# Improved Control of Mesh Density in Adaptive Tetrahedral Meshes for Finite Element Modeling

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Abstract: Tetrahedral meshing is the critical element of finite element modeling. Recently, adaptive meshing has been commonly used. In such meshing, according to the Delaunay method, mesh density is connected with the curvature of modeled object's edge. Such a method is especially efficient during the modeling of mechanical systems. However, the efficiency of commonly used meshing algorithms is strongly limited in surface-focused phenomena, such as eddy current induced by magneto-dynamic processes. The paper proposes the improved method of Delaunay meshing considering the specific requirements of magnetodynamic systems. In the proposed method, tetrahedral mesh density may be flexibly modified according to the needs of modeled physical phenomena, such as eddy current density. As a result, physical effects may be efficiently and accurately described in finite element models. The paper presents the example of implementing the proposed solution for cylindrical wire. The complete source code is available as open-source software for further practical use and development.

**Keywords:** adaptive meshing, tetrahedral meshing, FEM modeling, magnetodynamic systems

## 1. Wprowadzenie

The finite element method (FEM) is widely used for modeling solid-state mechanics [4], flow [3], and magnetodynamic systems, including microwave [9] and magnetoinductive [7] systems. Recently developed open-source FEM-based toolchains give the unprecedented possibility of flexible modeling and development of advanced and sophisticated systems, such as eddy current tomography [8], resistive tomography [6, 11], or airplane mechanics optimization [13].

On the other hand, the essential tool for efficient and robust FEM-based models is a meshing algorithm. Despite the fact that uniform grids are the most efficient from a physical point of view [12], such a meshing strategy leads to a radical increase in the number of elements [9]. As a result, adaptive meshes that utilize first-order tetrahedral elements are commonly used. In such meshes, considering the Delaunay algorithm [7], mesh density is connected with the curvature of the nearest edges of a meshed object. Such an approach is especially efficient during the modeling of mechanical systems, where strongly non-uniform mechanical stress distribution occurs in the close neighborhood of indents. In such a case, a decrease in the radius of indent edges causes an increase in mesh density. This mechanism is especially beneficial for the accuracy of mechanical models.

On the other hand, in the case of magnetodynamic systems, the accuracy of models is mainly connected with the accuracy of eddy currents modeling. Eddy current distribution is mostly coupled with driving signal frequency [5] as well as material magnetic permeability and resistivity instead of the radius of the nearest edge. As a result, commonly used Delaunay algorithm-based approaches to meshing for mechanical systems are not efficient for magnetodynamic modeling.

It should be highlighted that a more specific approach is essential for advanced analyses of magneto-mechanical systems utilizing magnetodynamic phenomena [14]. Such systems are especially promising in the case of haptic robots [10], where user-friendly and robust coil-less magneto-mechanical external mechanical forces sensors might be developed. However, precise FEM models of such sensors, necessary for their efficient application, require balancing of mesh density considering both mechanical and eddy current-based phenomena.

The paper presents both the method and implementation of adaptive meshing strategies suitable for magnetodynamic systems. The proposed method is implemented based on an open-source NETGEN mesher [17], considering the example of

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a cylindrical wire subjected to a high frequency driving current. Such a system is especially suitable for mechanical forces haptic sensors of human cooperating robots.

Importantly, open-source tools used for the study allow further free development of the code. Even though solutions to adapt local mesh for each model are available in other commercial software such as Ansys [18], they are limited to predefined specific cases. Open access to the source code removes such limitations and allows better control of user-defined meshes.

## 2. Delunay adaptive meshing principles

In meshes generated according to the Delaunay algorithm [2], mesh density is directly connected with the radius of the surrounding edges of an object. An example of such an object is presented in fig. 1. The tetrahedral mesh of a rectangular body with a cylinder-shaped indent is much denser nearly the indent, which can be observed in fig. 1b.

On the other hand, the Delaunay algorithm applied for meshing cylindrical bodies creates uniform meshes. It is connected with the fact that the curvature of a cylinder is constant regardless of the direction. The examples of such tetrahedral meshes with lower and higher mesh density are presented in figures 2a and 2b, respectively. Despite the fact that the lower density mesh consists of tetrahedral 3 137 elements, whereas the higher density mesh consists of 1 421 307 elements, both meshes are created uniformly. Such a meshing algorithm would be strongly inefficient for high-frequency magnetodynamic systems, where physical phenomena are observed nearly the surface of a modeled element.

## 3. The proposition of adaptive meshing for magnetodynamic modeling

For the efficient proposition of tetrahedral meshing algorithms for the magnetodynamic modeling of a cylindrical object, the eddy current distribution in such a wire has to be analyzed. In terms of a complex value, the eddy current density $J(r)$, dependent on a distance from the wire axis $r$ in the cylindrical wire, is given according to the following equations [15, 16]:

$$J(r) = \frac{k \cdot I}{2\pi \cdot R} \cdot \frac{J_0(k \cdot r)}{J_1(k \cdot r)} \tag{1}$$

$$k = \sqrt{\frac{-2\pi f \mu_r \mu_0 j}{\rho}} \tag{2}$$

where $f$ is the driving current $I$ frequency, $m_r$ is the relative magnetic permeability of the material, $m_0$ is the magnetic constant, ρ is the resistivity of the material, whereas $J_0(x)$ and $J_1(x)$ are Bessel functions of the first kind, 0 and 1 order respectively:

$$J_0(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-i \cdot x \cdot \sin(t)} dt \tag{3}$$

$$J_1(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{i \cdot (t - x \cdot \sin(t))} dt \tag{4}$$

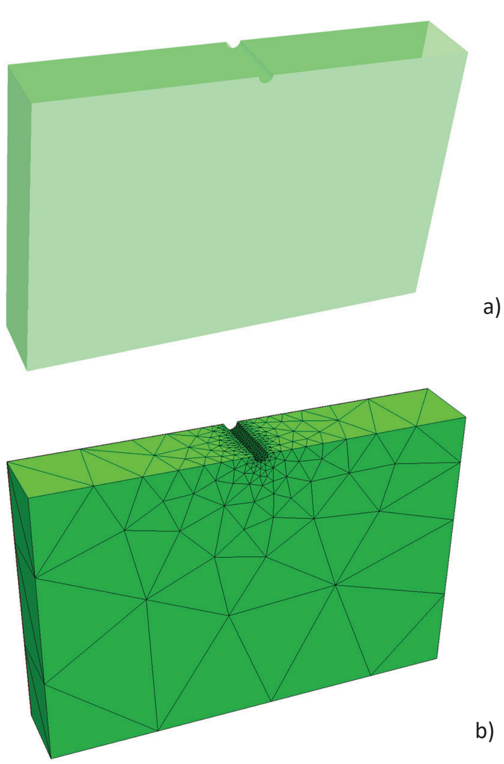


**Fig. 1. The example of a tetrahedral mesh generated according to the Delaunay algorithm: a) the shape of a modeled rectangular body with a cylinder indent, b) tetrahedral mesh created on the base of the Delaunay algorithm**

Rys. 1. Przykład siatki tetrahedralnej wygenerowanej według algorytmu Delaunaya: a) kształt zamodelowanej bryły prostokątnej z wcięciem walcowym, b) siatka tetrahedralna utworzona na podstawie algorytmu Delaunaya

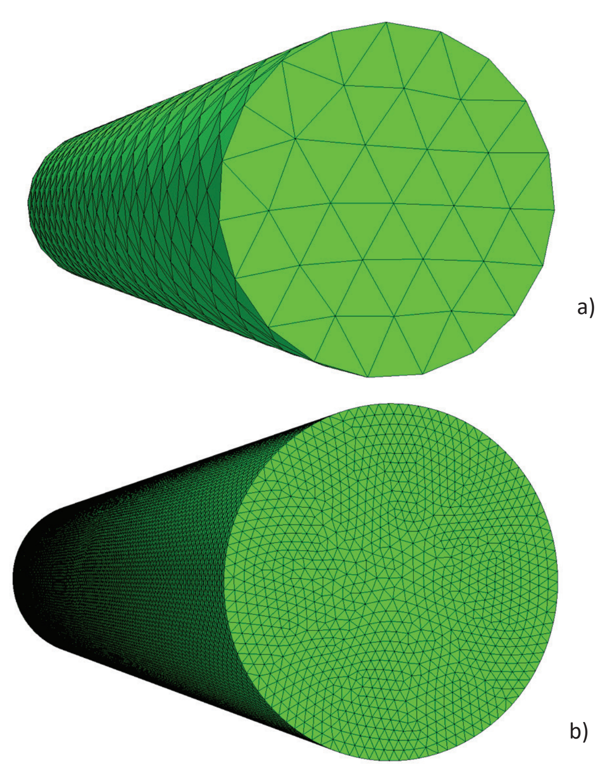


**Fig. 2. The examples of Delaunay algorithm-based tetrahedral meshes: a) low-density mesh (3 137 elements), b) high-density mesh (1 421 307 elements)**

Rys. 2. Przykłady siatek tetrahedralnych opartych na algorytmie Delaunaya: a) siatka o małej gęstości (3 137 elementów), b) siatka o dużej gęstości (1 421 307 elementów)

Due to the fact that $J(r)$ function is complex, its amplitude $|J(r)|$ can be calculated as:

$$\left|J(r)\right| = \sqrt{\left(re\,J(r)\right)^2 + \left(im\,J(r)\right)^2} \tag{5}$$

Figure 3 presents the amplitude of current density $|J(r)|$ in the wire calculated for the different values of driving frequency $f$ in the wire with diameter $D = 2$ mm, made of typical steel with relative magnetic permeability $m_r = 1000$ and resistivity $\rho = 1.6 \cdot 10^{-7}$ Wm.

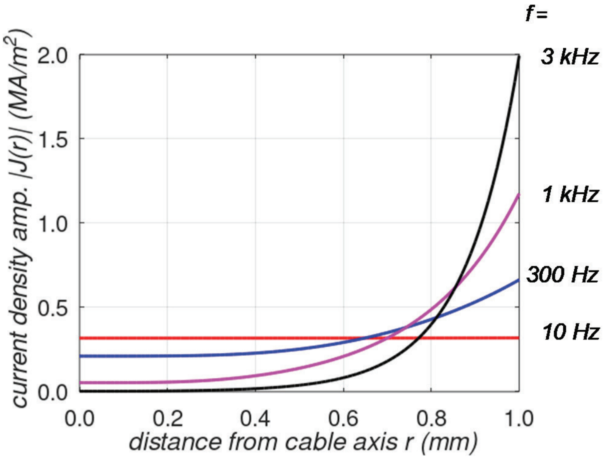


**Fig. 3. The amplitude of current density $|J(r)|$ in the wire calculated for driving frequency $f$ = 10 Hz, 300 Hz, 1 kHz, and 3 kHz. The relative magnetic permeability of wire material $\mu_r$ = 1000, the resistivity of a wire $\rho = 1.6 \cdot 10^{-7}$ Ωm**

Rys. 3. Amplituda gęstości prądu $|J(r)|$ w przewodzie obliczonym dla częstotliwości sterowania $f$ = 10 Hz, 300 Hz, 1 kHz i 3 kHz. Względna przenikalność magnetyczna materiału drutu $\mu_r$ = 1000, rezystywność drutu $\rho = 1{,}6 \cdot 10^{-7}$ Ωm

After defining all of the model's magnetomechanical parameters, the corresponding mesh with required refinement can be implemented. However, if necessary, after the analysis of a set of simulations with a refined mesh, meshing details can be modified manually by the user accordingly in order to perform more accurate calculations.

During the meshing utilizing NETGEN open-source software, mesh density is controlled by *maxH* parameter describing the maximal acceptable height of a tetrahedral element [17]. Parameter *maxH* might be provided as a global parameter for the element (in .geo file geometry description) or by a special .msz file. In the .msz file, mesh densities might be defined in terms of *maxH* parameters in specified points (where the position of a point and the *maxH* value is given) or on the specified lines (where the position of the beginning and endpoint of the line together with the *maxH* value is given) [17]. The example and the explanation of the structure of .msz file for NETGEN are provided in fig. 4.

The principles of the control of mesh density parameter *maxH* in the wire are presented in fig. 5. First, the difference between the maximal and minimal value of eddy current density $J(r)$ is divided by $m_1$ points into $m_1 - 1$ sections. Next, each section is divided into $m_2$ layers, where the maximal height *maxH* of any tetrahedral element is equal to the distance between layers.

It should be highlighted that in the proposed algorithm calculated value of *maxH* might only increase the value of mesh density. If the computed value of *maxH* is higher than the previously defined global value of the maximal height of a tetrahedral element, the global *maxH* value is dominant.

a)

```
3
1.000 0.000 5.000 0.017
1.000 0.006 5.000 0.017
1.000 0.011 5.000 0.017
5
0.122 0.993 5.000 0.122 0.993 -5.000 0.017
0.117 0.993 5.000 0.117 0.993 -5.000 0.017
0.111 0.994 5.000 0.111 0.994 -5.000 0.017
0.106 0.994 5.000 0.106 0.994 -5.000 0.017
0.100 0.995 5.000 0.100 0.995 -5.000 0.017
```

b)

```
<no. of points defined>
<x> <y> <z> <maxH>
<x> <y> <z> <maxH>
<x> <y> <z> <maxH>
<no. of lines defined>
<x1> <y1> <z1> <x2> <y2> <z2> <maxH>
<x1> <y1> <z1> <x2> <y2> <z2> <maxH>
<x1> <y1> <z1> <x2> <y2> <z2> <maxH>
<x1> <y1> <z1> <x2> <y2> <z2> <maxH>
<x1> <y1> <z1> <x2> <y2> <z2> <maxH>
```

**Fig. 4. NETGEN .msz file: a) the example of a file, b) the explanation of the file's structure**

Rys. 4. Plik NETGEN .msz: a) przykładowy plik, b) objaśnienie struktury pliku

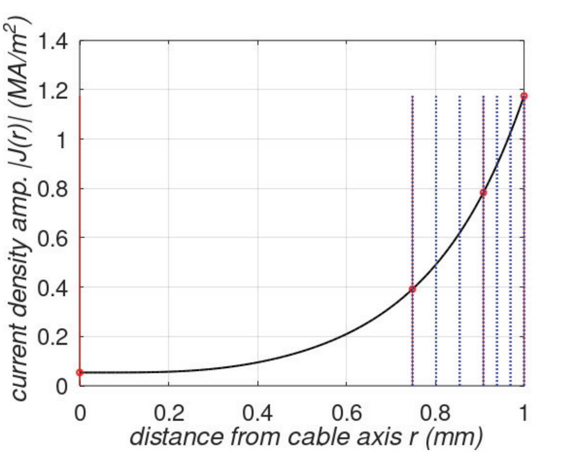


**Fig. 5. The principles of the control of mesh density parameter *maxH* in the wire for driving frequency $f$ = 1 kHz, the relative magnetic permeability of the wire material $\mu_r$ = 1000, resistivity of the wire $\rho = 1.6 \cdot 10^{-7}$ Ωm, $m_1 = 4$, $m_2 = 3$**

Rys. 5. Zasady kontroli parametru gęstości siatki maxH w drucie dla częstotliwości zasilania $f$ = 1 kHz, względnej przenikalności magnetycznej materiału drutu $\mu_r$ = 1000, rezystywności drutu $\rho = 1{,}6 \cdot 10^{-7}$ Ωm, $m_1 = 4$, $m_2 = 3$

To enable the further development of the proposed method as well as its validation, the source code generating results presented in the paper is freely available at: www.github.com/romanszewczyk/magdyn_meshes.

The examples of meshes for the cylindrical wire made of steel ($\mu_r = 1000$, the resistivity of the wire $\rho = 1.6 \cdot 10^{-7}$ Wm, $m_1 = 4$, $m_2 = 3$) are presented in figure 6. Meshes were generated according to the described method for the driving frequencies $f$ = 10 Hz, 300 Hz, 1 kHz, and 3 kHz, respectively. It should be observed that mesh density increases mostly nearly the surface, where the highest values of eddy current density $J(r)$ occur, whereas central elements remain relatively large. As a result, the significant reduction of the required number of tetrahedral elements may be reached without significant degradation of the quality of magnetodynamic modeling.

For the same high granularity of the model throughout its volume, for 300 Hz – 1 095 817 elements would be needed, for 1 kHz – 7 647 323 elements, and for 3 kHz – 26 359 587 elements. The presented method enables for the number of elements reduction to 696 609 for 300 Hz, 3 097 356 for 1 kHz and 9 413 330 for 3 kHz, which reduces the need for computa-

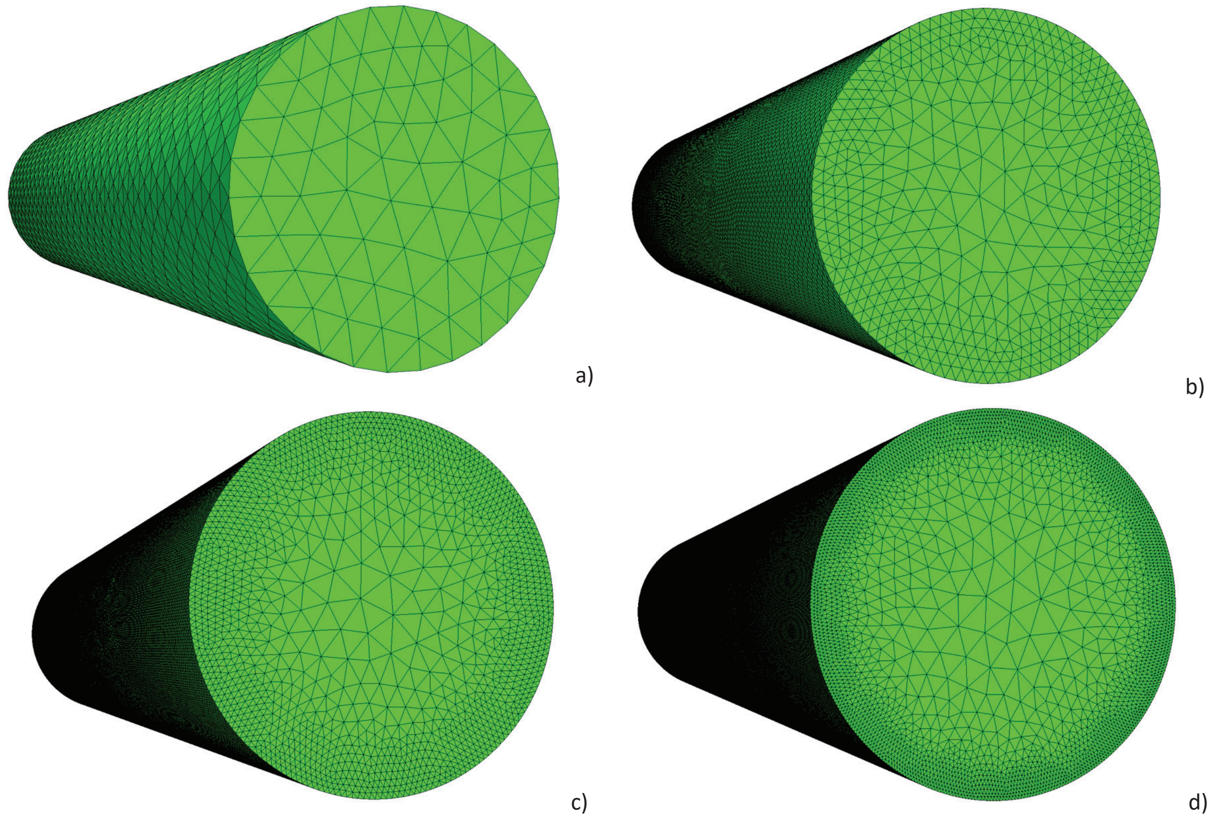


**Fig. 6. The examples of meshes for the cylindrical wire made of steel ($\mu_r$ = 1000, the resistivity of the wire $\rho$ = 1.6·10$^{-7}$ Ωm, $m_1$ = 4, $m_2$ = 3) generated accordingly to the presented method for different values of the driving frequency *f*: a) 10 Hz, b) 300 Hz, c) 1 kHz, d) 3 kHz**
Rys. 6. Przykładowe siatki dla drutu cylindrycznego wykonanego ze stali ($\mu_r$ = 1000, rezystywność drutu $\rho$ = 1,6·10$^{-7}$ Ωm, $m_1$ = 4, $m_2$ = 3) wygenerowane zgodnie z prezentowaną metodą dla różnych wartości częstotliwości sterowania *f*: a) 10 Hz, b) 300 Hz, c) 1 kHz, d) 3 kHz

tional memory from 2 (for 300 Hz) through 6 (for 1 kHz) up to 9 times for 3 kHz.

## 4. Conclusions

The efficient generation of adaptive tetrahedral meshes for magnetodynamic models requires taking into account eddy current distribution. Such meshes can significantly differ from Delaunay tetrahedral meshes suitable for mechanical models.

The method proposed in the paper directly connects the amplitude of eddy current density in a cylindrical wire with the maximal acceptable height of a tetrahedral element. As a result, generated meshes are adequate to the requirements of magnetodynamic modeling. In addition, the proposed algorithm was implemented with the use of an open-source code, including NETGEN software. As a result, the proposed solution might be easily developed and validated in further applications.

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# Udoskonalona kontrola gęstości siatki w adaptacyjnych siatkach tetrahedralnych dla metody elementów skończonych

Streszczenie: Siatki tetrahedralne są kluczowym elementem w metodzie elementów skończonych. Ostatnio powszechnie stosowane są siatki adaptacyjne, w których zgodnie z metodą Delaunay'a gęstość powiązana jest z krzywizną krawędzi modelowanego obiektu. Metoda ta jest wyjątkowo efektywna w przypadku modelowania układów mechanicznych. Niemniej jednak wydajność powszechnie stosowanych metod siatkowania jest mocno ograniczona w przypadku zjawisk skupionych na powierzchni modelu, takich jak prądy wirowe indukowane przez procesy magnetodynamiczne. Artykuł przedstawia propozycję ulepszonej metody siatkowania metodą Delaunay'a, uwzględniającą specyficzne wymagania układów magnetodynamicznych. W proponowanej metodzie gęstość siatki tetrahedralnej może być elastycznie modyfikowana odpowiednio do potrzeb modelowanego zjawiska, takiego jak gęstość prądów wirowych. W rezultacie efekty fizyczne mogą być efektywnie i dokładnie opisane za pomocą modeli stworzonych metodą elementów skończonych. W artykule przedstawiono przykład implementacji zaproponowanego rozwiązania dla przewodu cylindrycznego. Pełny kod źródłowy dostępny jest w formie otwartej licencji do dalszego rozwoju i użycia w praktyce.

**Słowa kluczowe:** siatki adaptacyjne, siatki tetrahedralne, modelowanie MES, układy magnetodynamiczne

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