Robust Flow Controllers for a Single Virtual Circuit in Data Transmission Networks with Lossy Links

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Abstract

The paper concerns an application of regulation theory methods to modeling and effective control of connection-oriented data transmission networks. In particular the problem of congestion control in a single virtual circuit of such a network is considered and new discrete-time sliding mode data flow rate controllers are proposed. The controllers are designed in such a way that packet losses are explicitly accounted for. The closed-loop system stability and finite-time error convergence are proved. Moreover, a number of favorable properties of the proposed controllers are stated as theorems, formally proved and verified in a simulation example. It is demonstrated that the proposed controllers guarantee full utilization of the available bandwidth and eliminates the risk of bottleneck node buffer overflow. Application of time-varying sliding hyperplanes helps avoid excessive transmission rates at the beginning of the control process.

Key words: data transmission networks, congestion control, sliding-mode control, discrete-time systems

1 Introduction

The problem of congestion control in data transmission networks has recently become one of the most extensively studied research issues. Due to bandwidth variations, packet losses, round trip time uncertainty and users' constraints, the solution of the problem is not an easy task. On the other hand, the control theoretic approach [14, 19] to the congestion elimination offers many well developed tools and methods which can turn out to be very useful in the design of flow management strategies. Therefore, in this paper we introduce a discrete time model of a single virtual circuit in connection-oriented network and we apply sliding mode methodology [13–18, 20] to solve the congestion problem in the circuit.

The difficulty of the congestion control in modern data transmission networks is mainly caused by long propagation delays in the system. If congestion occurs at a specific node, information about this condition must be conveyed to all the sources transmitting data through that node, which involves feedback propagation delays. The congestion control in connection-oriented networks has recently been studied in several papers [1–12]. The control algorithms proposed in those papers employ a proportional plus derivative [10], stochastic [7], adaptive [9] and Smith predictor based control strategies [1], [2], [11], [12]. Recently a number of sliding mode congestion control algorithms have also been proposed [3–6]. However, not many results on congestion control in networks with lossy links are available. Therefore, this paper presents a sliding mode flow controller for a single connection which looses some packets during the transmission process. In other words, in this paper – on the contrary to the previously published results – we consider not only data losses caused by the bottleneck buffer link overflow, but also those which for other reasons happen on the transmission way from the source to the bottleneck link.

In the next section we introduce the state space model of the network, and then in section 3 we use this model to design a feasible sliding mode congestion control strategies.

2 Network Model

In this paper we consider a virtual circuit in a connection-oriented network which consists of a single data source, intermediate nodes and a destination. The block diagram of the circuit is shown in Figure 1. It is assumed that there is only one bottleneck node in the network. A controller which determines data transmission rate of the source is placed at the bottleneck node. The output signal of the controller (denoted by u) is sent back to the source, and reaches it after backward delay T_B. The source then sends the specified amount of data, which is passed from node to node until it reaches the bottleneck queue after forward delay T_F. We assume that somewhere along that line a known, fixed percentage of data packets are lost so that only αu (where $\alpha \in (0,1)$) data packets arrive at the bottleneck node. The round trip time RTT, i.e. the delay between generating a signal by the controller and the requested data arriving at the bottleneck queue, is a sum of the forward and backward propagation delays

$$RTT = T_B + T_F \qquad (1)$$

Further in the paper, T represents the discretisation period, $x(kT)$ is the bottleneck queue length at time instant kT, and $x_d > 0$ is the demand value of $x(kT)$. It is assumed that before the start of data transmission, the buffer is empty, i.e.

$x(kT < 0) = 0$. We also assume that the round trip time is a multiple of the discretisation period, i.e. $RTT = m_{RTT}T$, where m_{RTT} is a positive integer.

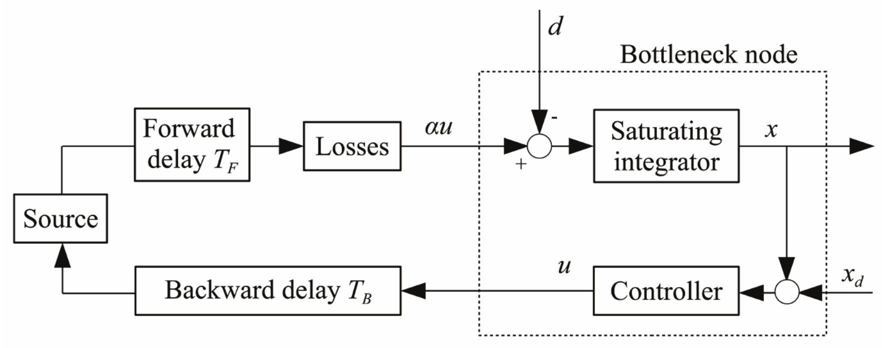

Figure 1. The network model

The controller output at time kT is denoted as $u(kT)$.The first data will reach the queue after RTT so for any time $kT \leq RTT$ the queue length

$$x(kT) = 0 \tag{2}$$

The amount of data which may leave the bottleneck buffer is modeled as an *a priori* unknown bounded function of time $d(kT)$. The maximum value of $d(kT)$ is represented by d_{max}. The amount of data actually leaving the bottleneck node at time kT is denoted by $h(kT)$. For any $k \geq 0$

$$0 \leq h(kT) \leq d(kT) \leq d_{max} \tag{3}$$

The queue length for $kT > RTT$ may be expressed as

$$x(kT) = \alpha \sum_{j=0}^{k-1} u(jT - RTT) - \sum_{j=0}^{k-1} h(jT) = \alpha \sum_{j=0}^{k-m_{RTT}-1} u(jT) - \sum_{j=0}^{k-1} h(jT) \tag{4}$$

and the network can be formulated in the state space in the following form

$$\begin{aligned} \boldsymbol{x}\left[(k+1)T\right] &= \boldsymbol{A}\boldsymbol{x}(kT) + \boldsymbol{b}u(kT) + \boldsymbol{o}h(kT) \\ y(kT) &= \boldsymbol{q}^T \boldsymbol{x}(kT) \end{aligned} \tag{5}$$

where $\boldsymbol{x}(kT) = [x_1(kT)\ x_2(kT)\ \ldots\ x_n(kT)]^T$ is the state vector, $y(kT) = x_1(kT)$ is the queue length, and $x_i(kT) = u[(k - n + i - 1)T]$ for any $i = 2, \ldots, n$. Furthermore, $\boldsymbol{A}$ is an n x n state matrix

$$\boldsymbol{A} = \begin{bmatrix} 1 & \alpha & 0 & & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & & 0 \end{bmatrix} \tag{6}$$

b, ***o*** and ***q*** denote $n \times 1$ vectors

$$\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad \mathbf{o} = \begin{bmatrix} -1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{q} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \tag{7}$$

and $n = m_{RTT} + 1$. The state space equation can also be rewritten as follows

$$\begin{cases} x_1\left[(k+1)T\right] = x_1(kT) + \alpha x_2(kT) - h(kT) \\ x_2\left[(k+1)T\right] = x_3(kT) \\ \vdots \\ x_{n-1}\left[(k+1)T\right] = x_n(kT) \\ x_n\left[(k+1)T\right] = u(kT) \end{cases} \tag{8}$$

with the output signal $y(kT) = x_1(kT)$. The desired state of the system is denoted by $\boldsymbol{x_d} = [x_{d1} \;\; x_{d2} \;\; \ldots \;\; x_{dn}]^T$. The first state variable x_{d1} is the demand queue length, and further in the paper it is represented by x_d. It can be noticed from equations (8) that for $h(kT) = 0$ all other components of the demand state vector are equal to zero.

3 Congestion Control Strategies

In this section the flow control problem for the described network is considered. In chapter 3.1 a chattering-free discrete-time sliding mode controller is designed that guarantees finite-time error convergence to zero. Important properties of the proposed control strategy are then formulated and proved. Since the strategy proposed in chapter 3.1 may lead to large values of control signal in the starting phase of the control process, in chapter 3.2 a time-varying sliding hyperplane is introduced that minimizes this effect. Then important properties of the modified control strategy are also formulated and proved.

3.1 Time-Invariant Sliding Hyperplane

For the sliding mode controller design purpose we neglect the disturbance $h(kT)$ and introduce a sliding hyperplane described by the following equation

$$s(kT) = \boldsymbol{c}^T \boldsymbol{e}(kT) = 0 \tag{9}$$

where vector $\boldsymbol{c}^T = [c_1 \;\; c_2 \;\; \ldots \;\; c_n]$ satisfies $\boldsymbol{c}^T\boldsymbol{b} \neq 0$. Error of the closed loop system is denoted by $\boldsymbol{e}(kT) = \boldsymbol{x_d} - \boldsymbol{x}(kT)$. Substituting (5) into $\boldsymbol{c}^T\boldsymbol{e}[(k+1)T] = 0$ we obtain the following control law

$$u(kT) = \left(\boldsymbol{c}^T\boldsymbol{b}\right)^{-1} \boldsymbol{c}^T \left[\boldsymbol{x_d} - \boldsymbol{A}\boldsymbol{x}(kT)\right] \tag{10}$$

When this control signal is used, the closed-loop system state matrix has the form $\boldsymbol{A_c} = [\mathbf{1} - \boldsymbol{b}(\boldsymbol{c}^T\boldsymbol{b})^{-1}\boldsymbol{c}^T]\boldsymbol{A}$. The characteristic polynomial of this matrix

$$det\left(z\boldsymbol{I_n} - \boldsymbol{A_c}\right) = z^n + \frac{c_{n-1} - c_n}{c_n} z^{n-1} + \ldots + \frac{\alpha c_1 - c_2}{c_n} z \tag{11}$$

which gives the condition $c_n \neq 0$. A discrete-time system is asymptotically stable if and only if all of its eigenvalues are located inside the unit circle. Furthermore, to ensure finite-time error convergence to zero the characteristic polynomial (11) has to satisfy

$$det\left(z\boldsymbol{I_n} - \boldsymbol{A_c}\right) = z^n \tag{12}$$

Comparing the coefficients of (11) and (12) we find the following form of vector $\boldsymbol{c}$

$$\boldsymbol{c}^T = \left[\, 1/\alpha \quad 1 \quad \cdots \quad 1 \,\right] c_n \tag{13}$$

Using (6), (7) and (13) we can rewrite (10) as follows

$$u(kT) = \frac{1}{\alpha}\left[x_d - x(kT)\right] - \sum_{i=2}^{n} x_i(kT) \tag{14}$$

From (8) we notice that all the state variables except x_1 are the delayed values of the control signal, i.e. for $i = 2, \ldots, n$

$$x_i(kT) = u\left[(k - n + i - 1)T\right] \tag{15}$$

Substituting (15) into (14) we get

$$u(kT) = \frac{1}{\alpha}\left[x_d - x(kT)\right] - \sum_{i=k-m_{RTT}}^{k-1} u(iT) \tag{16}$$

This completes the design of a flow control algorithm with a time-invariant sliding hyperplane.

Properties of the Proposed Strategy

In the previous section a time-invariant sliding hyperplane has been designed to guarantee stability and finite-time error convergence of the closed-loop system. The amount of data to be sent is given by (16). Consequently

$$u(0) = \frac{x_d}{\alpha} \tag{17}$$

Lemma 1: If the designed sliding mode controller is applied, then its output for any $k \geq 0$ satisfies

$$u(kT) = \frac{1}{\alpha} h\left[(k-1)T\right] \tag{18}$$

Proof: Substituting (4) into (16) we obtain

$$\begin{aligned} u(kT) &= \frac{1}{\alpha}\left[x_d - \alpha \sum_{j=0}^{k-m_{RTT}-1} u(jT) + \sum_{j=0}^{k-1} h(jT)\right] - \sum_{j=k-m_{RTT}}^{k-1} u(jT) \\ &= \frac{1}{\alpha}\left[x_d + \sum_{j=0}^{k-1} h(jT)\right] - \sum_{j=0}^{k-1} u(jT) \end{aligned} \tag{19}$$

By mathematical induction: first we check if (18) holds for $k = 1$

$$u(T) = \frac{1}{\alpha}\left[x_d + \sum_{j=0}^{0} h(jT)\right] - \sum_{j=0}^{0} u(jT) = \frac{1}{\alpha}\left[x_d + h(0)\right] - \frac{x_d}{\alpha} = \frac{1}{\alpha} h(0) \tag{20}$$

Now we assume that (18) holds for some $k = m$, where m is a positive integer, i.e.

$$u(mT) = \frac{1}{\alpha} h\left[(m-1)T\right] \tag{21}$$

Then using this assumption we can find from (19) that for $k = m + 1$

$$\begin{aligned} u\left[(m+1)T\right] &= \frac{1}{\alpha}\left[x_d + \sum_{j=0}^{m} h(jT)\right] - \sum_{j=0}^{m} u(jT) \\ &= \frac{1}{\alpha}\left[x_d + \sum_{j=0}^{m} h(jT)\right] - \frac{x_d}{\alpha} - \sum_{j=1}^{m} u(jT) \\ &= \frac{1}{\alpha}\sum_{j=0}^{m} h(jT) - \frac{1}{\alpha}\sum_{j=0}^{m-1} h(jT) = \frac{1}{\alpha} h(mT) \end{aligned} \tag{22}$$

which means that if (18) holds for $k = m$, then it also holds for $k = m + 1$.

Finally, taking into account (21) and (22) we can conclude that (18) indeed holds for any integer $k \geq 0$. This ends the proof.

Lemma 1 clearly shows that the output of the proposed controller is always nonnegative and bounded, i.e. for any $k \geq 1$

$$0 \leq u(kT) \leq \frac{1}{\alpha} d_{max} \tag{23}$$

Theorem 1: If the proposed strategy is used, then the queue length will never exceed its demand value, i.e. for any $k \geq 0$

$$x(kT) \leq x_d \tag{24}$$

Proof: From (2) for any $k < (m_{RTT} + 1)$ the queue length $x(kT) = 0$. Hence to prove the theorem we only need to check if (24) holds for $k \geq m_{RTT} + 1$. Using (18) we can rewrite (4) as

$$x(kT) = x_d - \sum_{j=k-m_{RTT}-1}^{k-1} h(jT) \leq x_d \tag{25}$$

This ends the proof.

From the first equation in set (8) we notice that if $x[(k+1)T]$ is greater than zero, then the available bandwidth $d(kT)$ is fully used. Theorem 2 gives the necessary condition to guarantee that the queue length is strictly positive.

Theorem 2: If the proposed strategy is used, and the demand queue length satisfies

$$x_d > (m_{RTT} + 1) d_{max} \tag{26}$$

then for any $k > m_{RTT}$ the queue length is always strictly positive.

Proof: From (3) we see that for any $k \geq 0$ the consumed bandwidth is always upper bounded $h(kT) \leq d_{max}$. Using (25) for $k > m_{RTT}$, we obtain

$$x\left(kT\right)=x_d-\sum_{j=k-m_{RTT}-1}^{k-1} h\left(jT\right)\geq x_d-\left(m_{RTT}+1\right)d_{max}>0 \tag{27}$$

This ends the proof.

Theorem 2 shows that for any $k > m_{RTT}$ the queue length is strictly greater than zero, which implies that the available bandwidth is fully used for any $k \geq m_{RTT}$.

3.2 Time-Varying Sliding Hyperplane

A disadvantage of the control strategy proposed in chapter 3.1 is a large value of the control signal at the first time instant. Therefore, in this subsection we introduce a time-varying hyperplane that reduces this effect. The properties of this modified strategy are then formulated and proved.

We replace equation (9) describing the sliding hyperplane with the following one

$$s\left(kT\right)=c^T e\left(kT\right)+f\left(kT\right)=0 \tag{28}$$

where $f(kT)$ is an *a priori* known function of time chosen to satisfy $s(0) = 0$ (the representative point at time instant $k = 0$ is positioned on the sliding hyperplane). This gives the following condition

$$f\left(0\right)=-c^T e\left(0\right) \tag{29}$$

Because the previously proposed controller exhibits very good dynamic performance after the starting phase of the regulation process, there should exist such a k_0 that $f(kT) = 0$ for any $k > k_0$. Furthermore function $f(kT)$ should be strictly monotonic in the time interval $[0, k_0]$ .With the use of such a function the sliding hyperplane moves monotonically towards the origin of the coordinate frame, intersects it after k_0, and remains fixed for any $k > k_0$.

We chose $f(kT)$ to be linear in the interval $[0, k_0]$. Thus it can be written as follows

$$f\left(kT\right)=\begin{cases}\dfrac{k-k_0}{k_0}\boldsymbol{c}^T\boldsymbol{e}\left(0\right) & \text{for} \quad k\leq k_0\\ 0 & \text{for} \quad k>k_0\end{cases} \tag{30}$$

Now substituting $\boldsymbol{e}(kT) = \boldsymbol{x_d} - \boldsymbol{x}(kT)$ into $s[(k + 1)T] = 0$ we obtain

$$u(kT)=\left(\boldsymbol{c}^T\boldsymbol{b}\right)^{-1}\left\{\boldsymbol{c}^T\left[\boldsymbol{x}_d-\boldsymbol{A}\boldsymbol{x}(kT)\right]+f\left[(k+1)T\right]\right\} \tag{31}$$

where vector $\boldsymbol{c}$ is given by (13) in order to maintain the desirable properties of the previous control strategy for $k > k_0$.

Using (6), (7), (13) and (15) we rewrite (31) as follows

$$\begin{aligned} u(kT)&=\frac{1}{\alpha}\left[x_d-x_1(kT)\right]-\sum_{i=2}^{n}x_i(kT)+\frac{1}{c_n}f\left[(k+1)T\right] \\ &=\frac{1}{\alpha}\left[x_d-x_1(kT)\right]-\sum_{j=k-m_{RTT}}^{k-1}u(jT)+\frac{1}{c_n}f\left[(k+1)T\right] \end{aligned} \tag{32}$$

This completes the design of a flow control algorithm with the proposed time-varying sliding hyperplane.

Properties of the Proposed Strategy

In the previous subsection we modified the strategy proposed in chapter 3.1, introducing a time-varying hyperplane. The goal of this modification is to reduce the control signal in the starting phase of the data transmission process. In this section, we formulate and prove the properties of this altered algorithm. Lemma 2 shows that the control signal is nonnegative and upper bounded. Theorems 3 and 4 (analogous to Theorems 1 and 2) show that the queue length will not exceed its demand value and that after some initial time the queue length will always be strictly positive, which implies that the available bandwidth will be fully used.

Lemma 2: If the designed sliding mode controller is applied, then its output for any $k \geq 0$ satisfies

$$u(kT)=\frac{1}{\alpha}h\left[(k-1)T\right]+\frac{1}{c_n}\left\{f\left[(k+1)T\right]-f(kT)\right\} \tag{33}$$

Proof: Substituting (4) into (32)

$$u(kT)=\frac{1}{\alpha}\left[x_d+\sum_{j=0}^{k-1}h(jT)\right]-\sum_{j=0}^{k-1}u(jT)+\frac{1}{c_n}f\left[(k+1)T\right] \tag{34}$$

By mathematical induction, first we check if (33) holds for $k = 0$

$$u(0)=\frac{1}{\alpha}\left[x_d+\sum_{j=0}^{-1}h(jT)\right]-\sum_{j=0}^{-1}u(jT)+\frac{1}{c_n}f(T)$$
$$=\frac{1}{\alpha}x_d+\frac{1}{c_n}f(T)=\frac{1}{c_n}\left[f(T)-f(0)\right]=\frac{1}{\alpha}h(-T)+\frac{1}{c_n}\left[f(T)-f(0)\right] \tag{35}$$

Then we assume that (33) holds for $k = m$

$$u(mT)=\frac{1}{\alpha}h\left[(m-1)T\right]+\frac{1}{c_n}\left\{f\left[(m+1)T\right]-f(mT)\right\} \tag{36}$$

Using this assumption from (32) we can find, that for $k = m + 1$

$$\begin{aligned}
u\left[(m+1)T\right]&=\frac{1}{\alpha}\left[x_d+\sum_{j=0}^{m}h(jT)\right]+\frac{f\left[(m+2)T\right]}{c_n}-\sum_{j=0}^{m}u(jT)\\
&=\frac{x_d}{\alpha}+\frac{1}{\alpha}\sum_{j=0}^{m}h(jT)+\frac{1}{c_n}f\left[(m+2)T\right]-u(0)\\
&-\sum_{j=1}^{m}\left\{\frac{1}{\alpha}h\left[(j-1)T\right]+\frac{1}{c_n}\left[f\left((j+1)T\right)-f(jT)\right]\right\}\\
&=\frac{x_d}{\alpha}+\frac{1}{\alpha}\sum_{j=0}^{m}h(jT)+\frac{1}{c_n}f\left[(m+2)T\right]-\frac{x_d}{\alpha}\frac{1}{k_0}\\
&-\frac{1}{\alpha}\sum_{j=0}^{m-1}h(jT)-\left\{\frac{1}{c_n}f\left[(m+1)T\right]-\frac{1}{c_n}f(T)\right\}\\
&=\frac{1}{\alpha}h(mT)+\frac{1}{c_n}\left\{f\left[(m+2)T\right]-f\left[(m+1)T\right]\right\}
\end{aligned} \tag{37}$$

which means that if (32) holds for $k = m$, then it also holds for $k = m + 1$. Taking into account (35) and (37), we conclude that equation (33) actually holds for any $k \geq 0$. This ends the proof.

It is easy to notice, that because $\max\{[f((k+1)T)-f(kT)]/c_n\} = x_d/\alpha k_0$ and $h(kT) \leq d_{max}$ for any $k \geq 0$, then $u(kT) \leq (x_d/\alpha k_0) + d_{max}$ for any $k \geq 0$. Moreover, as $\min\{[f((k+1)T)-f(kT)]/c_n\} = 0$ and $h(kT) \geq 0$ for any $k \geq 0$ then $u(kT) \geq 0$ for any $k \geq 0$. This shows that the designed controller determines data transmission rate which is always nonnegative and upper-bounded. Furthermore, choosing $f(kT)$ to be linear in the interval $[0, k_0]$ we obtained a constant upper bound of the control signal, which is quite practical from application point of view.

Theorem 3: If the proposed controller is applied, then the queue length will never exceed its demand value, i.e. for any $k \geq 0$

$$x\left(kT\right) \leq x_d \tag{38}$$

Proof: Transforming (32) we obtain

$$\frac{1}{\alpha}\left[x_d - x\left(kT\right)\right] = u\left(kT\right) + \sum_{j=k-m_{RTT}}^{k-1} u\left(jT\right) - \frac{1}{c_n} f\left[\left(k+1\right)T\right] \tag{39}$$

From the second Lemma $u(kT) \geq 0$ for any $k \geq 0$, and from (30) $f[(k+1)T]/c_n \leq 0$ also for any $k \geq 0$. From this follows that the right hand side of (39) is nonnegative, which gives $x_d - x(kT) \geq 0$. This ends the proof.

Theorem 4: If the proposed control strategy is applied and the demand queue length satisfies inequality

$$x_d > \left(m_{RTT} + 1\right) d_{max} \tag{40}$$

then the queue length is strictly greater than zero for any $k > k_0 + m_{RTT}$.
Proof: Using Lemma 2 we can rewrite (4) as follows

$$\begin{aligned} x\left(kT\right) &= -\sum_{j=0}^{k-1} h\left(jT\right) + \alpha u\left(0\right) \\ &+ \alpha \sum_{j=1}^{k-m_{RTT}-1} \left\{ \frac{1}{\alpha} h\left[\left(j-1\right)T\right] + \frac{1}{c_n}\left[f\left(\left(j+1\right)T\right) - f\left(jT\right)\right] \right\} \\ &= \frac{\alpha}{c_n}\left\{f\left[\left(k-m_{RTT}\right)T\right] - f\left(T\right)\right\} + \sum_{j=0}^{k-m_{RTT}-2} h\left(jT\right) - \sum_{j=0}^{k-1} h\left(jT\right) + \frac{1}{k_0} x_d \\ &= \frac{1}{k_0} x_d + \frac{\alpha}{c_n}\left\{f\left[\left(k-m_{RTT}\right)T\right] - f\left(T\right)\right\} - \sum_{j=k-m_{RTT}-1}^{k-1} h\left(jT\right) \end{aligned} \tag{41}$$

Then using (30), for any $k > k_0 + m_{RTT}$ from (41) we obtain

$$\begin{aligned} x\left(kT\right) &= \frac{1}{k_0} x_d + \frac{\alpha}{c_n}\left\{f\left[\left(k-m_{RTT}\right)T\right] - f\left(T\right)\right\} - \sum_{j=k-m_{RTT}-1}^{k-1} h\left(jT\right) \\ &= \frac{1}{k_0} x_d - \frac{1-k_0}{k_0} x_d - \sum_{j=k-m_{RTT}-1}^{k-1} h\left(jT\right) \geq x_d - \left(m_{RTT}+1\right) d_{max} > 0 \end{aligned} \tag{42}$$

This shows that the queue length is indeed strictly greater than zero for any $k > k_0 + m_{RTT}$.

4 Simulation Example

In order to verify the properties of both proposed strategies computer simulations of the network are performed. The discretisation period T is selected as 1 ms. The round trip time RTT is assumed to be 9 ms. Therefore $m_{RTT} = 9$, and $n = 10$. The maximum available bandwidth of the bottleneck node is $d_{max} = 80$ kb. According to Theorems 2 and 4, the demand queue length x_d in both control algorithms should be greater than 800 kb. Therefore x_d has been chosen as 810 kb. In the presented simulation example, coefficient $\alpha = 0.97$, which means that 97% of the data sent by the source arrive at the bottleneck node. For the time-varying hyperplane, parameter k_0 was chosen equal to 7.

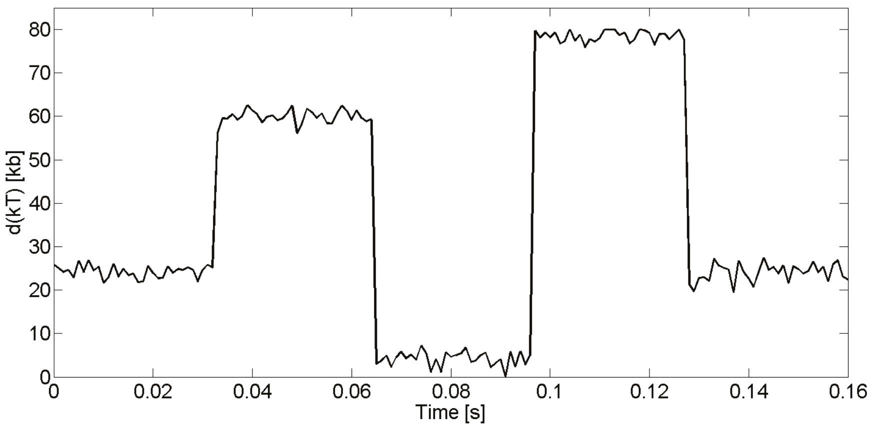

Figure 2. Available bandwidth

The available bandwidth is shown in Figure 2. It changes rapidly between small and large values, which reflects the most adverse possible conditions, that could exist in the network. Figure 3 shows the output signal of controller (16). It can be seen from this figure that the control signal is always strictly positive and upper bounded. Then, Figure 4 shows the bottleneck link queue length for the same control strategy. We can observe, that the queue length never exceeds its demand value, and is strictly positive for any $k > m_{RTT}$. This implies that the proposed strategy eliminates the risk of buffer overflow and ensures full bandwidth utilization.

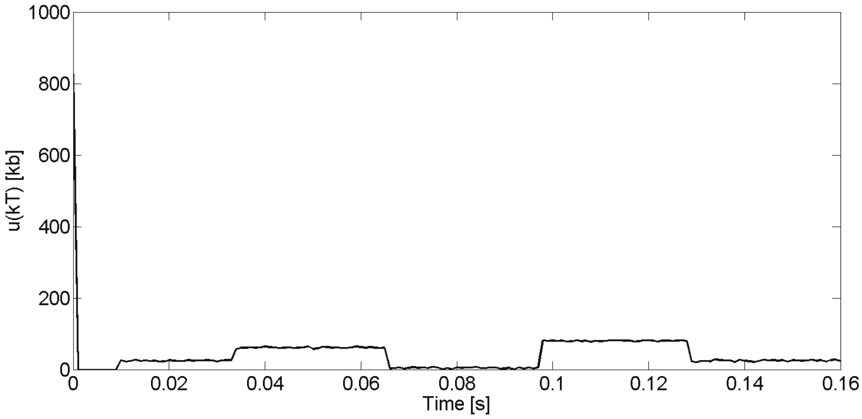

Figure 3. Output signal of the controller with the time-invariant sliding hyperplane

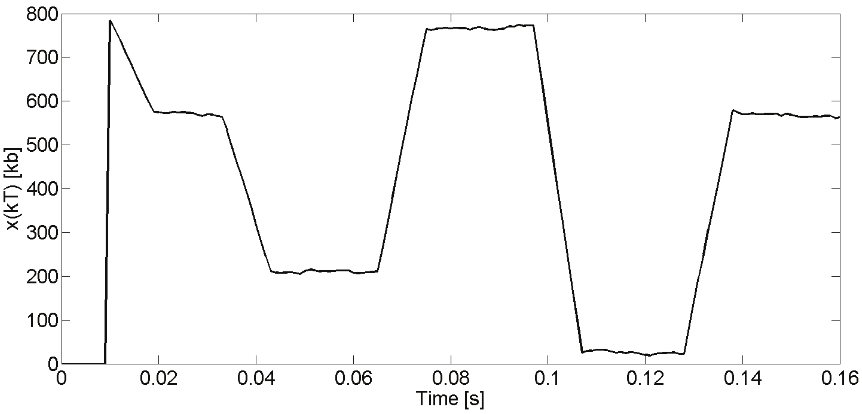

Figure 4. Queue length with the application of the controller with the time-invariant sliding hyperplane

Figures 5 and 6 show the respective simulation results for the network controlled according to strategy (32). Comparing figures 3 and 5 we notice that the introduction of a time-varying sliding hyperplane significantly reduces the maximum value of the control signal in the starting phase of the control process. Furthermore, as can be seen from figure 6, all the advantages of the previous controller with the time-invariant sliding hyperplane are maintained.

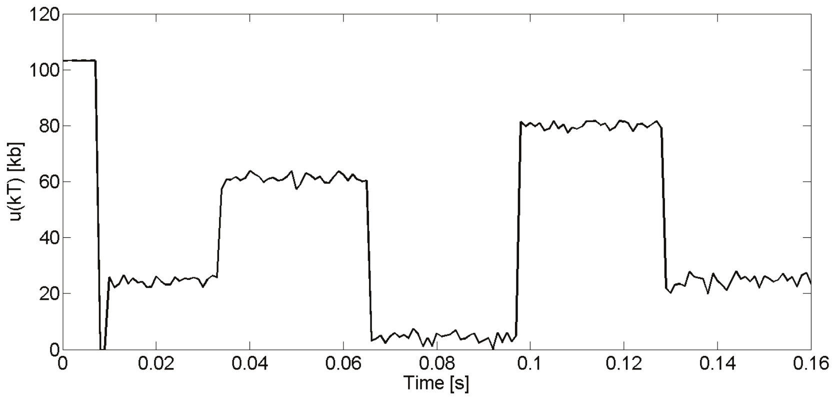

Figure 5. Output signal of the controller with the time-varying sliding hyperplane

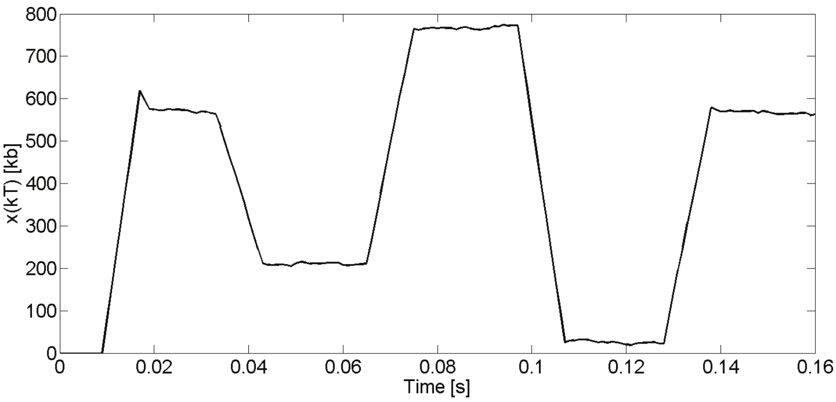

Figure 6. Queue length with the application of the controller with the time-varying sliding hyperplane

5 Conclusions

In this paper two sliding mode control strategies for a single virtual connection in a network with lossy links have been presented. The first strategy, which uses a time-invariant sliding hyperplane, has been designed to ensure closed-loop system stability and finite time error convergence. Then it has been modified by introducing a time-varying sliding hyperplane in order to reduce the maximum value of the control signal in the starting phase of data transmission. Flow rates generated by both strategies are proved to be always non-negative and upper bounded. Moreover, both control algorithms eliminate the risk of buffer overflow and for each of the algorithms conditions that guarantee full bottleneck link bandwidth consumption have been derived.

Acknowledgments

This work has been performed in the framework of a project "Optimal sliding mode control of time delay systems" financed by the National Science Centre of Poland – decision number DEC 2011/01/B/ST7/02582.

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