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ANALYSIS OF PROPERTIES OF INDUCTION MACHINE WITH COMBINED PARALLEL STAR-DELTA STATOR WINDING

ANALIZA WŁAŚCIWOŚCI MASZYNY INDUKCYJNEJ Z UZWOJENIEM STOJANA POŁĄCZONYM GWIAZDA-TRÓJKĄT

Abstract: The paper analyses the properties of a machine with a parallel combined stator winding. The analysis has been carried out with regard to the numbers of conductors of delta- and star- connected windings. The paper shows the influence of the ratio of the number of conductors of these windings on the possibility of suppressing higher spatial harmonics and on the efficiency of the machine. The derived equations of the machine with the combined stator winding describe the behaviour of the machine both in a steady state and in transient operating and fault states.

Streszczenie: W artykule przeanalizowano właściwości maszyny z równoległym połączeniem uzwojenia stojana. Analizę prowadzono w zakresie liczby przewodów delta i połączonego w gwiazdę uzwojenia. Przedstawiono wpływ stosunku liczby przewodów tych uzwojeń do możliwości stłumienia wyższych harmonicznych przestrzennych i dotyczących sprawności urządzenia. Pochodne równania maszyny z połączonym uzwojeniem stojana opisują zachowanie maszyny, zarówno w stanie ustalonym, nieustalonym oraz podczas awarii.

Keywords: *induction machine, parallel combined stator winding*
Słowa kluczowe: *maszyna indukcyjna, równolegle połączone uzwojenie stojana*

1. Introduction

At present, a considerable emphasis is put all over the world on power reduction. In the field of electric machines, one of the possibilities of energy consumption is increasing the efficiency of electric motors. One of the ways how to increase the efficiency of induction machines, which are mostly used electric machines, is the use of the so-called combined stator winding [1], [2]. This winding makes it possible to substantially decrease the content of higher spatial harmonics of the current layer, magnetic flux density in the air gap and flux in yoke. The reduction of the content of higher spatial harmonics results in decreasing the losses and thus improving the energy efficiency [3] and [4]. In the literature, there are mentioned two basic variants of connection of the combined winding. These variants are schematically depicted in Figs. 1a and 1b. In this paper, the properties of the combined parallel winding in Fig. 1b will be analysed.

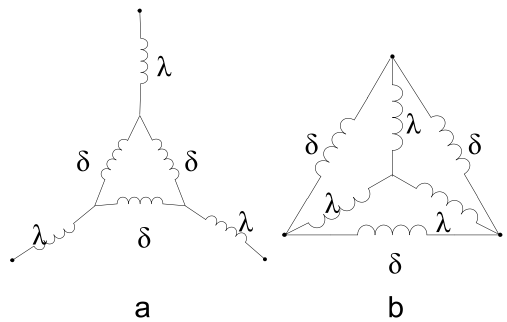

Fig. 1. Basic variants of combined winding

2. Analysis of parallel winding

According to [1], [2], and [3], the higher space harmonics of the lowest orders (i.e. orders 5 and 7) can be eliminated if two basic conditions are fulfilled. The ratio of the number of conductors of a single phase of the delta-connected winding to the number of the conductors of a single phase of the star-connected winding must be $\sqrt{3}$ and the axes of the two windings must be shifted by 30 electric degrees in the space of the machine. For the analysis of this winding, the theory of space vectors and symmetrical components of instantaneous values of currents and voltages has been used. According to [4], a space vector of the considered physical quantity (e.g. current layer, magnetic flux density in air gap, flux in yoke) is a complex vector lying in the plane perpendicular to the axis of the machine. The absolute value of this vector equals or is proportional to the maximum (amplitude) of this wave. The position of the vector in the complex plane corresponds to the position of this maximum. For example, space vector $\boldsymbol{I}_\nu$ of the ν-th wave of the current layer of one conductor is, according to [4], defined as

$$\boldsymbol{I}_\nu = i e^{j\nu\alpha}\ , \quad (1)$$

where i is a conductor current, ν is a harmonic order and α determines the position of a conductor in the complex plane of the given harmonic. If there are N_d conductors in a slot, then the vector of the ν-th current wave of this slot $\boldsymbol{I}_{Nd\nu}$ is

$$\boldsymbol{I}_{Nd\nu} = N_d i e^{j\nu\alpha}. \quad (2)$$

The space vector of the current wave of the order ν of the single phase winding with the number of slots Q is given by the sum of the current waves of the individual slots.

$$\boldsymbol{I}_{N\nu} = \kappa_\nu Q N_d i e^{j\nu\alpha}. \quad (3)$$

In this case, the angle α represents the position of the axis of this winding in the space of the machine and κ_ν is the winding factor of the ν-th harmonic. The vector of the current wave of the multi-phase winding is given by the sum of the vectors of individual windings. As it will be further shown, higher spatial harmonics have, in comparison with the fundamental wave, a small influence on the waveforms of currents and for the sake of simplicity, they will not be considered. The fundamental current wave of the delta-connected winding in Fig. 1 can be written as

$$\boldsymbol{I}_{N\delta} = \kappa_\delta N_\delta (i_{A\delta} + \boldsymbol{a} i_{B\delta} + \boldsymbol{a}^2 i_{C\delta}), \quad (4)$$

Where $i_{A\delta}$, $i_{B\delta}$ and $i_{C\delta}$ are currents of individual phases, κ_δ is a winding factor of the fundamental harmonic, N_δ is a number of all conductors of this winding, given by the product of the numbers of conductors in one slot and the number of slots of this winding Q_δ. Quantity $\boldsymbol{a}$ is the complex operator $e^{j\frac{2}{3}\pi}$. The expression in the brackets in equation (4) is proportional to the first symmetrical component of the stator currents of the winding δ, for which it can be written

$$\boldsymbol{i}_{1\delta} = {}^1/_3\,(i_{A\delta} + \boldsymbol{a} i_{B\delta} + \boldsymbol{a}^2 i_{C\delta}). \quad (5)$$

The axis of the winding star can be generally shifted by the angle α. The current wave of this winding is

$$\boldsymbol{I}_{N\lambda} = \kappa_\lambda N_\lambda (i_{A\lambda} + \boldsymbol{a} i_{B\lambda} + \boldsymbol{a}^2 i_{C\lambda}) e^{j\alpha}. \quad (6)$$

The individual symbols represent similar quantities as in the case of the winding δ. The first symmetrical component of currents of the winding delta is

$$\boldsymbol{i}_{1\lambda} = {}^1/_3\,(i_{A\lambda} + \boldsymbol{a} i_{B\lambda} + \boldsymbol{a}^2 i_{C\lambda}) e^{j\alpha}. \quad (7)$$

The voltage components u_δ and u_λ can be introduced similarly as it was in the case of currents. The voltage over a single phase winding, e.g. the winding of the delta-connected phase A, consists of the decrease in its resistance R_δ, leakage inductance $L_{\sigma\delta}$ and the voltage induced in this winding by the flux in the yoke $\boldsymbol{\Phi}_j$

$$u_{A\delta} = R_\delta i_{A\delta} + L_{\sigma\delta}\frac{di_{A\delta}}{dt} + \kappa_\delta N_\delta Re\left[\frac{d\boldsymbol{\Phi}_j}{dt}\right]. \quad (8)$$

The vector $\boldsymbol{\Phi}_j$ is given by the sum of flux vectors induced by the currents of the winding delta

$$\boldsymbol{\Phi}_{j\delta} = \kappa_\delta N_\delta \boldsymbol{i}_\delta L_1, \quad (9)$$

by the currents of the winding star

$$\boldsymbol{\Phi}_{j\lambda} = \kappa_\lambda N_\lambda \boldsymbol{i}_\lambda L_1, \quad (10)$$

and the flux vector excited by the rotor currents. This vector $\boldsymbol{\Phi}_{jR\lambda}$ after the transformation into stator co-ordinates is

$$\boldsymbol{\Phi}_{jR\lambda} = m_R N_R \boldsymbol{i}_{R\lambda} L_1, \quad (11)$$

where $\boldsymbol{i}_{R\lambda}$ is the symmetrical component of rotor currents transformed into stator coordinates. The symbol L_1 represents the so-called inductance of a single conductor for the fundamental harmonic. For the inductance of one conductor of the harmonic of the order ν can be, according to [7], written

$$L_\nu = \frac{\mu_0 k_\nu D l}{2\pi\delta\nu^2}, \quad (12)$$

where μ_0 is the permeability of vacuum, k is the factor of a slot opening, D is the bore of the stator, l is the active length of iron, δ is the length of an air gap including the Carter factor. The magnitude of the quantity $L\nu$ decreases with the second power of the order of the harmonic, i. e., the influence of higher spatial waves on the current waveforms dramatically decreases with their order. Equation (8) can be written for the individual phase windings δ and λ. After substitution of equations (9) to (11) for $\boldsymbol{\Phi}_j$ in these equations and after substitution of symmetrical component for phase voltages and currents and after manipulations, equations for the windings delta and star can be written as

$$\boldsymbol{u}_{\lambda\delta} = R_{\lambda\delta}\boldsymbol{i}_{\lambda\delta} + L_{\lambda\delta}\frac{d\boldsymbol{i}_{\lambda\delta}}{dt} + L_h\frac{d\boldsymbol{i}_{\delta}}{dt} + L_h\frac{d\boldsymbol{i}_{R\lambda\delta}}{dt}, \tag{13}$$

$$\boldsymbol{u}_{\delta} = R_{\delta}\boldsymbol{i}_{\delta} + L_{\delta}\frac{d\boldsymbol{i}_{\delta}}{dt} + L_h\frac{d\boldsymbol{i}_{\delta}}{dt} + L_h\frac{d\boldsymbol{i}_{R\lambda\delta}}{dt}, \tag{14}$$

where

$$\boldsymbol{i}_{\lambda\delta} = \boldsymbol{i}_{\lambda}\frac{\kappa_\lambda N_\lambda}{\kappa_\delta N_\delta}, \tag{15}$$

$$\boldsymbol{u}_{\lambda\delta} = \boldsymbol{u}_{\lambda}\frac{\kappa_\delta N_\delta}{\kappa_\lambda N_\lambda}, \tag{16}$$

$$R_{\lambda\delta} = R_{\lambda}\left(\frac{\kappa_\delta N_\delta}{\kappa_\lambda N_\lambda}\right)^2, \tag{17}$$

$$L_h = {}^3/_2\,(\kappa_\delta N_\delta)^2 L_1, \tag{18}$$

$$L_\lambda = L_{\sigma\lambda\delta} + L_h, \tag{19}$$

$$L_{\sigma\lambda\delta} = L_{\sigma\lambda}\left(\frac{\kappa_\delta N_\delta}{\kappa_\lambda N_\lambda}\right)^2, \tag{20}$$

$$L_\delta = L_{\sigma\delta} + L_h. \tag{21}$$

The rotor equation is

$$= R_{R\delta}\boldsymbol{i}_{R\lambda\delta} + L_{R\delta}\frac{d\boldsymbol{i}_{R\lambda\delta}}{dt} + L_h\frac{d\boldsymbol{i}_{\delta}}{dt} + L_h\frac{d\boldsymbol{i}_{\lambda\delta}}{dt} - -j\frac{d\rho}{dt}\left(L_{R\delta}\boldsymbol{i}_{R\lambda\delta} + L_h\boldsymbol{i}_{\delta} + L_h\boldsymbol{i}_{\lambda\delta}\right) \tag{22}$$

where $\boldsymbol{i}_{R\lambda\delta}$ is the symmetrical component of rotor currents transformed into the stator co-coordinate system by multiplying $e^{j\rho}$ and rated to the number of the conductors winding delta according to equation

$$\boldsymbol{i}_{R\lambda\delta} = \frac{m_R\kappa_R N_R}{3\kappa_R N_R}. \tag{23}$$

In the case of a cage rotor, the number of the rotor phases m_R equals the number of bars and the number of the conductors N_R equals 1. The symbol ρ is the angle between the stator and rotor symmetrical systems. Further,

$$R_{R\delta} = R_R\frac{3}{m_R}\left(\frac{\kappa_\delta N_\delta}{\kappa_R N_R}\right)^2, \tag{24}$$

where R_R is the resistance of a single rotor phase. The total inductance of the rotor $L_{R\delta}$ is

$$L_{R\delta} = L_{R\sigma\delta} + L_h, \tag{25}$$

where $L_{\sigma R\delta}$ is a leakage inductance of the rotor $L_{\delta R}$ rated to the number of conductors of the winding delta

$$L_{\sigma R\delta} = L_{\sigma R}\frac{3}{m_R}\left(\frac{\kappa_\delta N_\delta}{\kappa_R N_R}\right)^2. \tag{26}$$

The torques generated by the winding delta and star are

$$T_\delta = 6pL_h Re[j\boldsymbol{i}_\delta^*\boldsymbol{i}_{R\lambda\delta}]. \tag{27}$$

$$T_\lambda = 6pL_h Re[j\boldsymbol{i}_\lambda^*\boldsymbol{i}_{R\lambda\delta}]. \tag{28}$$

The resulting torque T is given by the sum of the torques T_δ and T_λ. The motion equation is

$$\frac{d\omega_m}{dt} = {}^1/_J\,(T - T_l). \tag{29}$$

where ω_m is the mechanical angular speed. The symbol J is the inertia torque and T_l is the load torque.

3. Experimental machine

The experimental induction machine has been designed and produced in cooperation with the ATAS elektromotory Náchod a.s. The stator combined two-pole winding of this machine is formed by three windings with 159 conductors in a slot and three windings with 92 conductors in a slot. The number of slots of the stator is 24. Each winding is distributed in four slots. The winding with 159 conductors in a slot is a delta-connected winding. The remaining three windings are star-connected windings. The beginnings and ends of all the windings are connected to the terminals. This makes it possible to realize the connection of the two fundamental variants of the combined winding. The delta-connected windings are designed for the voltage of 100 V, the star-connected windings for the voltage of 57.5 V. The nominal frequency is 50 Hz. The total reactance of the delta-connected winding $X_\delta = 193.77\ \Omega$, the leakage reactance $X_{\sigma\delta} = 8.07\ \Omega$ and the resistance $R_\delta = 19.6\ \Omega$. The parameters of the star-connected winding are $X_\lambda = 65.73\ \Omega$, $X_{\sigma\lambda} = 2.77\ \Omega$ and $R_\lambda = 6.8\ \Omega$. The leakage reactance and the rotor resistance rated to the number of conductors of the winding delta are $X_{\sigma R\delta} = 14.5\ \Omega$ and $R_{R\delta} = 8.11\ \Omega$. The number of slots per pole and phase of the delta winding as well as the star winding are 2. That means, both the windings have the same winding factors. The factors of the winding of the order 1 – 13 are in Table 1.

Table 1.

ν	1	5	7	11	13
κ_ν	0.991	0.793	0.609	0.131	0.131

For comparison, in Table 2, there are shown the winding factors of the two-pole three-phase machine with the stator with 24 stator slots.

Table 2.

ν	1	5	7	11	13
κ_ν	0.958	0.205	0.158	0.126	0.126

4. Examples of results of simulation and conclusion

On the basis of equations (13), (14), (22), (27), (28) and (29), a numerical model of an induction machine with the parallel combined stator winding has been set forth. The results of simulations of the experimental machine loaded by the torque T_l = 0,525 Nm are in Figs. 2 – 12. Figure 2 shows two periods of terminal voltages. In Fig. 3, there are phase currents of the star winding in a steady state, in Fig. 4, there are currents of the delta winding.

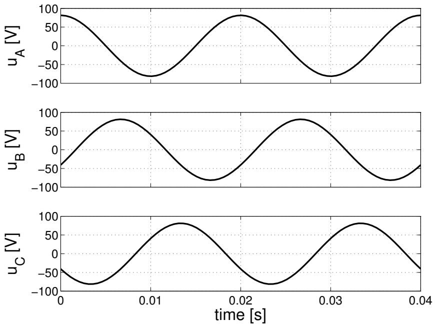

Fig. 2. Terminal voltages

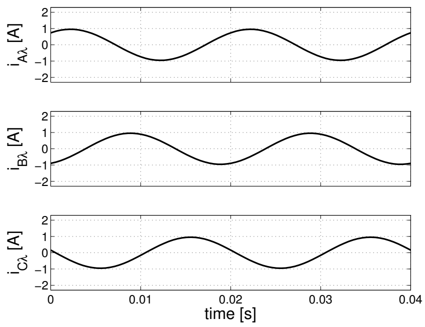

Fig. 3. Currents of winding star

In Fig. 5, there are currents flowing to the stator from the feeding source. The magnitude of currents of the source is 1.39 A, of the currents of the winding star is 0.675 A and the winding delta is 0.395 A.

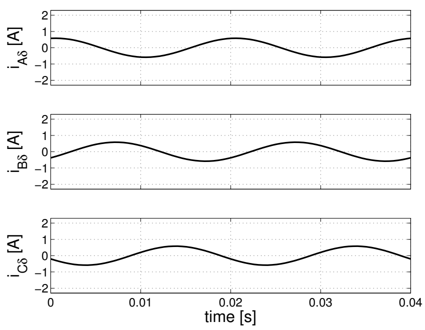

Fig. 4. Currents of winding delta

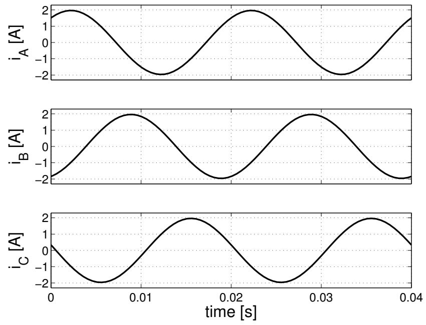

Fig. 5. Terminal currents

The waveforms in Figs. 2 – 4 are in good accordance with measurement. In Fig. 6, there are shown the waveforms of the real part α and of the imaginary part β of the vector of the current wave $\boldsymbol{I}_{N\lambda 1}$ of the currents of the star winding and in Fig. 7, there are components of the vector $\boldsymbol{I}_{N\delta 1}$.

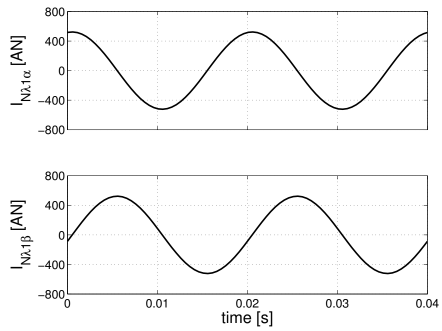

Fig. 6. Real and imaginary part of $\boldsymbol{I}_{N\lambda 1}$

The final current wave $\boldsymbol{I}_{N1}$ is given by the sum of the waves generated by the winding star and the winding delta. Its components are in Fig. 8. The components of the waves of the fifth harmonic of particular windings and the components of the resulting wave $\boldsymbol{I}_{N5}$ are in Figs. 9 - 11. The current waves of the fifth harmonic are given by the relations

$$\boldsymbol{I}_{N\lambda 5} = \kappa_{\lambda 5} N_\lambda (i_{A\lambda} + \boldsymbol{a}^2 i_{B\lambda} + \boldsymbol{a} i_{C\lambda}) e^{j5\alpha}, \quad (30)$$

$$\boldsymbol{I}_{N\delta 5} = \kappa_{\delta 5} N_\delta (i_{A\delta} + \boldsymbol{a}^2 i_{B\delta} + \boldsymbol{a} i_{C\delta}). \quad (31)$$

The waveforms of the components of particular current waves and the final current wave of the seventh harmonic are in Figs. 12 - 14. For the current waves $\boldsymbol{I}_{N\lambda 7}$ and $\boldsymbol{I}_{N\delta 7}$ it can be written

$$\boldsymbol{I}_{N\lambda 7} = \kappa_{\lambda 7} N_\lambda (i_{A\lambda} + \boldsymbol{a} i_{B\lambda} + \boldsymbol{a}^2 i_{C\lambda}) e^{j7\alpha}, \quad (32)$$

$$\boldsymbol{I}_{N\delta 7} = \kappa_{\delta 7} N_\delta (i_{A\delta} + \boldsymbol{a} i_{B\delta} + \boldsymbol{a}^2 i_{C\delta}). \quad (33)$$

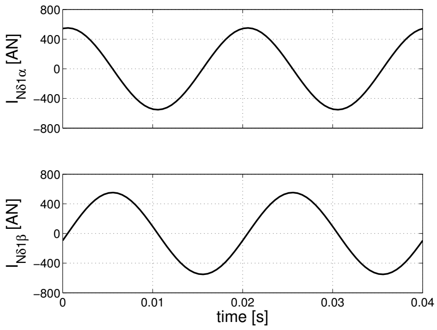

Fig. 7. Real and imaginary part of $\boldsymbol{I}_{N\delta 1}$

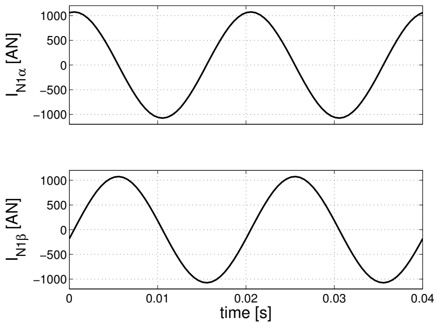

Fig. 8. Real and imaginary part of $\boldsymbol{I}_{N1}$

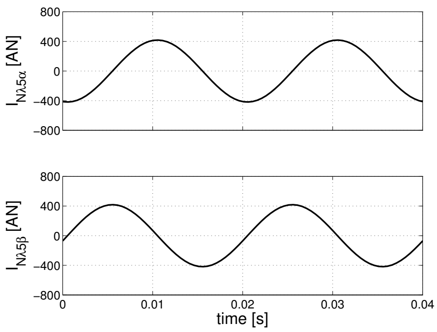

Fig. 9. Real and imaginary part of $\boldsymbol{I}_{N\lambda 5}$

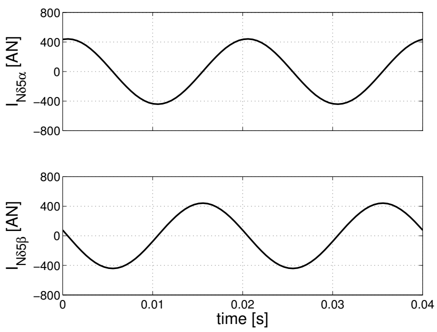

Fig. 10. Real and imaginary part of $\boldsymbol{I}_{N\delta 5}$

It is obvious that neither the fifth nor the seventh waves are quite eliminated. The fifth and seventh waves of individual windings star and delta can mutually eliminate themselves if the values of corresponding constants in equations (13) and (14) equal.

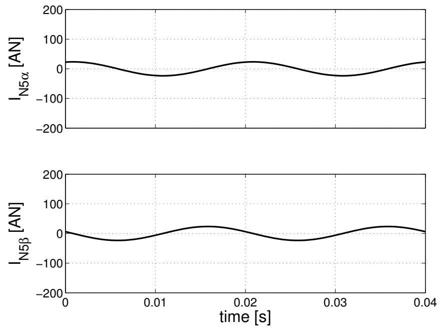

Fig. 11. Real and imaginary part of $\boldsymbol{I}_{N5}$

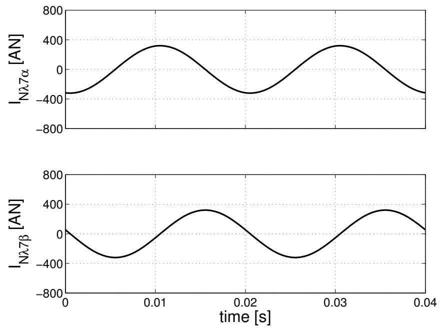

Fig. 12. Real and imaginary part of $\boldsymbol{I}_{N\lambda 7}$

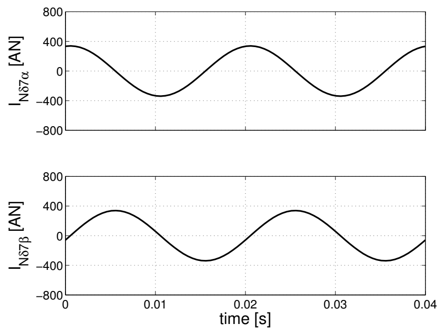

Fig. 13. Real and imaginary part of $\boldsymbol{I}_{N\delta 7}$

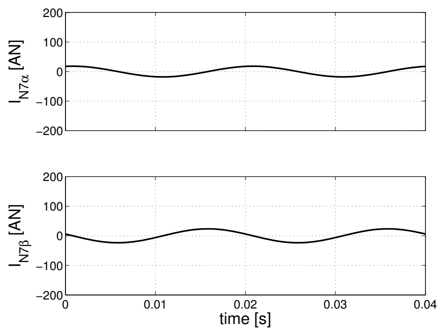

Fig. 14. Real and imaginary part of $\boldsymbol{I}_{N7}$

That means that besides the ratios of the number of conductors $\sqrt{3}$, also the ratio of leakage inudctances and resistances of the two windings must equal $\sqrt{3}$. The ratio of conductors of the experimental machine is 1.728. The above mentioned conditions cannot be completely fulfilled. Due to this fact, the amplitudes of the vectors of the waves of the fundamental harmonic differ and in the space of the machine, these vectors are mutually shifted. The final current wave is thus smaller than the sums of the absolute magnitudes of the waves of the individual windings. The torque T_λ is 0.2547 Nm, the T_δ is 0.2705 Nm, the coper losses in the winding λ are $P_\lambda = 9.37$ W and the coper losses P_δ in the winding delta are 10.01 W. In order to show the influence of the ratio of the windings on the situation in the machine, the number of conductors in the slot of the winding delta is supposed to be decreased by four conductors. The currents of the source, the winding star and the winding delta are similar to waveforms in Figs. 2 – 4. The magnitude of currents of the source has slightly increased to the value of 1.41 A, the magnitude of the star currents has decreased to 0.583 A, and the magnitude of the delta currents has increased to the value of 0.479 A. The components of the current waves $\boldsymbol{I}_{N\lambda 1}$, $\boldsymbol{I}_{N\delta 1}$ and the resulting wave $\boldsymbol{I}_{N1}$ are shown in Figs. 15 - 17.

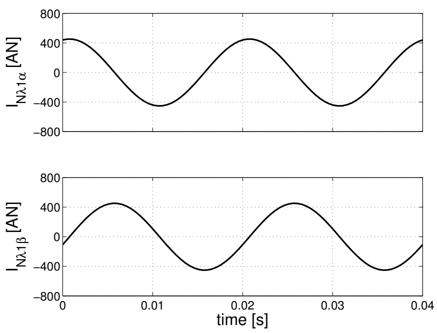

Fig. 15. Real and imaginary part of $\boldsymbol{I}_{N\lambda 1}$

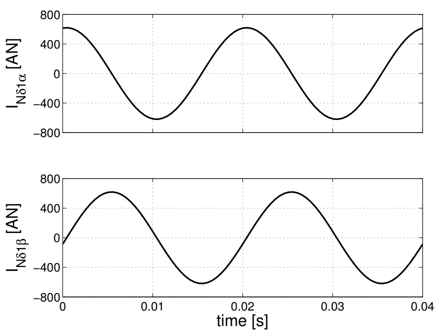

Fig. 16. Real and imaginary part of $\boldsymbol{I}_{N\delta 1}$

The components of the waves $\boldsymbol{I}_{N\lambda 5}$, $\boldsymbol{I}_{N\delta 1}$ and $\boldsymbol{I}_{N5}$ are in Figs. 18 – 20. The rise of the magnitude of the quantities $\boldsymbol{I}_{N5}$ in comparison with the previous case is considerable. The increase of magnitude of the current wave $\boldsymbol{I}_{N7}$ is similar. The possibility of the suppression of the spatial harmonics is significantly limited.

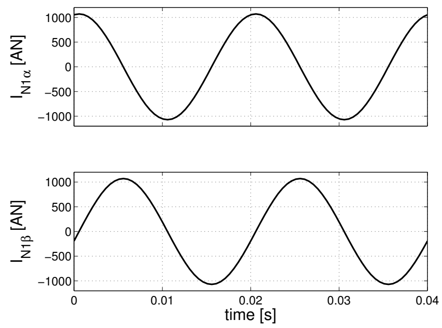

Fig. 17. Real and imaginary part of $\boldsymbol{I}_{N1}$

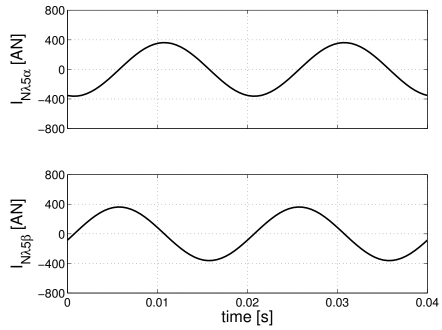

Fig. 18. Real and imaginary part of $\boldsymbol{I}_{N\lambda 5}$

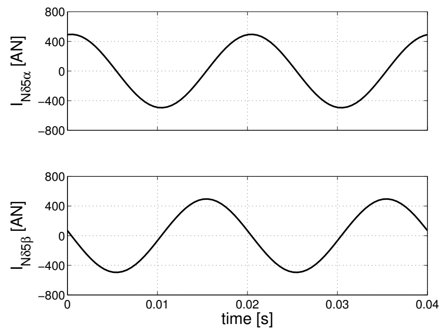

Fig. 19. Real and imaginary part of $\boldsymbol{I}_{N\delta 5}$

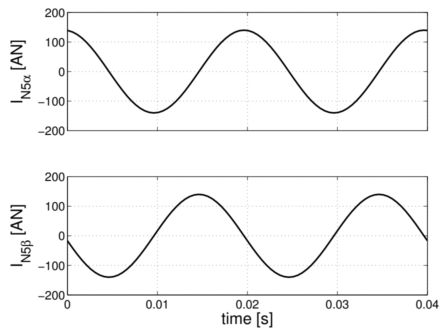

Fig. 20. Real and imaginary part of $\boldsymbol{I}_{N5}$

The winding star in this case produces the torque $T_\lambda = 0.238$ Nm and the torque of the winding delta is $T_\delta = 0.287$ Nm. The difference of the two values is considerable. The losses in the winding delta have increased by almost 30 % to the value of 19.99 W. The losses in the winding star have dropped to 6.98 W. The results of the analysis and the simulations show that the higher spatial harmonics of the orders 5 and 7 will be suppressed if the conditions under which their current layers generated by the winding star as well as by the winding delta mutually eliminate themselves are satisfied. One of these conditions is that the ratio of the number of the conductors of the winding delta and the winding star must be $\sqrt{3}$. This condition cannot be in practice completely fulfilled and even a small deviation from this value results not only in a considerable increase of the resulting current waves of the fifth and the seventh spatial harmonics but also in deterioration of the properties of the machine from the viewpoint of the fundamental harmonic.

5. Bibliography

[1]. Cistelecan M.V., Ferreira J.T.E.F., Cosan H.B.: *Generalized MMF space harmonics and performance analysis of combined multiple-step, star-delta, three-phase windings applied on induction motors*. ICEM 2008, Vilamoura, Portugal

[2]. Kasten H., Hofmann W.: *Combined Stator Windings in Electric Machines with Same Coils*. ICEM 2012, Marseille, France

[3]. Schreier L., Bendl J., Chomat M.: *Effect of combined stator winding on reduction of higher spatial harmonics in induction machine.* Electrical Engineering, 2016, pp. 1-9

[4]. Stepina J.: *Fundamental Equations of the Space Vector Analysis of Electrical Machines*. Acta Technica CSAV (6), 1968, pp.184-198

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