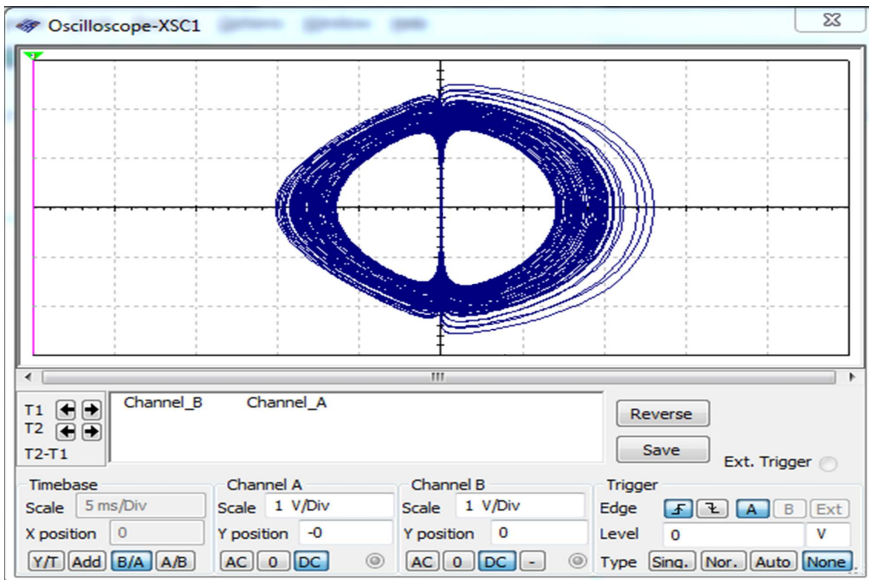


Figure 10: MultiSIM 10 simulation of the system in  $(y, z)$ -plane

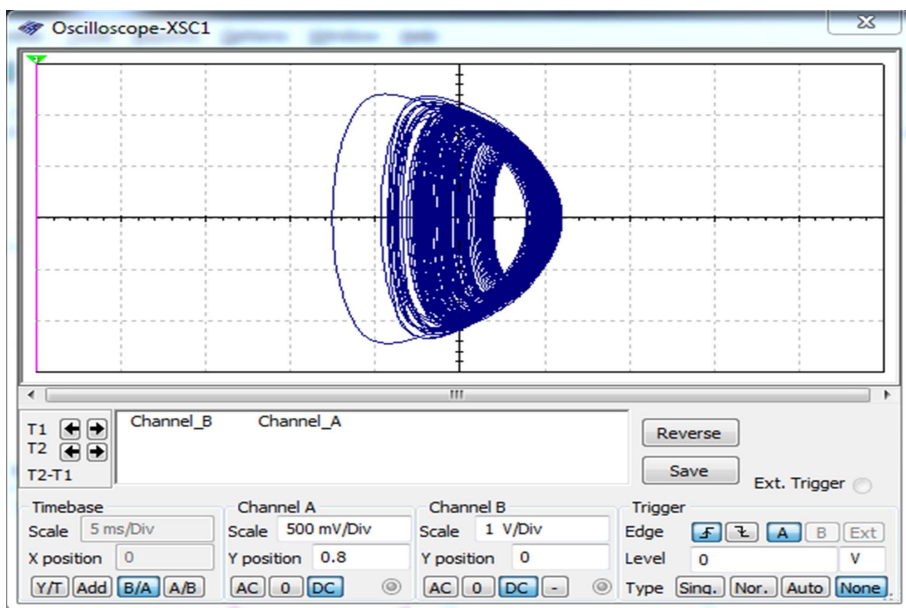


Figure 11: MultiSIM 10 simulation of the system in  $(x, z)$ -plane

#### 4. Synchronization results for the new chaotic system with axe-shaped equilibrium

In this section, we design an adaptive synchronizing law for achieving global synchronization of a pair of identical new chaotic systems with axe-shaped equilibrium points (called as the *drive* and *response* systems).

As the drive system, we take the new system with axe-shaped equilibrium given by the dynamics

$$\begin{cases} \dot{x}_1 = z_1, \\ \dot{y}_1 = -z_1(-x_1^2 - ay_1 + by_1^2 + cx_1z_1), \\ \dot{z}_1 = 4|x_1| + 4|y_1| + x_1y_1 - 10. \end{cases} \quad (10)$$

In Eq. (10),  $X_1 = (x_1, y_1, z_1)$  represents the state of the drive system and  $a, b$  are unknown parameters of the system.

As the response system, we take the new system with axe-shaped equilibrium with controls given by the dynamics

$$\begin{cases} \dot{x}_2 = z_2 + u_x, \\ \dot{y}_2 = -z_2(-x_2^2 - ay_2 + by_2^2 + cx_2z_2) + u_y, \\ \dot{z}_2 = 4|x_2| + 4|y_2| + x_2y_2 - 10 + u_z. \end{cases} \quad (11)$$

In Eq. (11),  $X_2 = (x_2, y_2, z_2)$  represents the state of the response system and  $u = (u_x, u_y, u_z)$  is the adaptive control to be determined.

We define the synchronization error between the state responses  $X_1$  and  $X_2$  as

$$\begin{cases} e_x = x_2 - x_1, \\ e_y = y_2 - y_1, \\ e_z = z_2 - z_1. \end{cases} \quad (12)$$

It is easy to determine the error dynamics as follows:

$$\begin{cases} \dot{e}_x = e_z + u_x, \\ \dot{e}_y = z_2x_2^2 - z_1x_1^2 + a(y_2z_2 - y_1z_1) - b(y_2^2z_2 - y_1^2z_1) \\ \quad - c(x_2z_2^2 - x_1z_1^2) + u_y, \\ \dot{e}_z = 4(|x_2| - |x_1| + |y_2| - |y_1|) + x_2y_2 - x_1y_1 + u_z. \end{cases} \quad (13)$$

As an adaptive control, we consider the feedback law

$$\begin{cases} u_x = -e_z - k_x e_x, \\ u_y = -z_2x_2^2 + z_1x_1^2 - \alpha(t)(y_2z_2 - y_1z_1) + \beta(t)(y_2^2z_2 - y_1^2z_1) \\ \quad + \gamma(t)(x_2z_2^2 - x_1z_1^2) - k_y e_y, \\ u_z = -4(|x_2| - |x_1| + |y_2| - |y_1|) - x_2y_2 + x_1y_1 - k_z e_z. \end{cases} \quad (14)$$

In (14),  $(\alpha(t), \beta(t), \gamma(t))$  is an estimate of  $(a, b, c)$  and  $k_x, k_y, k_z \in \mathbf{R}^+$ .

When we implement the feedback control law (14), we obtain the closed-loop synchronization error dynamics as

$$\begin{cases} \dot{e}_x = -k_x e_x, \\ \dot{e}_y = [a - \alpha(t)](y_2z_2 - y_1z_1) - [b - \beta(t)](y_2^2z_2 - y_1^2z_1) \\ \quad - [c - \gamma(t)](x_2z_2^2 - x_1z_1^2) - k_y e_y, \\ \dot{e}_z = -k_z e_z. \end{cases} \quad (15)$$

To simplify the closed-loop dynamics (15), we set

$$\begin{aligned} e_a &= a - \alpha(t), \\ e_b &= b - \beta(t), \\ e_c &= c - \gamma(t). \end{aligned} \quad (16)$$

Then (15) simplifies into the error dynamics

$$\begin{cases} \dot{e}_x = -k_x e_x, \\ \dot{e}_y = e_a(y_2 z_2 - y_1 z_1) - e_b(y_2^2 z_2 - y_1^2 z_1) \\ \quad - e_c(x_2 z_2^2 - x_1 z_1^2) - k_y e_y, \\ \dot{e}_z = -k_z e_z. \end{cases} \quad (17)$$

It is also easy to verify that

$$\dot{e}_a = -\dot{\alpha}, \quad \dot{e}_b = -\dot{\beta}, \quad \dot{e}_c = -\dot{\gamma}. \quad (18)$$

We consider the quadratic Lyapunov function candidate

$$V(e_x, e_y, e_z, e_a, e_b, e_c) = 0.5(e_x^2 + e_y^2 + e_z^2 + e_a^2 + e_b^2 + e_c^2). \quad (19)$$

Time-derivative of  $V$  along the dynamics (17) and (18) is calculated as follows.

$$\begin{aligned} \dot{V} &= -k_x e_x^2 - k_y e_y^2 - k_z e_z^2 + e_a[e_y(y_2 z_2 - y_1 z_1) - \dot{\alpha}], \\ &\quad + e_b[-e_y(y_2^2 z_2 - y_1^2 z_1) - \dot{\beta}] + e_c[-e_y(x_2 z_2^2 - x_1 z_1^2) - \dot{\gamma}]. \end{aligned} \quad (20)$$

We consider the following dynamics for the parameter updates.

$$\begin{cases} \dot{\alpha} = e_y(y_2 z_2 - y_1 z_1), \\ \dot{\beta} = -e_y(y_2^2 z_2 - y_1^2 z_1), \\ \dot{\gamma} = -e_y(x_2 z_2^2 - x_1 z_1^2). \end{cases} \quad (21)$$

**Theorem 1** *The new chaotic systems with axe-shaped equilibrium points given by the dynamics (10) and (11) are globally and asymptotically synchronized by the adaptive control law (14) and the parameter update law (21) where  $k_x, k_y, k_z \in \mathbf{R}^+$ .*

**Proof.** It is an easy observation that the candidate Lyapunov function  $V$  defined via Eq. (19) is positive definite on  $\mathbf{R}^6$ .

Furthermore, when we substitute (21) into (20), we obtain the time-derivative of  $V$  as

$$\dot{V} = -k_x e_x^2 - k_y e_y^2 - k_z e_z^2. \quad (22)$$

Thus,  $\dot{V}$  is a negative semi-definite function on  $\mathbf{R}^3$ .

We set  $K = \min\{k_x, k_y, k_z\}$  and  $\mathbf{e} = (e_x, e_y, e_z)$ .

Then we get  $\dot{V} \leq -K|\mathbf{e}|^2$ , which can be expressed as

$$K|\mathbf{e}(t)|^2 \leq -\dot{V}. \tag{23}$$

By integration of the above inequality, we deduce that

$$K \int_0^t |\mathbf{e}(\tau)|^2 d\tau \leq V(0) - V(t). \tag{24}$$

Thus, it follows that  $\mathbf{e}(t) \in \mathbf{L}_2$ .

Using (17), it can be deduced that  $\dot{\mathbf{e}} \in \mathbf{L}_\infty$ .

As a consequence of Barbalat's lemma [50],  $\mathbf{e}(t) \rightarrow 0$  as  $t \rightarrow \infty$  for all values of  $\mathbf{e}(0) \in \mathbf{R}^3$ . □

For numerical simulations, we take the gain constants as  $(k_1, k_2, k_3) = (10, 10, 10)$  and the system parameter values as  $(a, b, c) = (6, 0.2, 0.1)$ .

As initial states of the systems (10) and (11), we take  $(x_1(0), y_1(0), z_1(0)) = (1.5, 0.6, 0.1)$  and  $(x_2(0), y_2(0), z_2(0)) = (0.4, 0.3, 2.4)$ .

Also, we take  $(\alpha(0), \beta(0), \gamma(0)) = (1.2, 0.5, 3.1)$ .

Figures 12–14 show the complete synchronization of (10) and (11), while Figure 15 shows the time-history of the synchronization error  $\mathbf{e}$ .

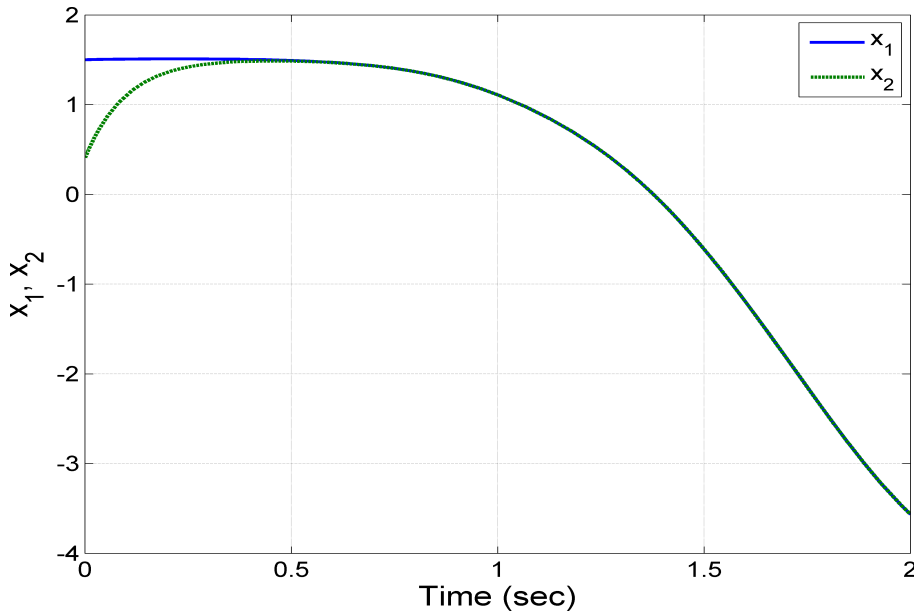


Figure 12: MATLAB simulation showing synchronization of the states  $x_1$  and  $x_2$  of the systems (10) and (11)

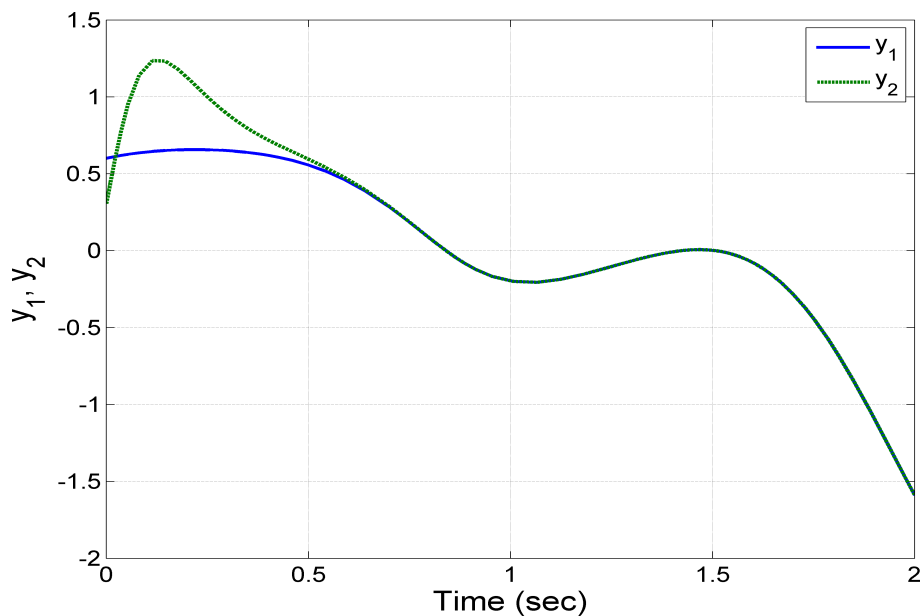


Figure 13: MATLAB simulation showing synchronization of the states  $y_1$  and  $y_2$  of the systems (10) and (11)

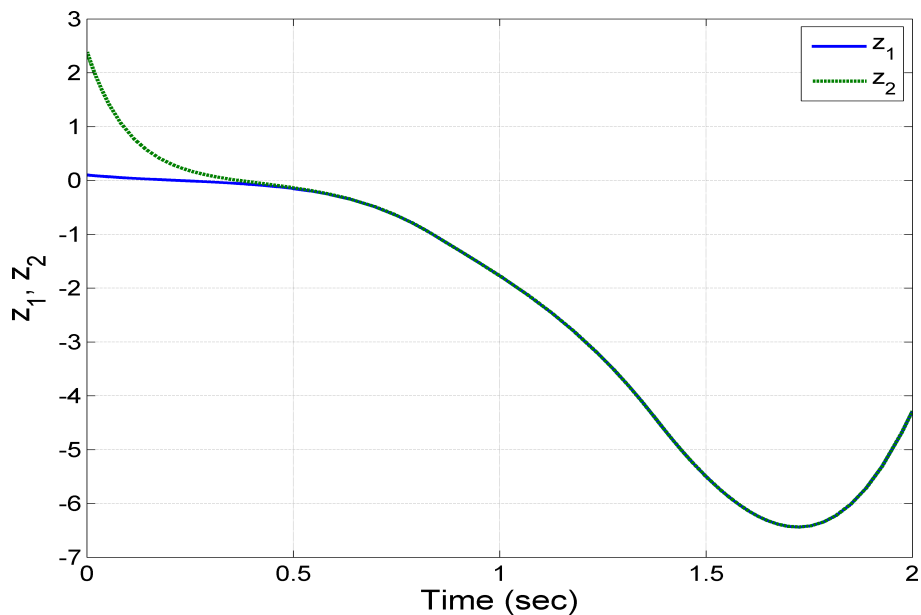


Figure 14: MATLAB simulation showing synchronization of the states  $x_1$  and  $x_2$  of the systems (10) and (11)

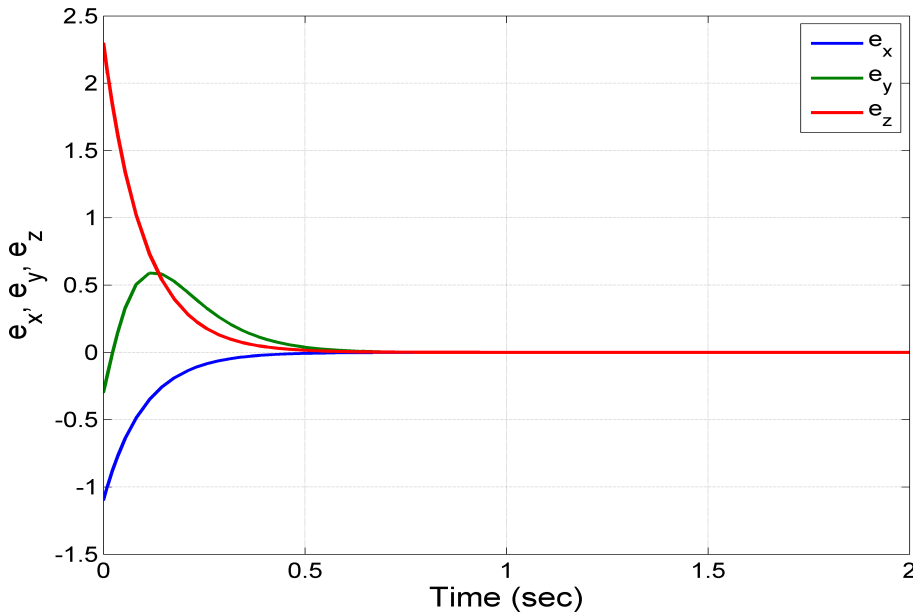


Figure 15: MATLAB simulation showing the time-history of the synchronization error  $e$  between the systems (10) and (11)

## 5. Conclusions

In this research work, we reported a chaotic system with an axe-shaped curve of equilibrium points. In addition to detailing the dynamic properties of the new system, an electronic circuit implementation of the new chaotic system with axe-shaped equilibrium was also reported to check its feasibility. As a control application, we gave results for the synchronization of the new system possessing an axe-shaped curve of equilibrium points.

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