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THE THEORETICAL MODEL OF HUMAN — WEAPON SYSTEM

ABSTRACT

Rapid firing of two bullets from the Skorpion submachine gun vz. 61 cal. 7,65 mm towards the target located at a distance of 25 [m] was registered at a police shooting range. The tests confirmed the opinion of the users regarding the presence of significant weapon dispersion. This undesirable effect prompted the authors to undertake work in order to improve the dynamic properties of weapons.

Experimental studies provided information on the kinematics of movement of human — weapon system. On the basis of the courses of variation of displacement, velocity and acceleration over time, the behaviour of a machine gun in thirteen characteristic points of its operation was specified. The verification of bullet holes and analysis of recorded images showed a slight deviation of movement of the objects under examination from the vertical plane. Therefore, a physical model of human — weapon system with five degrees of freedom in the vertical plane was formulated. On the basis of the physical model, a mathematical model of human — weapon system was derived, which can be classified as linear, determined, variable over time, dissipative and constrained.

Key words:
weapon, human, physical model, mathematical model.

INTRODUCTION

The object of research is the system of man-submachine gun Scorpion wz. 61 cal. 7,65 mm, as in figure 1. Its long term use indicated large dispersion of bullets

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fired, especially when shooting from a distance of several meters. In order to confirm the unfavourable phenomenon of vast spread of bullets fired, rapid firing of two bullets towards the target located at a distance of 25 [m] was conducted at a police shooting range. Shooting the Skorpion submachine gun was executed by an experienced policeman from the secret service, as in Figure 1. The tests demonstrated the presence of dispersion between two bullet holes of more than 1.3 [m].

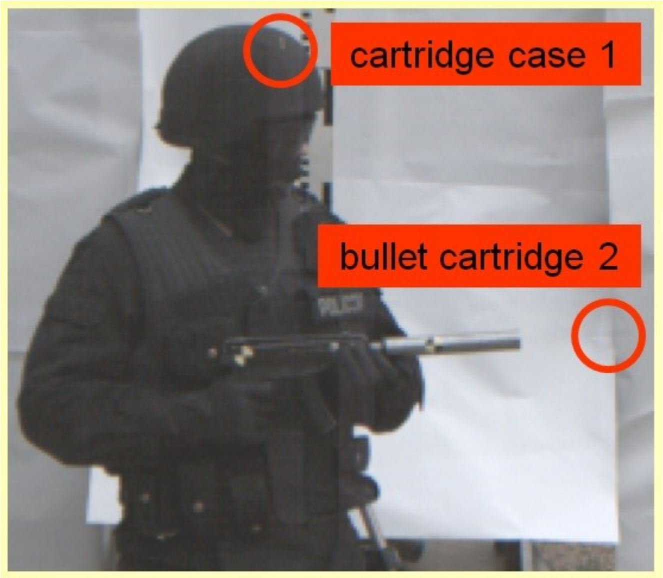

Fig. 1. A police officer from the special service during rapid firing the Skorpion submachine gun vz. 61 cal. 7,65 mm [own work]

The presence of large dispersion while shooting prompted the authors to undertake work to improve this unfavourable phenomenon. At the beginning empirical studies were conducted with the aim to analyze the variability of kinematic quantities characterizing the weapon movement during firing and then, a theoretical model of human — weapon system was formulated. **In this article, the authors present the physical and mathematical model developed on the basis of considerations arising from the experimental studies**. Ultimately, the task of considerations undertaken is to develop guidelines to shape the dynamic properties of automatic weapons so as to reduce the dispersion of bullets fired rapidly. For this purpose, a study of the system will be carried out based on the empirical research and theoretical analysis of the suggested models, taking into account the identification of parameters and the validation process.

THE ELEMENTS OF EXPERIMENTAL RESEARCH

At the police shooting range the process of rapid firing of two bullets was registered. For this purpose a fast digital camera Phantom v 9.1 was used. After the acquisition of the data obtained, the recorded image was analysed with the use of the specialized TEMA software. On the basis of the course of variation of kinematic quantities over time describing the weapon movement, the behaviour of the machine gun in thirteen characteristic points of its operation was specified. With the use of the TEMA software, a moment of time was achieved where a change in structure, weight or weight distribution of the system under examination takes place. Physical phenomena occurring while firing every shot affect the weapon movement and thus, spread of bullets fired one after another [4]. The characteristic moments of time are as follows:

1. The first shot — I
    1) $t = 0{,}0448$ [s] — lock movement to the rear position,
    2) $t = 0{,}0462$ [s] — cartridge bullet 1 leaves the barrel,
    3) $t = 0{,}0532$ [s] — cartridge shell 1 is thrown out of the receiver,
    4) $t = 0{,}0581$ [s] — lock reached the extreme rear position,
    5) $t = 0{,}0749$ [s] — lock movement to the front position,
    6) $t = 0{,}0973$ [s] — lock reached the extreme front position.
2. The second shot — II
    7) $t = 0{,}1022$ [s] — lock movement to the rear position,
    8) $t = 0{,}1036$ [s] — cartridge bullet 2 leaves the barrel,
    9) $t = 0{,}1134$ [s] — cartridge case 2 is thrown out of the receiver,
    10) $t = 0{,}1162$ [s] — lock reached the extreme rear position,
    11) $t = 0{,}1484$ [s] — lock movement to the front position,
    12) $t = 0{,}1743$ [s] — lock reached the extreme front position.
3. The shooter leaves the weapon to the initial position
    13) $t = 0{,}2604$ [s] — weapon is in the initial position.

Figure 2 presents the shooter with a gun in two characteristic moments of time specified in point 2 and 8.

Having compared the location of the weapon in the hands of the shooter when leaving the barrel by the bullet 1 and 2, a clear difference in the inclination angle of the machine gun might be noticed. Each bullet receives different values of track factors at the starting point [6, 12]. If the same initial velocity values of both bullets are adopted, then the most important factor influencing the trajectory will be the departure angle.

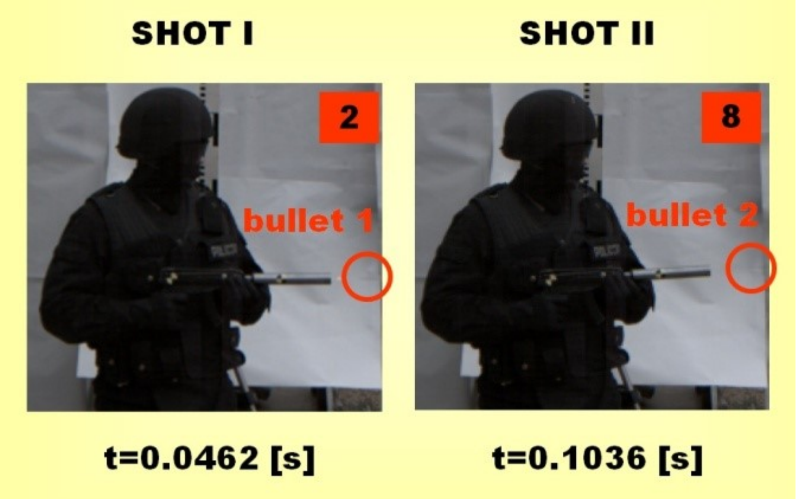

Fig. 2. A police officer during firing two shots from the Skorpion submachine gun [own work]

For the purpose of quantitative interpretation of track factors presented in figures 3, 4 and 5, the courses of variation of kinematic quantities over time characterizing the movement of weapon tilt are presented. The graphs include points at which the previously discussed phenomena occur.

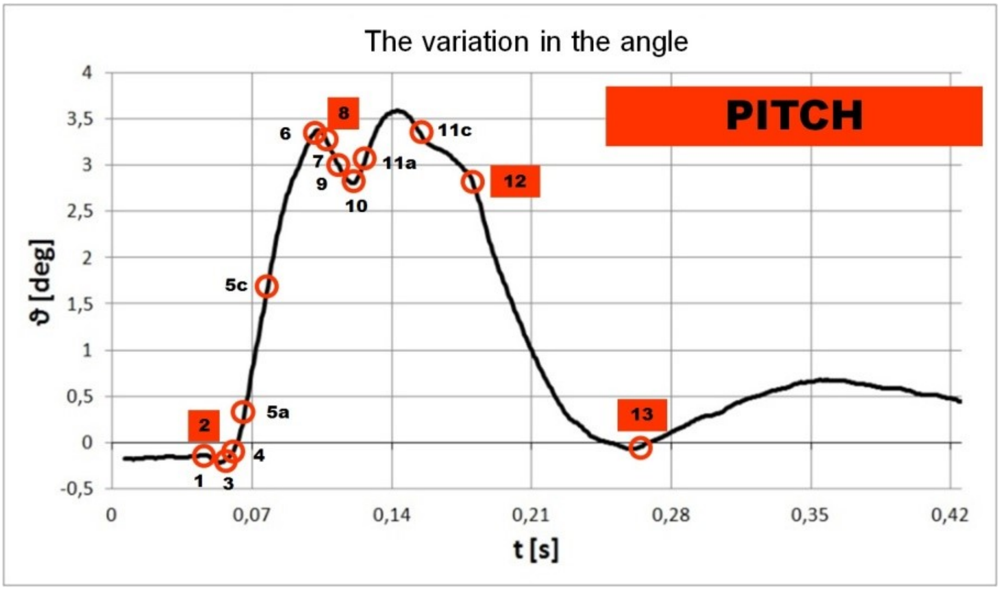

Fig. 3. The course of variation of weapon tilt over time [own work]

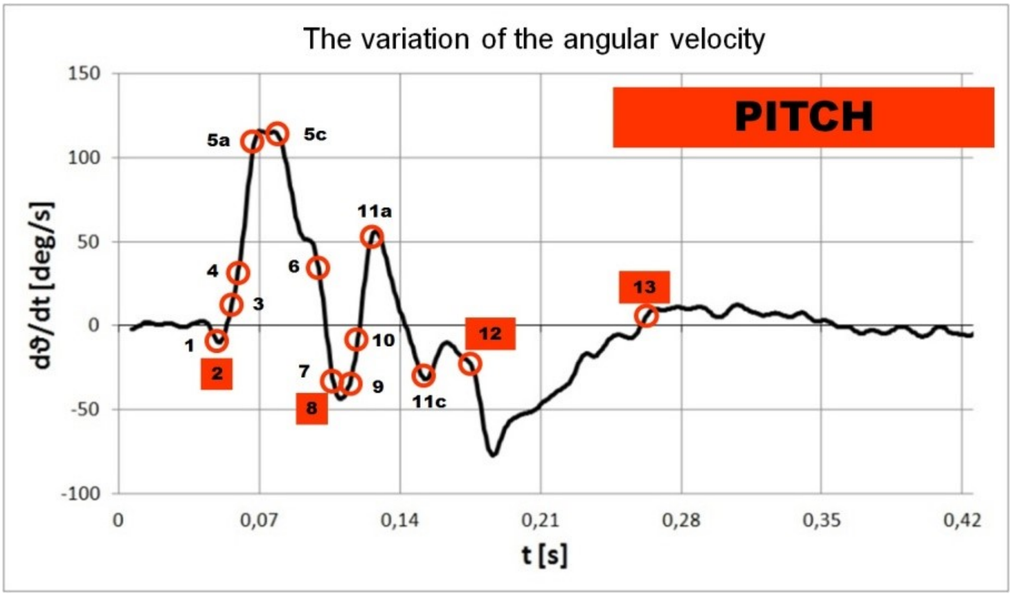

Fig. 4. The course of variation of angular velocity in the movement
of weapon tilt over time [own work]

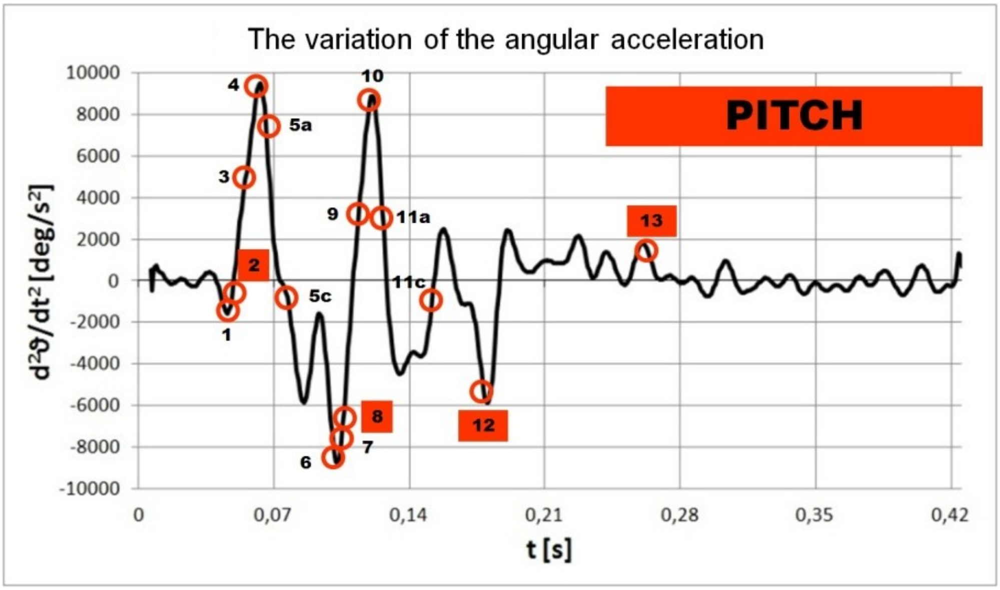

Fig. 5. The course of variation of angular acceleration in the movement
of weapon tilt over time [own work]

In the present case, the difference between the values of the departure angle for both bullets exceeds 3 [deg]. Due to the difference in value of the angular velocity of weapon tilt amounting to around 30 [deg/sec], the initial speed for both bullets should also be adjusted slightly. It follows from all of the above presented considerations that the trajectories of both bullets should differ from each other. This conclusion was confirmed with the tests that were conducted which showed the presence of clear dispersion between two bullet holes emerging on the target located at a distance of 25 [m] from the shooter. **The verification of bullet holes and analysis of recorded images showed a slight deviation of movement of objects examined from the vertical plane. In this respect the theoretical model of human — weapon system in the vertical plane was formulated**.

PHYSICAL MODEL OF HUMAN — WEAPON SYSTEM

Figure 6 presents a physical model of human — weapon system that was formulated with the use of considerations resulting from the shooting performed with the use of the Skorpion submachine gun vz. 61 that was recorded at the police shooting range. The physical model represents the movement of the system under examination in the vertical plane [10].

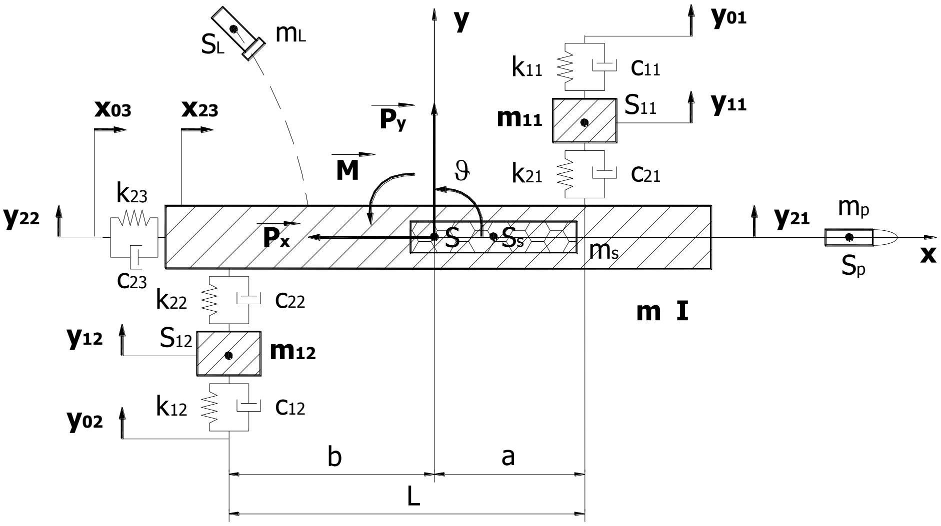

Fig. 6. Physical model of human — weapon system [own work]

When developing the physical model of the Skorpion submachine gun vz. 61, the following issues were determined and implemented:

1. Inertial components:
   - Material points,
   - Rigid body,
   - Objects variable over time.
2. Non-inertial components:
   - Restitution elements,
   - Dissipative elements,
   - Mathematical points.
3. Cartesian orthogonal clockwise systems of reference:
   - Galileo system,
   - Non-inertial coordinate systems,
   - Isometric transformations of coordinate systems.
4. Space:
   - Two-dimensional Euclidean space,
   - Homegenous field of gravity,
   - Earth's atmosphere.

The physical model that was developed consists of a rigid body of mass m and moment of inertia I representing the weapon and two point masses m_{11} and m_{12}, five weightless elements in the form of Kelvin-Voigt models of parameters k_{11}, c_{11} i k_{21}, c_{21} i k_{12}, c_{12} i k_{22}, c_{22} i k_{23}, c_{23}, three kinematic excitations x_{03}, y_{01} i y_{02} representing the anthropodynamic model of a human being [2, 3, 7, 8, 9]. The occurrence of physical phenomena, including the processes of firing bullets, lock movement, throwing the cartridge case and loading a new cartridge was simplified by introducing a dynamic excitations in the form of forces $\overrightarrow{P_x}$, $\overrightarrow{P_y}$ and torque $\overrightarrow{M}$ [5]. This model has five degrees of freedom.

MATHEMATICAL MODEL OF HUMAN — WEAPON SYSTEM

On the basis of the physical model and with the use of so-called energy method, a mathematical model was derived [10]. While defining the mathematical model of the Skorpion submachine gun vz.61, the following issues were defined and implemented [11]:

1. Kinematic correlations:
   - Location of inertial elements and mathematical points,

- Deformation of restitution elements,
- Static displacements,
- Velocity of inertial elements and mathematical points,
- Velocity of displacement of dissipative elements,
- Constraint equations.

2. Energy:
   - Kinetic,
   - Potential:
     - Elastic,
     - Field of gravitational forces,
3. Non-potential forces:
   - Rayleigh dissipation function,
   - Forces resulting from the movement of an object variable over time.
4. Equations of motion — the Lagrange equations of 2nd type.
5. Parameters described with functions.

In order to determine the system movement, five independent coordinates were adopted $x,\ y,\ \vartheta,\ y_{11},\ y_{12}$.

Equations of motion of human — weapon system:

$$\begin{gathered}
m\ddot{x} + c_{23}\dot{x} + k_{23}x = c_{23}\dot{x}_{03} + k_{23}x_{03} - P_x \\
m\ddot{y} - c_{21}\dot{\lambda}_{21} - k_{21}\lambda_{21} + c_{22}\dot{\lambda}_{22} + k_{22}\lambda_{22} = P_y - mg \\
I\ddot{\vartheta} - c_{21}a\dot{\lambda}_{21} - k_{21}a\lambda_{21} - c_{22}b\dot{\lambda}_{22} - k_{22}b\lambda_{22} = M \\
m_{11}\ddot{y}_{11} + c_{11}\dot{y}_{11} + k_{11}y_{11} + c_{21}\dot{\lambda}_{21} + k_{21}\lambda_{21} = c_{11}\dot{y}_{01} + k_{11}y_{01} - m_{11}g \\
m_{12}\ddot{y}_{12} + c_{12}\dot{y}_{12} + k_{12}y_{12} - c_{22}\dot{\lambda}_{22} - k_{22}\lambda_{22} = c_{12}\dot{y}_{02} + k_{12}y_{02} - m_{12}g
\end{gathered} \tag{1}$$

where:

$\lambda_{21} = y_{11} + y_{11st} - y - y_{st} - a(\vartheta + \vartheta_{st})$

$\lambda_{22} = y + y_{st} - b(\vartheta + \vartheta_{st}) - y_{12} - y_{12st}$

$\dot{\lambda}_{21} = \dot{y}_{11} - \dot{y} - a\dot{\vartheta}$

$\dot{\lambda}_{22} = \dot{y} - b\dot{\vartheta} - \dot{y}_{12}$

Equilibrium equations of human — weapon system:

$$\begin{gathered}
k_{22}(y_{st} - b\vartheta_{st} - y_{12st}) - k_{21}(y_{11st} - y_{st} - a\vartheta_{st}) + mg = 0 \\
-k_{22}b(y_{st} - b\vartheta_{st} - y_{12st}) - k_{21}a(y_{11st} - y_{st} - a\vartheta_{st}) = 0 \\
k_{11}y_{11st} + k_{21}(y_{11st} - y_{st} - a\vartheta_{st}) + m_{11}g = 0 \\
k_{12}y_{12st} - k_{22}(y_{st} - b\vartheta_{st} - y_{12st}) + m_{12}g = 0
\end{gathered} \tag{2}$$

where:

$y_{st},\ \vartheta_{st},\ y_{11st},\ y_{12st}$ — static displacements.

SUMMARY

The analysis of the empirical results confirmed the opinion of the users regarding the presence of significant dispersion of the Skorpion submachine gun vz. 61 cal. 7,65 mm while shooting towards the target located at a distance of 25 [m]. The undesirable effect encouraged the authors to undertake work with the aim to improve the dynamic properties of weapons.

Thanks to the experience gained during studying the results obtained at the shooting range, the physical model of human — weapon system with five degrees of freedom in the vertical plane was formulated. The verification of bullet holes and analysis of the registered images with the use of a fast digital camera Phantom v 9.1 proved that the movement of the tested objects slightly deviates from the vertical plane. In this respect, the theoretical model of human — weapon in the vertical plane was formulated. If there is a reasonable need, a spatial model will be developed. Undoubtedly, the system under examination is an object variable over time [1]. The structure, weight and weight distribution of the system undergo changes. It results from:

- lock movement together with the resistance-return system, which takes place in the receiver;
- leaving the gun by the bullet moving in the barrel;
- leaving the gun by shell being thrown out;
- pushing the second bullet into the receiver.

When formulating the model of human — weapon system, only the physical phenomena that have the nature of mechanical interactions were taken into consideration.

On the basis of the physical model, the mathematical model of human — weapon system was derived, which can be classified as linear, determined, variable over time, dissipative and constrained. If there is a reasonable need, a model that takes the physical and geometric nonlinearity and active elements resulting from human impact on the weapon into account will be developed.

Ultimately, the task of these considerations is to develop guidelines with the aim to shape the dynamic properties of automatic weapons so as to reduce the dispersion of fired bullets. For this purpose, a study based on the empirical research and theoretical analysis of the models developed will be conducted, taking the identification of parameters and validation process into consideration. The model suggested in this paper should be considered as a poll variant.

REFERENCES

[1] Bessonov A. P., *Fundamentals of dynamics mechanisms with variable mass units*, Nauka, Moscow 1967.

[2] Dziopa Z., Florin-Dziopa I., *Analiza komfortu użytkowania przeciwlotniczego zestawu rakiet umieszczonych na samochodzie z wykorzystaniem antropodynamicznego modelu operatora*, 'Mechanika', 2004, No. 5, pp.91–98 [*The analysis of comfort of using the set of anti-aircraft missiles located on a vehicle with the use of the anthropodynamic model of the operator* — available in Polish].

[3] Dziopa Z., Florin-Dziopa I., *Wpływ startu rakiety z wyrzutni samochodowej na organizm operatora*, 2004, 'Mechanika', No. 7, pp.153–160 [*The influence of the start of the missile from the launcher on the body of the operator* — available in Polish].

[4] Ejsmont J. A., *Celność broni strzeleckiej. Praktyczny poradnik*, WKiŁ, Warszawa 2012 [*The accuracy of small arms. Practical guide* — available in Polish].

[5] Ewertowski J., *Analiza siły oddziaływania broni ramiennej na strzelca w czasie strzału*, 'Biuletyn WAT', 2007, Vol. LVI, No. 1, pp. 207–221 [*The analysis of the force of influence of small calibre weapons on the shooter during firing* — available in Polish].

[6] Gacek J., *Balistyka zewnętrzna*, Part I, *Modelowanie zjawisk balistyki zewnętrznej i dynamiki lotu*; Part II, *Analiza dynamicznych właściwości obiektów w locie*, WAT, Warszawa 1997 and 1998 [*External ballistics*, Part I, *Modelling the phenomena of outer ballistics and flight dynamics*; Part II, *The analysis of dynamic properties of objects during flight* — available in Polish].

[7] Griffin M. J., *Handbook of human vibration*, Academic Press, London 1990.

[8] Książek M., *Analiza istniejących modeli biodynamicznych układu ręka-ramię pod kątem wibroizolacji człowieka-operatora od drgań emitowanych przez narzędzia ręczne*, 'Czasopismo Techniczne', 1996, 2M, Wyd. Politechniki Krakowskiej, Krakow 1996, pp. 87–113 [*The analysis of existing biodynamic models of hand-arm system with respect to the vibroisolation of human-operator from the vibrations emitted by the manual processing tool* — available in Polish].

[9] Książek M., *Modelowanie i optymalizacja układu człowiek — wibroizolator — maszyna*, 'Mechanika', 1999, Monografia 244, Politechnika Krakowska [*The modelling and optimalisation of human — vibroisolation — device system* — available in Polish].

[10] Suchocki C., Ewertowski J., *Modeling and numerical simulation of semi-automatic pistol Dynamics*, 'Journal of Theoretical and Applied Mechanics', 2015, Vol., 53, No., 1, pp. 81–91.

[11] Silva C. W. de, *Vibration Fundamentals and Practice*, Taylor & Francis Group, Boca Raton, London — New York 2007.

[12] Szapiro J., *Balistyka zewnętrzna*, MON, Warsaw 1956, pp. 468 [*External ballistics* — available in Polish].

MODEL TEORETYCZNY UKŁADU CZŁOWIEK — BROŃ

STRESZCZENIE

Na strzelnicy policyjnej przeprowadzono rejestrację procesu wystrzelenia z pistoletu maszynowego Skorpion wz. 61 kal. 7,65 mm ogniem seryjnym dwóch pocisków do tarczy znajdującej się w odległości 25 [m]. Przeprowadzone próby potwierdziły opinię użytkowników o występowaniu znacznego rozrzutu broni. Występowanie niepożądanego zjawiska skłoniło autorów do podjęcia prac zmierzających do poprawienia właściwości dynamicznych broni.

Badania doświadczalne dostarczyły informacji na temat kinematyki ruchu układu człowiek — broń. Na podstawie otrzymanych przebiegów zmienności przemieszczenia, prędkości i przyspieszenia w funkcji czasu określono zachowanie się pistoletu maszynowego w trzynastu charakterystycznych punktach jego działania. Weryfikacja przestrzelin i analiza zarejestrowanych obrazów wykazała nieznaczne odstępstwo ruchu badanych obiektów od płaszczyzny pionowej. W związku z tym sformułowano w płaszczyźnie pionowej model fizyczny układu człowiek — broń o pięciu stopniach swobody. Na podstawie modelu fizycznego wyprowadzono model matematyczny układu człowiek — broń, który można zakwalifikować do liniowych, zdeterminowanych, zmiennych w czasie, dyssypatywnych i nieswobodnych.

Słowa kluczowe:
broń, człowiek, model fizyczny, model matematyczny.