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Kołowrocki Krzysztof

Kuligowska Ewa

Soszyńska-Budny Joanna
Maritime University, Gdynia, Poland

Reliability of complex system under operation process influence – Monte Carlo simulation approach

Keywords

multistate system, operation process, complex system, reliability, Monte Carlo simulation method

Abstract

The paper presents Monte Carlo simulation method applied to the reliability evaluation of a multistate system subjected to a variable multistate operation process. The system operation process model is linked with the system reliability model and proposed to get a general reliability model of the complex system operating at varying in time operation conditions and to find its reliability characteristics. The Monte Carlo simulation algorithm based on the integrated general model of a complex multistate system reliability, linking its reliability model and its operation process model and considering variable at different operation states its reliability structure and its components reliability parameters is applied to the reliability evaluation of port grain transportation system. Next the results of this simulation method application are illustrated and compared with the results obtained by the analytical method.

1. Introduction

The reliability analysis of a system subjected to a varying in time its operation process very often leads to complicated calculations, especially in the case when we assume the system multistate reliability model and the multistate model of its operation process [1]-[7]. On the other hand, the complexity of the systems' operation processes and their influence on changing in time the systems' reliability structures and their components reliability parameters are very often met in real practice [4]-[7]. Thus, the practical importance of an approach linking the system reliability models and the system operation processes models into an integrated general model in reliability assessment of real technical systems is evident. All cited here publications presents general results obtained under a strong assumption that the system components have exponential conditional reliability functions at different operation states. To omit this assumption that narrows the investigation down and to get general solutions of the problem, a Monte Carlo simulation method is proposed to test the possibility of finding more general solutions better convergent to real technical problems. The computer simulation modeling approach to the reliability analysis of multistate systems subjected to multistate operation processes called complex systems is presented and practically applied to a port grain transportation system reliability characteristics determination. Next, the general analytical model of the reliability of multistate systems subjected to multistate operation processes is compared with the analytical method proposed in [5].

2. System operation process

We assume that a system during its operation at the fixed moment t, $t \in \langle 0,+\infty)$, may be at one of ν, $\nu \geq 2$, operations states z_b, $b, l = 1,2,\ldots,\nu$. Consequently, we mark by $Z(t)$, $t \in \langle 0,+\infty)$, the system operation process, that is a function of a continuous variable t, taking discrete values at the set $\{z_1, z_2, \ldots, z_\nu\}$ of the system operation states. We assume a semi-Markov model [1]-[6], of the system operation process $Z(t)$ and we mark by Θ_{bl} its random conditional sojourn times at the operation states z_b, when its next operation state is z_l, $b, l = 1,2,\ldots,\nu,\ b \neq l$.

Consequently, the operation process may be described by the following parameters [3]:

- the vector $[p_b(0)]_{1\times\nu}$, of the initial probabilities of the system operation process $Z(t)$ staying at the particular operation states z_b, $b = 1,2,\ldots,\nu$, at the moment $t = 0$;
- the matrix $[p_{bl}]_{\nu\times\nu}$ of the probabilities of the system operation process $Z(t)$ transitions between the operation states z_b and z_l, $b, l = 1,2,\ldots,\nu,\ b \neq l$;
- the matrix $[H_{bl}(t)]_{\nu\times\nu}$ of the conditional distribution functions of the system operation process $Z(t)$ conditional sojourn times θ_{bl} at the operation states, $b, l = 1,2,\ldots,\nu, b \neq l$.

As the mean values $E[\theta_{bl}]$ of the conditional sojourn times θ_{bl} are given by

$$M_{bl} = E[\theta_{bl}] = \int_0^\infty t dH_{bl}(t),\ b,l = 1,2,\ldots,\nu,\ b \neq l, \quad (1)$$

then from the formula for total probability, it follows that the unconditional distribution functions of the sojourn times θ_b, $b = 1,2,\ldots,\nu$, of the system operation process $Z(t)$ at the operation states z_b, $b = 1,2,\ldots,\nu$, are given by [3], [6]

$$H_b(t) = \sum_{l=1}^{\nu} p_{bl} H_{bl}(t),\ t \in \langle 0,+\infty),\ b = 1,2,\ldots,\nu. \quad (2)$$

Hence, the mean values $E[\theta_b]$ of the system operation process $Z(t)$ unconditional sojourn times θ_b, $b = 1,2,\ldots,\nu$, at the operation states are given by

$$M_b = E[\theta_b] = \sum_{l=1}^{\nu} p_{bl} M_{bl},\ b = 1,2,\ldots,\nu, \quad (3)$$

where M_{bl} are defined by the formula (1).

The limit values of the system operation process $Z(t)$ transient probabilities at the particular operation states

$$p_b(t) = P(Z(t) = z_b),\ t \in \langle 0,+\infty),\ b = 1,2,\ldots,\nu, \quad (4)$$

are given by [1], [3], [6]

$$p_b = \lim_{t\to\infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^{\nu} \pi_l M_l},\ b = 1,2,\ldots,\nu, \quad (5)$$

where M_b, $b = 1,2,\ldots,\nu$, are given by (3), while the steady probabilities π_b of the vector $[\pi_b]_{1x\nu}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^{\nu} \pi_l = 1. \end{cases} \quad (6)$$

3. Reliability of multistate complex system

3.1. General theoretical backgrounds

In the multistate reliability analysis to define the system with degrading components, we assume that:

- n is the number of the system components,
- E_i, $i = 1,2,...,n$, are components of a system,
- all components and a system under consideration have the reliability state set $\{0,1,...,z\}$, $z \geq 1$,
- the reliability states are ordered, the reliability state 0 is the worst and the reliability state z is the best,
- $T_i(u)$, $i = 1,2,...,n$, are independent random variables representing the lifetimes of components E_i in the reliability state subset $\{u,u+1,...,z\}$, while they were in the reliability state z at the moment $t = 0$,
- $T(u)$ is a random variable representing the lifetime of a system in the reliability state subset $\{u,u+1,...,z\}$ while it was in the reliability state z at the moment $t = 0$,
- the system states degrades with time t,
- $E_i(t)$ is a component E_i reliability state at the moment t, $t \in \langle 0,+\infty)$, given that it was in the reliability state z at the moment $t = 0$,
- $s(t)$ is a system S reliability state at the moment t, $t \in \langle 0,+\infty)$, given that it was in the reliability state z at the moment $t = 0$.

Under the above assumptions, we denote the component E_i, $i = 1,2,...,n$, reliability function by the vector

$$R_i(t,\cdot) = [1,\ R_i(t,1),\ ...,\ R_i(t,z)], \quad (7)$$

with the coordinates defined by

$$R_i(t,u) = P(T_i(u) > t) \text{ for } t \in \langle 0,+\infty),\ u = 1,2,\ldots,z, \quad (8)$$

Similarly, we denote the system reliability function by the vector

$$\boldsymbol{R}(t,\cdot) = [1, \ \boldsymbol{R}(t,1), ..., \ \boldsymbol{R}(t,z)], \tag{9}$$

with the coordinates defined by

$$\boldsymbol{R}(t,u) = P(T(u) > t) \text{ for } t \in \langle 0,+\infty), \tag{10}$$

$u = 1,2,\ldots,z$

Further, we assume that the changes of the operation states of the system operation process $Z(t)$ have an influence on the system multistate components E_i, $i = 1,2,\ldots,n,$ reliability and the system reliability structure as well. Consequently, we denote the system multistate component E_i, $i = 1,2,\ldots,n,$ conditional lifetime in the reliability state subset $\{u, u+1,\ldots,z\}$ while the system is at the operation state z_b, $b = 1,2,\ldots,v,$ by $T_i^{(b)}(u)$ and its conditional reliability function by the vector

$$[R_i(t,\cdot)]^{(b)} = [1, \ [R_i(t,1)]^{(b)}, ..., \ [R_i(t,z)]^{(b)}], \tag{11}$$

with the coordinates defined by

$$[R_i(t,u)]^{(b)} = P(T_i^{(b)}(u) > t | Z(t) = z_b) \tag{12}$$

for $t \in \langle 0,+\infty),$ $u = 1,2,\ldots,z,$ $b = 1,2,\ldots,v.$
Similarly, we denote the system conditional lifetime in the reliability state subset $\{u, u+1,\ldots,z\}$ while the system is at the operation state z_b, $b = 1,2,\ldots,v,$ by $T^{(b)}(u)$ and the conditional reliability function of the system by the vector

$$[\boldsymbol{R}(t,\cdot)]^{(b)} = [1, \ [\boldsymbol{R}(t,1)]^{(b)}, ..., \ [\boldsymbol{R}(t,z)]^{(b)}], \tag{13}$$

with the coordinates defined by

$$[\boldsymbol{R}(t,u)]^{(b)} = P(T^{(b)}(u) > t | Z(t) = z_b) \tag{14}$$

for $t \in \langle 0,+\infty),$ $u = 1,2,\ldots,z,$ $b = 1,2,\ldots,v.$

Definition 1. A multistate system is called series if its lifetime $T(u)$ in the reliability state subset $\{u, u + 1,...,z\}$ is given by

$$T(u) = \min_{1 \le i \le n}\{T_i(u)\}, \ u = 1,2,...,z. \tag{15}$$

Definition 2. A multistate system is called series-parallel if its lifetime $T(u)$ in the reliability state subset $\{u, u + 1,...,z\}$ is given by

$$T(u) = \max_{1 \le i \le k}\{\min_{1 \le j \le l_i}\{T_{ij}(u)\}\}, \ u = 1,2,...,z, \tag{16}$$

where k is the number of series subsystems linked in parallel and l_i is the number of components in the i^{th} series subsystem.

3.2. System reliability states changing process with memory

We assume that the changes of the operation states of the system operation process $Z(t)$ have an influence on the system multistate components E_i, $i = 1,2,\ldots,n,$ reliability and the system reliability structure as well. Moreover, in particular, we assume that the system components' reliability depend on the number of operation states changes of the system operation process. Consequently, we denote the system multistate component E_i, $i = 1,2,\ldots,n,$ conditional lifetime in the reliability state subset $\{u, u+1,\ldots,z\}$ while the system is at the operation state z_b, $b = 1,2,\ldots,v,$ after k, $k = 0,1,\ldots,$ changes of the system operation states by $[T_i \ (u)]_k^{(b)}$ and its conditional reliability function by the vector

$$[R_i(t,\cdot)]_k^{(b)} = [1, \ [R_i(t,1)]_k^{(b)}, ..., \ [R_i(t,z)]_k^{(b)}], \tag{17}$$

with the coordinates defined by

$$[R_i(t,u)]_k^{(b)} = P([T_i \ (u)]_k^{(b)} > t | Z(t) = z_b) \tag{18}$$

for $t \in \langle 0,+\infty),$ $u = 1,2,\ldots,z,$ $b = 1,2,\ldots,v,$ $k = 0,1,\ldots.$
Similarly, we denote the system conditional lifetime in the reliability state subset $\{u, u+1,\ldots,z\}$ while the system is at the operation state z_b, $b = 1,2,\ldots,v,$ after k, $k = 0,1,\ldots,$ changes of the system operation states by $[T(u)]_k^{(b)}$ and the conditional reliability function of the system by the vector

$$[\boldsymbol{R}(t,\cdot)]_k^{(b)} = [1, \ [\boldsymbol{R}(t,1)]_k^{(b)}, ..., \ [\boldsymbol{R}(t,z)]_k^{(b)}], \tag{19}$$

with the coordinates defined by

$$[\boldsymbol{R}(t,u)]_k^{(b)} = P([T(u)]_k^{(b)} > t | Z(t) = z_b) \tag{20}$$

for $t \in \langle 0,+\infty),$ $u = 1,2,\ldots,z,$ $b = 1,2,\ldots,v,$ $k = 0,1,\ldots.$
Under those assumptions, we want to find the

system unconditional lifetime in the reliability state subset $\{u,u+1,\ldots,z\}$ by $T(u)$ and the unconditional reliability function of the system by the vector

$$\boldsymbol{R}(t,\cdot) = [1,\ \boldsymbol{R}(t,1),\ldots,\ \boldsymbol{R}(t,z)\,], \tag{21}$$

with the coordinates defined by

$$\boldsymbol{R}(t,u) = P(T(u) > t) \ \text{ for } t \in \langle 0,+\infty), \tag{22}$$

$u = 1,2,\ldots,z.$

We assume that the changes of the operation states of the system operation process $Z(t)$ have an influence on the system multistate components E_i, $i = 1,2,\ldots,n,$ reliability and the system reliability structure as well. Moreover, in particular, we assume that the system components' reliability depend on the number of operation states changes of the system operation process. Consequently, we denote the system multistate component E_i, $i = 1,2,\ldots,n,$ conditional lifetime in the reliability state subset $\{u,u+1,\ldots,z\}$ while the system is at the operation state z_b, $b = 1,2,\ldots,v,$ after $k,$ $k = 0,1,\ldots,$ changes of the system operation states by $[T_i\ (u)]_k^{(b)}$ and its conditional reliability function by the vector

$$[R_i(t,\cdot)]_k^{(b)} = [1,\ [R_i(t,1)]_k^{(b)},\ldots,\ [R_i(t,z)]_k^{(b)}\,], \tag{23}$$

with the coordinates defined by

$$[R_i(t,u)]_k^{(b)} = P([T_i\ (u)]_k^{(b)} > t \,|\, Z(t) = z_b) \tag{24}$$

for $t \in \langle 0,+\infty),$ $u = 1,2,\ldots,z,$ $b = 1,2,\ldots,v,$ $k = 0,1,\ldots.$
Similarly, we denote the system conditional lifetime in the reliability state subset $\{u,u+1,\ldots,z\}$ while the system is at the operation state z_b, $b = 1,2,\ldots,v,$ after $k,$ $k = 0,1,\ldots,$ changes of the system operation states by $[T(u)]_k^{(b)}$ and the conditional reliability function of the system by the vector

$$[\boldsymbol{R}(t,\cdot)]_k^{(b)} = [1,\ [\boldsymbol{R}(t,1)]_k^{(b)},\ldots,\ [\boldsymbol{R}(t,z)]_k^{(b)}\,], \tag{25}$$

with the coordinates defined by

$$[\boldsymbol{R}(t,u)]_k^{(b)} = P([T(u)]_k^{(b)} > t \,|\, Z(t) = z_b) \tag{26}$$

for $t \in \langle 0,+\infty),$ $u = 1,2,\ldots,z,$ $b = 1,2,\ldots,v,$ $k = 0,1,\ldots.$

4. Monte Carlo simulation approach to reliability evaluation

We denote by $z_b(q)$, $b = 1,2,\ldots,v,$ the realization of the system operation process initial operation state at the moment $t = 0$ generated from the distribution $[p_b(0)]_{1\times v}$. This realization is generated according to the formula

$$z_b(q) = \begin{cases} z_1, & 0 \le q < p_1(0), \\ z_2, & p_1(0) \le q < p_1(0) + p_2(0), \\ \vdots & \quad \vdots \\ z_v, & \sum_{i=1}^{v-1} p_i(0) \le q \le 1, \end{cases} \tag{27}$$

where q is a randomly generated number from the uniform distribution on the interval $\langle 0,1\rangle$.
We denote by $z_{bl}(g)$, $l = 1,2,\ldots,v,$ $b \ne l,$ the sequence of the realizations of the system operation process consecutive operation states generated from the distribution defined by $[p_{bl}]_{v\times v}$. Those realizations are generated according to the formula

$$z_{1l}(g) = \begin{cases} z_2, & 0 \le g < p_{12}, \\ z_3, & p_{12} \le g < p_{12} + p_{13}, \\ \vdots & \quad \vdots \\ z_v, & \sum_{i=1}^{v-1} p_{bi} \le g \le 1, \end{cases} \tag{28}$$

$$z_{bl}(g) = \begin{cases} z_1, & 0 \le g < p_{b1}, \\ \vdots & \vdots \\ z_{b-1}, & \sum_{i=1}^{b-2} p_{bi} \le g < \sum_{i=1}^{b-1} p_{bi}, \\ z_{b+1}, & \sum_{i=1}^{b-1} p_{bi} \le g < \sum_{i=1}^{b+1} p_{bi}, \\ \vdots & \vdots \\ z_v, & \sum_{i=1}^{v-1} p_{bi} \le g \le 1, \end{cases} \tag{29}$$

for $b = 2,3,\ldots,v,$

$$z_{v\,l}(g) = \begin{cases} z_1, & 0 \le g < p_{v1}, \\ z_2, & p_{v1} \le g < p_{v1} + p_{v2} \\ \vdots & \quad \vdots \\ z_{v-1}, & \sum_{i=1}^{v-2} p_{v\,i} \le g \le 1, \end{cases} \tag{30}$$

where g is a randomly generated number from the uniform distribution on the interval $\langle 0,1\rangle$.

We denote by $\theta_{bl}^{(i)}$, $b,l=1,2,\ldots,\nu$, $i=1,2,\ldots,n_{bl}$, $b\neq l$, the realizations of the conditional sojourn time θ_{bl} of the system operation process generated from the distribution H_{bl}, where n_{bl} is the number of those sojourn time realizations during the experiment time $\tilde{\theta}$.

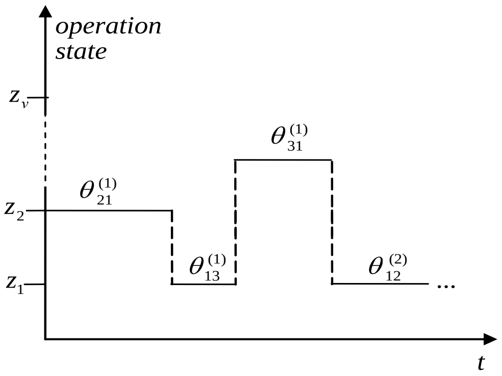

Figure 1. Realizations of the system operation process

Those realizations are generated according to the formulae

$$\theta_{bl}^{(i)} = H_{bl}^{-1}(h),\ b,l=1,2,\ldots,\nu, b\neq l \tag{31}$$

where $H_{bl}^{-1}(h)$ is the inverse function of the distribution function $H_{bl}(t)$ and h is a randomly generated number from the uniform distribution on the interval $\langle 0,1\rangle$, which in the case of exponential distribution

$$\mathrm{H}_{bl}(\mathrm{t}) = 1-\exp[-\alpha_{bl}\, t],\ t\in\langle 0,\infty),\tag{32}$$

takes the following form

$$\theta_{bl} = -\frac{1}{\alpha_{bl}}\ln(1-h),\ b,l=1,2,\ldots,\nu, b\neq l.\tag{33}$$

The realizations $[t_i(u)]_k^{(b)}$, of the components E_i, $i=1,2,\ldots,n,$ conditional lifetimes $[T_i(u)]_k^{(b)}$, $i=1,2,\ldots,n,$ $u=1,2,\ldots,z,$ $b=1,2,\ldots,\nu,$ after k, $k=0,1,\ldots,$ changes of the system operation states, are generated according to the distribution (18), i.e. they are generated by the sampling formula

$$[t_i(u)]_k^{(b)} = \left([F_i(f,u)]_k^{(b)}\right)^{-1} = \left(1-[R_i(f,u)]_k^{(b)}\right)^{-1},\tag{34}$$

where $[R_i\,(t,u)]_k^{(b)}$ is defined by (18). In the case of exponential distribution we have the following form

$$[F_i(t,u)]_k^{(b)} = 1-\exp[-[\lambda_i(u)]_k^{(b)} t],\tag{35}$$

for $t\in\langle 0,+\infty)$, , $i=1,2,\ldots,n,$ $u=1,2,\ldots,z,$ $b=1,2,\ldots,\nu,$ $k=0,1,\ldots.$

In the case of the above exponential distribution the realizations of the system component conditional lifetimes take the following form

$$[t_i(u)]_k^{(b)} = -\frac{1}{[\lambda_i(u)]_k^{(b)}}\ln(1-f),\tag{36}$$

for $i=1,2,\ldots,n,$ $b=1,2,\ldots,\nu,$ $k=0,1,\ldots.$

The realizations $t(u)$, $u=1,2,\ldots,z,$ of the system unconditional lifetime $T(u)$, $u=1,2,\ldots,z,$ in the reliability state subset $\{u,u+1,\ldots,z\}$ depend on the realizations $[t_i(u)]_k^{(b)}$, $i=1,2,\ldots,n,$ $b=1,2,\ldots,\nu,$ $k=0,1,\ldots,$ the system component conditional lifetimes and are calculated from the expression

$$t(u) = t([t_i(u)]_k^{(b)};\ i=1,2,\ldots,n,\ b=1,2,\ldots,\nu,\ k=0,1,\ldots)\tag{37}$$

For $u=1,2,\ldots,z,$ taking suitable explicit form dependent of the system structure.

Using the procedure of Monte Carlo simulation [7]-[8], the histogram of the system unconditional lifetime can be found and the empirical mean value and the standard deviation of the system unconditional lifetime can be calculated.

5. Port grain transportation system reliability evaluation

The considered transportation system reliability analysis is performed in [5]. The port grain transportation system subsystems S_υ, $\upsilon=1,2,3.4,$ are composed of three-state, i.e. z = 3, components $E_{ij}^{(\upsilon)}$, $\upsilon=1,2,3,4,$ having the conditional reliability functions while the system is at the operation state z_b, $b=1,2,\ldots,\nu,$ after k, $k=0,1,\ldots,$ changes of the system operation process states given by the vectors

$$[R_{ij}^{(\upsilon)}(t,\cdot)]_k^{(b)} = [1,[R_{ij}^{(\upsilon)}(t,1)]_k^{(b)},[R_{ij}^{(\upsilon)}(t,2)]_k^{(b)}],\tag{38}$$

with the exponential coordinates

$$[R_{ij}^{(\upsilon)}(t,u)]_k^{(b)} = \exp[-[\lambda_{ij}^{(\upsilon)}(u)]_k^{(b)} t],\tag{39}$$

for $i=1,2,\ldots,k^{(b)}$, $j=1,2,\ldots,l_i^{(b)}$, $\upsilon=1,2,3,4,$ $b=1,2,3,$ $k=0,1,\ldots,$ $u=1,2,$

different at various operation states z_b, $b = 1,2,3,$ and with the intensities of departure from the reliability state subsets {1,2}, {2},

$$[\lambda_{ij}^{(\upsilon)}(u)]_k^{(b)} = [\lambda_{ij}^{(\upsilon)}(u)]^{(b)} \cdot \frac{2k+1}{k+1}, \quad k = 0,1,..., \tag{40}$$

where the parameters $[\lambda_{ij}^{(\upsilon)}(u)]^{(b)}$, $i = 1,2,...,k^{(b)}$, $j = 1,2,...,l_i^{(b)}$, $\upsilon = 1,2,3,4,$ $b = 1,2,3,$ $u = 1,2,$ are given in *Table 1* in [5].

5.1. Monte Carlo simulation of port grain transportation system operation process

The simulation is performed according to data given in [5]. The first step is to select the initial operation state $z_b(q)$, $b = 1, 2, 3,$ at the moment $t = 0,$ using formula (27), which is given by

$$z_b(g) = \begin{cases} z_1, & 0 \le q < 0.531 \\ z_2, & 0.531 \le q < 0.641 \\ z_3, & 0.641 \le q \le 1, \end{cases} \tag{41}$$

where q is a randomly generated number from the uniform distribution on the interval $\langle 0,1 \rangle$. The next operation state z_l, $l = 1,2,3,$ is generated according to (28)-(30), from $z_{bl}(g)$, $b = 1,2,3,4,$ defined as

$$z_{1l}(g) = \begin{cases} z_2, & 0 \le g < 0.333 \\ z_3, & 0.333 \le g \le 1 \end{cases} \tag{42}$$

$$z_{2l}(g) = \begin{cases} z_1, & 0 \le g < 0.444 \\ z_3, & 0.444 \le g \le 1 \end{cases} \tag{43}$$

$$z_{3l}(g) = \begin{cases} z_1, & 0 \le g < 0.333 \\ z_2, & 0.333 \le g \le 1 \end{cases} \tag{44}$$

Applying (33), the realizations of the empirical conditional sojourn times are generated according to the formulae

$$\theta_{12}(h) = -0.2\ln[1-h],\ \theta_{13}(h) = -0.1\ln[1-h],$$

$$\theta_{21}(h) = -0.025\ln[1-h],\ \theta_{23}(h) = -0.02\ln[1-h],$$

$$\theta_{31}(h) = -0.1\ln[1-h],\ \theta_{32}(h) = -0.05\ln[1-h], \tag{45}$$

where h is a randomly generated number from the uniform distribution on the interval $\langle 0,1 \rangle$.

5.2. Monte Carlo approach to the port grain transportation system reliability modelling

The Monte Carlo simulation method uses a computational procedure and can provide the fairly accurate results in a relatively small amount of time [1], [7]-[8]. Obviously, the accuracy of the proposed Monte Carlo simulation method depends on the number of iterations.

We can apply the Monte Carlo simulation method, according to the scheme presented in *Figure 2*.

At the beginning, we fix the following parameters:

- the number $N \in \boldsymbol{N} \setminus \{0\}$ of iterations (runs of the simulation) equal to the number of the system lifetime realizations;
- the function generating initial operation state $z_b(q)$, $b = 1, 2, 3,$ at the moment $t = 0,$ defined by (41);
- the functions generating next operation state $z_{bl}(g)$, $b, l = 1,2,3, b \ne l$ defined by (42)-(44),
- the matrix $[\alpha_{bl}], \alpha_{bl} \in \langle 0,\infty), b, l = 1,2,3, b \ne l$, of the intensities of the system operation process transitions between the operation states existing in [5];
- the system reliability parameters $[[\lambda_{ij}^{(\upsilon)}(u)]_k^{(b)}]_n$, $b, l = 1,2,3, b \ne l$, $i = 1,2,...,k^{(b)}$, $j = 1,2,...,l_i^{(b)}$, $k = 0,1,...,$ $u = 1,2,$ $\upsilon = 1,2,3,4,$ according to the *Table 1* in [5].

We declare the conditional sojourn times formulae (36) and the system component's lifetimes exponential sampling formula

$$[[t_{ij}^{(\upsilon)}(u)]_k^{(b)}]_n := -\frac{\ln[1 - [[f_{ij}^{(\upsilon)}(u)]_k^{(b)}]_n]}{[[\lambda_{ij}^{(\upsilon)}(u)]_k^{(b)}]_n}, \tag{46}$$

where $[[\lambda_{ij}^{(\upsilon)}(u)]_k^{(b)}]_n$, are given according to *Table 1* in [5], and $[[f_{ij}^{(\upsilon)}(u)]_k^{(b)}]_n$, $b, l = 1,2,3,$ $b \ne l$, $i = 1,2,...,k^{(b)}$, $j = 1,2,...,l_i^{(b)}$, $k = 0,1,...,u = 1,2,$ $\upsilon = 1,2,3,4,$ is a randomly generated number from the uniform distribution on the interval $\langle 0,1 \rangle$.

In the next step we introduce:

- $n \in \{1,2,...,N\}$, as the subsequent iteration of the simulation and set $n = 1$;
- system component lifetimes exponential sampling formula $[[t_{ij}^{(\upsilon)}(u)]_k^{(b)}]_n$, according to (46) and set $[[t_{ij}^{(\upsilon)}(u)]_k^{(b)}]_n = 0$;

- the sum $\hat{\theta}$ of the realizations $\theta_{bl}^{(\kappa)}$, $\kappa = 1,2,...,$ of the empirical conditional sojourn times and set $\hat{\theta} = 0$;
- $u \in \{1,2,...,z\}$, as the subsequent iteration in the loop and set $z = 2$;

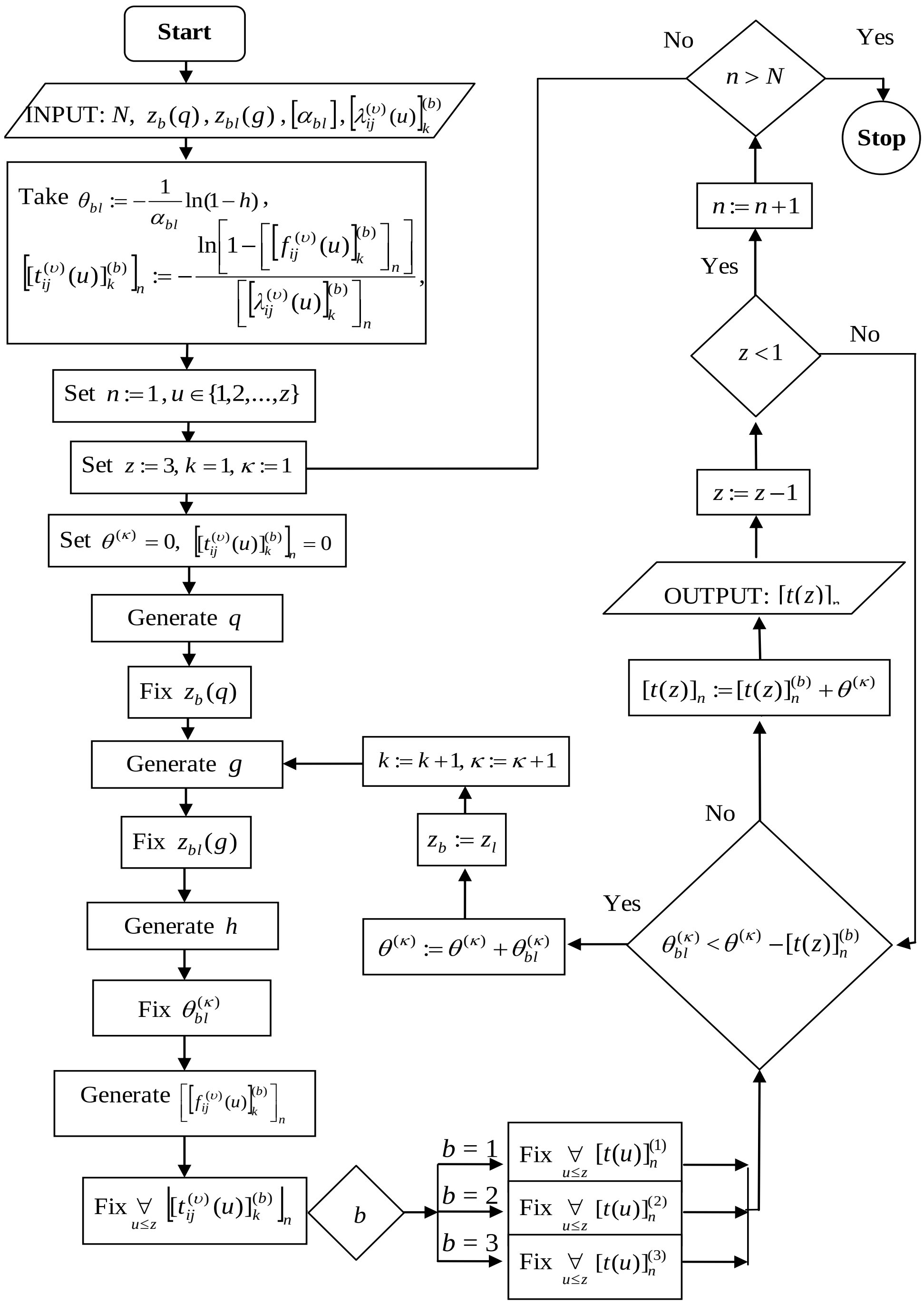

Figure 2. Monte Carlo algorithm for the exemplary system reliability evaluation.

As the algorithm progresses, we draw a random number q from the uniform distribution on the interval $\langle 0,1 \rangle$. Based on this random value, the realization

$$z_b(q),\ \mathrm{b} = 1, 2, 3,$$

of the system operation process initial operation state at the moment $t = 0$ is generated according to the formula (41).

Next, we draw a random number g uniformly distributed on the unit interval. Concerning this random value, the realization

$z_l(g)$, $l = 1,2,3, l \neq b,$

of the system operation process consecutive operation state is generated according to one of the formulae (42)-(44).

Further, we generate a random number h from the uniform distribution on the interval $\langle 0,1 \rangle$, which we put into the formula (45) obtaining the realisation $\theta_{bl}^{(\kappa)}$, $b, l = 1,2,3,$ $b \neq l, \kappa = 1.$ Subsequently, for a particular initial operation state z_b, $b, l = 1,2,3,$ $b \neq l$, we draw a random number $[[f_{ij}^{(\upsilon)}(u)]_k^{(b)}]_n$, $i = 1$, $j = 1$, $\upsilon = 1$, from the uniform distribution on the interval $\langle 0,1 \rangle$. Based on this random value, the realization $[[t_{ij}^{(\upsilon)}(u)]_k^{(b)}]_n$, $i = 1$, $j = 1$, $k = 0$, $n = 1$, $u = z$, $\upsilon = 1$, of the considered system component lifetime realization is generated according the formula (46). We generate another random numbers $[[f_{ij}^{(\upsilon)}(u)]_k^{(b)}]_n$, $i = 1,2,...,k^{(b)}$, $j = 1,2,...,l_i^{(b)}$, $\upsilon = 1,2,3,4$, from the uniform distribution on the interval $\langle 0,1 \rangle$ obtaining the realizations $[[t_{ij}^{(\upsilon)}(u)]_k^{(b)}]_n$ $i = 1,2,...,k^{(b)}$, $j = 1,2,...,l_i^{(b)}$, $k = 0,1,...,$ $n = 1$, $u = 1,2$, $\upsilon = 1,2,3,4$.

The realizations $[t^{(\upsilon)}(u)]_n^{(b)}$, $n = 1$, $u = z$, $\upsilon = 1,2,3,4$, of the system lifetime $[T^{(\upsilon)}(u)]_n^{(b)}$ in the reliability state subsets $\{u,u + 1,...,z\}$, $z = 2$, depend on the realizations $[[t_{ij}^{(\upsilon)}(u)]_k^{(b)}]_n$ given by (46) of the system component lifetimes $[T_{ij}^{(\upsilon)}(u)]_n^{(b)}$, $i = 1,2,...k^{(b)}$, $j = 1,2,...l_i^{(b)}$, $n = 1,2,...,N$, $u = z$, $\upsilon = 1,2,3,4$, and are calculated from the expression

$$[t(u)]_n = t([t_{ij}^{(\upsilon)}(u)]_n^{(b)}; b, l = 1,2,3, \ b \neq l,$$
$$i = 1,2,...,k^{(b)}, \ j = 1,2,...,l_i^{(b)}, \ n = 1,2,...,N, u = z,$$

taking suitable explicit form dependent on the system structure according to (15)-(16):

$$[t(u)]_n^{(b)} = \min_{1 \le \upsilon \le 4}\{ \max_{1 \le i \le k^{(b)}} \{ \min_{1 \le j \le l_i^{(b)}} \{[t_{ij}^{(\upsilon)}(u)]_n^{(b)}\}\},$$
$$b = 1,2,3, \ n = 1,2,...,N, \ u = 1,2.$$

If the realization of the empirical conditional sojourn time $\theta_{bl}^{(\kappa)}$, $b, l = 1,2,3,4,$ $b \neq l, \kappa = 1,$ is not greater than the realization of the difference between system conditional lifetime $[t(z)]_n^{(b)}$ and $\theta^{(\kappa)}$, we add to $\theta^{(\kappa)}$ the value $\theta_{bl}^{(\kappa)}$. The realization $[t(z)]_n$ is recorded, z_l is set as the initial operation state and we increase the value of κ. Otherwise, if the realization of the empirical conditional sojourn time $\theta_{bl}^{(\kappa)}$, $b, l = 1,2,3,$ $b \neq l, \kappa = 1,$ is greater than the realization of the difference between system conditional lifetime $[t(z)]_n^{(b)}$ and $\theta^{(\kappa)}$, we add to the system unconditional lifetime $[t(z)]_n$ the value $[t(z)]_n^{(b)}$, $b, l = 1,2,3,$ $b \neq l,$ $n = 1,2,...,N$. and record the realization $[t(z)]_n$. If the value of z is positive, we repeat the comparison for $z - 1$. each time, after completing all the steps in the loop, we record the rest realizations $[t(u)]_n$, for $u = 1,2,\ldots,z - 1$. Thus, if the value of z is negative, we can proceed replacing n with $n + 1$ and shift into the next iteration in the loop if $n < N$. In the other case, we stop the procedure.

The procedure of Monte Carlo simulation is performed with N = 100 000 runs. The expected values of the system lifetimes in the reliability states subsets {1,2}, {2}, calculated as an arithmetic mean of all system lifetime realizations for N iterations for u = 1,2, respectively are

$$\mu_s(1) \cong 0.0519, \mu_s(2) \cong 0.0444, \tag{47}$$

The approximate system lifetimes standard deviation lifetimes in the reliability states subsets {1,2}, {2},is calculated as a square root of the average squared deviation from the mean values (47), for u = 1,2, are given as follows:

$$\sigma_s(1) = 0.0383, \ \sigma_s(2) = 0,0328, \tag{48}$$

The system lifetimes in the particular reliability states 1, 2, 3, respectively are:

$$\bar{\mu}_s(1) \cong 0.0075, \ \ \bar{\mu}_s(2) \cong 0.0444$$

The histograms of the exemplary system lifetimes in the particular reliability state subsets are illustrated in *Figure 3*. It can be seen that their shapes are similar to the shapes of the Weibull density functions.

The approximate expected values of the system unconditional lifetimes in the reliability state subsets {1,2}, {2} and the mean values of the unconditional lifetimes in the particular reliability states 1, 2 calculated for port grain transportation system using the analytical approach presented in [5], respectively are

$$\mu(1) \cong 0.0404, \ \ \mu(2) \cong 0.0345. \tag{49}$$

$$\bar{\mu}(1) = \mu(1) - \mu(2) = 0.0059, \quad (50)$$

$$\bar{\mu}(2) = \mu(2) = 0.0345. \quad (51)$$

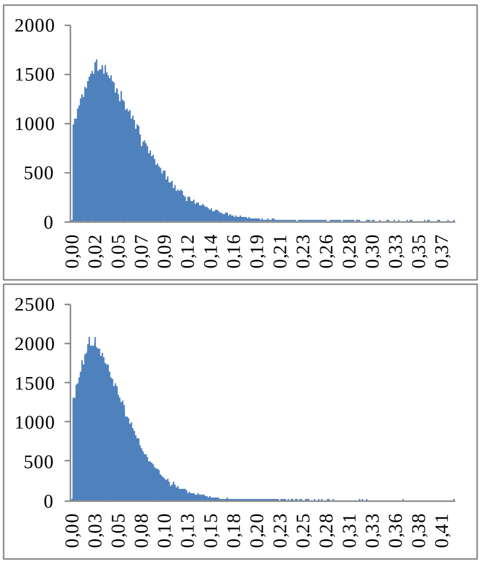

Figure 3. The graph of the histograms of the port grain transportation system lifetimes in reliability state subsets {1,2}, {2}

6. Conclusions

The Monte Carlo simulation algorithm based on the integrated general model of complex systems' reliability, linking their reliability models and their operation processes models and considering variable at different operation states their reliability structures and their components reliability parameters is constructed and applied to the reliability evaluation of the port grain transportation system composed of three series-parallel subsystems linked in series. The considered transportation system reliability analysis in the case of the system reliability states changing with memory that have influence on the system components reliability parameters is performed. The predicted reliability characteristics of this system differ not much from those determined for this system by approximate analytical method presented in [5]. This fact justifies the sensibility of using Monte Carlo simulation approach to reliability evaluation of a very wide class of real complex technical systems changing their reliability structures and reliability parameters at their variable operation processes. The approach, upon the good accuracy of the systems' operation processes and the systems' components reliability parameters identification, makes their reliability prediction more precise and convergent to reality.

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