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An approach to the evaluation of dynamic uncertainty in measurement using non-statistical techniques

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Abstract

The article discusses a new approach to the expression of dynamic uncertainty of experimental dynamic measurements based on a priori information about the frequency response measurement tool. This approach allows to measure the results of dynamic measurements with the international requirements for the characterization accuracy.

keywords: Measurement, uncertainty, dynamics, frequency characteristic.

Ocena niepewności przy pomiarach dynamicznych

Streszczenie

Pomiary dynamiczne są prowadzone w badaniach naukowych, przemyśle i medycynie. W artykule zaproponowano nowe podejście do oceny niepewności przy pomiarach dynamicznych które opiera się na wykorzystaniu funkcji spektralnych wejściowego sygnału pomiarowego i danych znanych a priori odnoszących się do częstotliwościowych charakterystyk stosowanej aparatury pomiarowej.

Słowa kluczowe: Pomiary dynamiczne, niepewność, charakterystyki częstotliwościowe.

1. Definitions of issues

Experiments conducted using Measuring Instruments (MI) under dynamic conditions are becoming more common in many fields including scientific research, technology, manufacturing industry, commerce, and medicine. Dynamic measurements are in the first instance concerned with the study of the conformity of the path of the physical processes in subjects under investigation. As a result, the role of such measurements is particularly significant, firstly in the areas of science related to the investigation of the structure of matter, the analysis and synthesis of new substances and materials, where the study takes place under experimental conditions, and secondly, in the fields of engineering, especially in manufacturing and medicine, which are characterized by the creation of new technological processes and the testing of new MI using high precision equipment.

When reporting on the results of dynamic measurements, it is necessary to provide a quantitative assessment of the quality of the experiment in order that its reliability may be correctly appraised [1-4]. Without such a reference value, the results of dynamic measurement can neither be compared with other equivalent studies, nor with standard reference values. It is therefore necessary to develop a uniform and understandable assessment methodology of the quality characteristics of dynamic measurements.

In this context, is necessary to take into account the fact that during dynamic measurement there is always a transition period during the operation of the MI, during which the output signal of the MI changes significantly over time. This set of circumstances may be explained given the inertial properties of the MI, because they comprise, on the whole, of components with different masses and springs, capacitance and inductance or other inertial elements, that lead to the emergence of dynamic uncertainty. This leads to the fact that the means of measuring the conversion equation that maps its statics is unacceptable in a dynamic mode. In this case, we need to use differential equations that describe the dynamic relationship of the output, $y(t)$, and the input $x(t)$ values of measuring instruments [5].

Given the above, it is necessary to develop a single approach to the expression of dynamic uncertainty in measurement means, which would meet international requirements that apply to the evaluation of the characteristics of the quality of measurements, and this is seen as the core scientific problem in the field of metrology.

The objective in writing this paper is the development and mathematical description of a new approach to the expression of the dynamic uncertainty of measurement means, that would allow us to take into account the inertial properties of the measurement means and the measurement signal passing through it, which will ensure a unified estimation of dynamic measurements in accordance with existing regulations in metrology, even if produced in different countries, different laboratories and different metrological organizations.

2. Analysis of the status of research and publications

In drawing up relevant differential equations, input signals are recorded on the right, i.e. the reason that led the MI to function, while the left side of the differential equation, describes the output signal (or response of the MI), and for linear transducers, it is written in the form [3, 5]

$$\sum_{i=0}^{n} a_i y^i(t) = \sum_{k=0}^{m} b_k x^k(t), \tag{1}$$

where $x(t)$, $y(t)$, are respectively the input and output values; i, k, are the derivative orders; and a, b are the coefficients that characterize the properties of the MI.

To express the differential equation in the area of frequency, the differentiation symbol jω may replace d/dt as the time coordinate, and then the equation (1) takes the form

$$\frac{y(j\omega)}{x(j\omega)} = S_0 \frac{b_m (j\omega)^m + b_{m-1}(j\omega)^{m-1} + ... + 1}{a_n (j\omega)^n + a_{n-1}(j\omega)^{n-1} + ... + 1}$$

or

$$y(j\omega) = S(j\omega)x(j\omega), \quad (2)$$

where y(jω), x(jω), are respectively the spectral functions of the input and output measurement signal; $S_0=b_0/a_0$ is the static sensitivity, i.e. the sensitivity to the constant input value (when jω=0); S(jω) is the transfer function of the MI or operational sensitivity.

The most typical properties of the MI are dynamic characteristics, which are described by differential equations of the first or second order, although in some cases, the third or higher order [5-9].

Information about the dynamic characteristics should be found in the regulary and technical documentation of the MI, although if data is not available, it can be obtained on the basis of a priori data on the MI.

To express the experimental uncertainty of the results of dynamic measurements, it may be convenient for practical use to refer to the frequency characteristics of the measuring instrument [5], listed in Tab. 1.

Tab. 1. Transmission functions for the most typical dynamic links
Tab. 1. Funkcje transmisji niektórych elementów dynamicznych

Frequency characteristics of the MI	Typical Units
$S(j\omega)=K$ where K is the transmission coefficient	Non-inertial (ideal measurement transducer)
$S(j\omega)=K/(1+j\omega\tau)$. where τ is the time constant determined by the parameters of the MI	Aperiodic (temperature transducer)
$S(j\omega)=K/j\omega$	Integrated (integrated amplification)
$S(j\omega)=K/(1+j\omega\tau)$	Forcing (differential amplification)
$S(j\omega)=\exp(-j\omega\tau)$	Delay (analog-to-digital converters)
$S(j\omega)=K/(1+j\omega\tau_1-\omega^2\tau_2^2) = K/(1+2j\omega\beta\tau-\omega^2\tau^2)$	Oscillating (electromechanical transducers)

It is also known that the existing international experience in the concept of evaluation and expression of measurement uncertainty [1] does not describes how to undertake estimation of dynamic uncertainties in the performance of metrological works (or experiments in dynamic modes of MI).

[1] only makes it apparent that in existence there are ways of estimation as demonstrated by type A and type B, and in addition ways to demonstrate uncertainties, which may be standard, combined or enhanced. The definitions of these uncertainties are given in [1]. A well-known approach, as investigated in the papers [3, 6-10], is that dynamic uncertainty is calculated as a standard uncertainty of type B, itself determined by the dynamic error value divided by the square root of 3 (assuming a uniform distribution law).

Using classical theory in the measurement of dynamic error in the expression of dynamic uncertainty is unacceptable, given the concept of measurement uncertainty expression, which, as set out in the international standard [1], is moving away from the concept of measurement error, as such, which does not use known values, and cannot have absolute values. This is as opposed to measurement uncertainty, which can be evaluated, and for a particular measurement result is not a single value, but has an infinite number of values, which are scattered around the result.

Consequently, there is a need to develop a new approach to the expression of dynamic uncertainty that can be evaluated without using the classic dynamic errors used in error theory.

3. Approach to the expression of dynamic uncertainty

The measurement of dynamic uncertainty depends on measurement uncertainty that is conditional on the responses of the measurement means to determine the speed (frequency) of the input signal, which is itself dependent both on the dynamic properties of the measurement means and on the frequency spectrum of the input signal.

Dynamic uncertainty measurement $u_D[y(t)]$ can be expressed by the square root of the integral of the product of the square of the spectral function of the input signal and the square of the modulus of the frequency response of the measuring instrument that is used during dynamic measurements over a wide range of frequencies

$$u_D[y(t)] = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty} |S(j\omega)|^2 |X(j\omega)|^2 d\omega}, \quad (3)$$

where $|S(j\omega)|$ is the modulus of the frequency characteristics of the MI that is used for dynamic measurement, or the amplitude frequency characteristics of the MI, which is defined by formula [7]

$$|S(j\omega)| = \sqrt{a^2(\omega) + b^2(\omega)}, \quad (4)$$

where $a(\omega)$, $b(\omega)$are respectively the real and imaginary parts of the frequency characteristics of $S(j\omega)$ of measuring instruments; $X(j\omega)$ is the spectral function of the input signal that is associated with the input time function $x(t)$ of the Laplace expansion [5]

$$X(j\omega) = \int_0^{\infty} x(t)e^{-j\omega_0 t} dt, \quad (5)$$

where ω_0 is the frequency of the input signal.

The upper limit of integral equation (5) on a finite time interval can be changed by the total observation time T.

If the measured signal $x(t)$ is determined by sampling, then the integration of equation (5) can be replaced by a summation operation, when the following substitutions are made: t is replaced by nT_a, where n varies from 0 to N–1, through T_a which designates a sampling period, then $x(t)$ has the form $x(nT_a)$, and $e^{-j\omega_0 t}$ is replaced by $e^{-j\omega_0 nT_a}$ [7].

Should such replacements be made in equation (5), it may then be written in a discrete form [5, 7]

$$X_d(j\omega) = \sum_{n=0}^{N-1} x(nT_a)e^{-j\omega_0 nT_a} =$$
$$= \sum_{n=0}^{N-1} x(nT_a)\cos\omega_0 nT_a - j\sum_{n=0}^{N-1} x(nT_a)\sin\omega_0 nT_a, \quad (6)$$

where $\omega_0=2\pi k/(nT_a)$, $k=0,1,\ldots,N-1$.

In this case, that the discrete spectral function value corresponds to a continuous spectral function, it needs to be multiplied by the sampling interval [7]

$$X(j\omega) = T_a X_d(j\omega). \quad (7)$$

During dynamic sampling measurements during the production of a signal, the equation expressing dynamic uncertainty (3), taking into account equations (6) and (7), may be written in the form

$$u_D[y(t_i)] = \sqrt{\frac{T_a}{N}\sum_{k=0}^{N-1}\sum_{n=0}^{N-1} x^2(nT_a)e^{-j\frac{4\pi nk}{N}} A^2\left(k\frac{2\pi}{NT_a}\right)}, \quad (8)$$

where $A\left(k\frac{2\pi}{NT_a}\right)=A(\omega)=|S(j\omega)|$ represents the amplitude frequency characteristics of the MI supported by the dynamic measurement values taken: $\Delta\omega=\frac{2\pi}{NT_a}$ is the interval of the discrete defined frequency values: T_a is the sampling time; N is the number of samples; NT_a is the total duration of observation.

If rapidly changing processes are measured, it is necessary to take this into account and to adjust the dynamic uncertainty of the measurement means used. This uncertainty is dependent on imperfections in the dynamic properties of the means of measurement and is associated with a non-zero response time of the MI. One of the main tasks of information-providing equipment is also the determination of the measured value of $x(t)$ through the output signal of the integrated circuit, $y(t)$. The correction of the dynamic uncertainty input provides a solution to the operator equation of dynamics

$$x(t)=L^{-1}[y(t)], \tag{9}$$

where $y(t)$ is the output signal of the MI and L^{-1} is the return operator equation of the MI.

Under these circumstances, let us imagine that the known values of the operator L and the output signal $y(t)$ and their uncertainties are $u(L)$ and $u_D[y(t)]$. The uncertainty $u(L)$ will now be either the uncertainty of found operators of a specific instance of the MI, or a deviation of a specific operator from standardized values. The uncertainty $u_D[y(t)]$ is an uncertainty of the dynamic output signal $y(t)$, which can be calculated by formula (3) or (8).

Compensation for dynamic uncertainty, unlike that for static, where corrections are introduced into the measurement results, may be put into practise only by complex processing of the output signal of the MI.

Regeneration of the input signal comprises the following [5, 7]:

a) receiving the output signal of the MI $y(t)$ of duration T;
b) resolution of the measured signal $y(t)$ in a Fourier series with the period of the first harmonic of the form:

$$y(t)=\frac{m_0}{2}+\sum_{k=1}^{K}m_k\cos(k\omega_0 t)+\sum_{k=1}^{K}n_k\sin(k\omega_0 t), \tag{10}$$

where $\omega_0=2\pi/T$; with T being the duration of the output signal $y(t)$; and with m_0, m_k, n_k being coefficients found by the formulae:

$$m_0=\frac{2}{N}\sum_{n=0}^{N-1}y_n; \tag{11}$$

$$m_k=\frac{2}{N}\sum_{n=0}^{N-1}y_n\cos(k\omega_0 t_n); \tag{12}$$

$$n_k=\frac{2}{N}\sum_{n=0}^{N-1}y_n\sin(k\omega_0 t_n); \tag{13}$$

c) experimental determination of the amplitude frequency $A(\omega)$ and phase frequency $\Phi(\omega)$ characteristics of the MI and discovery of the real and imaginary parts of the complex frequency behaviour:

$$a(\omega)=A(\omega)\cos\Phi(\omega); \tag{14}$$

$$b(\omega)=A(\omega)\sin\Phi(\omega); \tag{15}$$

d) calculation of the Fourier coefficients c_k and z_k of the input signal $x(t)$ from the chain of equations:

$$\begin{cases}c_k a(k\omega)+z_k b(k\omega)=m_k,\\ z_k a(k\omega)-c_k b(k\omega)=n_k;\end{cases} \tag{16}$$

$$m_o=\frac{c_0}{a(0)}; \tag{17}$$

e) determination of the input signal $x(t)$ in the form of a Fourier series

$$x(t)=\frac{c_0}{2}+\sum_{k=1}^{K}c_k\cos(k\omega t)+\sum_{k=1}^{K}z_k\sin(k\omega t). \tag{18}$$

The disadvantage of this sequence is in the need for redundant calculations. Since the actual output signal $y(t)$ is only defined at an interval of time [0, T], it is enough to assume that the function $y(t)$ is even and stated as cosines, or odd and stated as sines. This significantly reduces the number of calculations and simplifies the correction algorithm.

In actual MI, both digital and analog, sampling of input signals $x(t)$ and output signals $y(t)$ is output in the form of discrete readings $y(t_i)$. Therefore, in calculating the coefficients of a Fourier series, the integral sign is replaced by a summation sign. It may be assumed that the function $y(t_i)$ is even and may be expanded by its cosine, limiting the K-th harmonic.

$$y(t_i)=\frac{m_0}{2}+\sum_{k=1}^{K}m_k\cos(k\omega t). \tag{19}$$

The coefficients m_0, and m_k may be calculated by the formulae:

$$m_0=\frac{2}{N}\sum_{i=1}^{N}[y(t_i)T_a]; \tag{20}$$

$$m_k=\frac{2}{N}\cos(k\omega t)\sum_{i=1}^{N}[y(t_i)T_a], \tag{21}$$

where $N=T/T_a$; T_a is here the path of an incremental signal.

The input signal $x(t_i)$, found in a Fourier series by its cosine, has an analogous formula (19) in the form of [7-10]

$$x(t_i)=\frac{c_0}{2}+\sum_{k=1}^{K}c_k\cos(k\omega t), \tag{22}$$

where $c_0=\frac{2}{N}\sum_{i=1}^{N}[x(t_i)T_a]$; $c_k=\frac{2}{N}\cos(k\omega t)\sum_{i=1}^{N}[x(t_i)T_a]$.

The coefficients c_0 and c_k may be determined by the formulae:

$$c_0=\frac{m_0}{a(0)}; \tag{23}$$

$$c_k=\frac{m_k a(k\omega)}{a^2(k\omega)+b^2(k\omega)}. \tag{24}$$

In the majority of MI $\Phi(\omega)=0$, therefore formula (24) may be simplified to

$$c_k=\frac{m_k}{a(k\omega)}=\frac{m_k}{A(k\omega)}. \tag{25}$$

Consequently, the reconfigured input signal $x(t_i)$ may be demonstrated by the expression [10]

$$x(t_i)=\frac{T_a\sum_{i=1}^{N}y(t_i)}{A(0)}+\sum_{k=1}^{K}\frac{2T_a\sum_{i=1}^{N}[y(t_i)]}{A(k\omega)}\cos^2(k\omega t). \tag{26}$$

The uncertainty that occurs during an input signal reconfiguration may be one of two kinds: either methodological, associated with the replacement of the infinite limits of the summation of the signal harmonics into a finite number K, or instrumental, that creates uncertainty in the recording of the output signal $y(t)$ and the frequency characteristics of the measurement means.

Measuring input signal by a MI is indirect, since the required value $x(t)$ is determined on the basis of the results of direct measurements of the output signal of the MI $y(t)$, and the amplitude frequency characteristics (AFC) of the MI $A(\omega)$, which are functionally related. The most common values $y(t)$ and $A(\omega)$ are correlated with each other [8], since they are determined by one MI.

Taking into account the above, the following approach to the expression of combined dynamic uncertainty is proposed:

a) the value of the output signal is determined by formula (26);

b) the dynamic uncertainty is found with the help of formulae (3)-(8). In practice, the measurement results $y(t)$ are more likely to be observed only once, and the frequency response of the MI is determined a priori (during metrological investigation), or in advance during the acceptance of the MI following manufacture.

c) sensitivity coefficients are calculated by:

$$\frac{\partial x(t_i)}{\partial y(t_i)} = \frac{NT_a}{A(0)} + \sum_{k=1}^{K} \frac{2NT_a}{A(k\omega)} \cos^2(k\omega t); \tag{27}$$

$$\frac{\partial x(t_i)}{\partial |S(\mathrm{j}\omega)|} = \sum_{k=1}^{K} \frac{2T_a \sum_{i=1}^{N} [y(t_i)]}{A^2(k\omega)} \cos^2(k\omega t); \tag{28}$$

d) correlation coefficients are calculated by $r(y(t), |S(\mathrm{j}\omega|)$ by formula [8]

$$r(y(t), |S(\mathrm{j}\omega)|) = \frac{\sum_{q=1}^{G} \left(y(t_q) - \overline{y(t)}\right)\left(A(q\omega) - \overline{A(\omega)}\right)}{\sqrt{\sum_{q=1}^{G} \left(y(t_q) - \overline{y(t)}\right)^2 \sum_{q=1}^{G} \left(A(q\omega) - \overline{A(\omega)}\right)^2}}, \tag{29}$$

where G is the quantity of consistent pairs resulting from dynamic measurement;

e) the square of combined dynamic uncertainties resulting from dynamic measurement is calculated by

$$u_{Dc}^2 = \left[\left(\frac{\partial x(t_i)}{\partial y(t_i)}\right)^2 + \left(\frac{\partial x(t_i)}{\partial |S(\mathrm{j}\omega)|}\right)^2\right] u_D^2 + + 2\frac{\partial x(t_i)}{\partial y(t_i)} \frac{\partial x(t_i)}{\partial |S(\mathrm{j}\omega)|} u_D^2 r(y(t), |S(\mathrm{j}\omega)|); \tag{30}$$

f) expansion of dynamic uncertainty resulting from dynamic measurements is denoted by the formula

$$U_D = t_p(\nu_{eff}) \cdot u_{Dc}, \tag{31}$$

where $t_p(\nu_{eff})$ is the Student coefficient for a given confidence level p and the number of degree of freedom ν_{eff} [1].

Thus, the method or procedure to express dynamic uncertainty in dynamic measurement is as follows:

- obtain output readings of the MI;
- determine a priori or by experiment the frequency characteristics of the MI;
- define the input signal $x(t)$ by use of the expression (26), limiting the required number of harmonics K;
- evaluate dynamic uncertainty $u_D[y(t)]$, which is expressed by the frequency characteristics of the MI and the spectral functions of the input signal $x(t)$;
- find the combined and extended dynamic uncertainties taking into account the correlation coefficient between the results obtained of dynamic measurement and the frequency characteristics of the measurement means employed.

4. Evaluation of dynamic uncertainty in the measurement of dynamic torque in electric motors

The method of measuring dynamic torque is based on the fact that the stator of an electric motor (EM) has reactive momentum acting on it, which is equal to the torque on its rotor. The fundamental basis for implementing the measurement method is a transducer that quantifies the response of the stator of the EM under test. The transducer comprises a moving part that is attached to the balanced blade supports, and a bedplate, which is connected to the moving part by an energy sensor. The moment $M_C(t)$ that acts on the stator of the EM under test is transmitted through the movable part of the transducer, acting on the energy sensor and converted into the rotation angle $\varphi(t)$ of the mobile part of the transducer.

The differential equation that describes the means for measuring the transduced dynamic torque is given by

$$\frac{d^2\varphi(t)}{dt^2} + 2\upsilon\omega_p \frac{d\varphi(t)}{dt} + \omega_p^2 \varphi(t) = \frac{M_c(t)}{J_c}, \tag{32}$$

where J_C is the total moment of inertia of the stator of the EM and the travelling part of the measurement transducer; $\upsilon = P/\left(2\sqrt{J_c C}\right)$ is the degree of damping of free oscillations; $\omega_{\mathrm{p}} = \sqrt{C/J_c}$ is the natural frequency of free non-damped oscillations of the measurement transducer; C is the rigidity of the energy sensor; P is the damping coefficient.

The essence of the method of dynamic torque measurement is that at the instant of connection to the power supply of the rotor of the EM under test, torque $M_C(t)$ is created, which acts on the energy sensor through the measuring switch. Since the energy sensor is an resilient component, at this time it undergoes a transition process, which lasts for a duration t_1. After the completion of the transition process the EM is disconnected, at which point the supply voltage to the stator coil equalises, and the torque output of the transducer, as a result of the inertial properties of the energy sensor, reduces from $M_C(t)$ to zero during time t.

The characteristics of change in the measured dynamic moment of the EM (input signal for the instruments measuring dynamic torque) may be represented by the equation

$$M_{\mathrm{c}}(t) = M_k e^{-\nu\omega_0 t}, \tag{33}$$

where M_k is the value of the torque at the shutdown of the EM; ω_0 is the cyclical frequency of the input signal.

The transfer function of the measurement of dynamic torque of the EM is represented by the expression

$$S(s) = \frac{K}{s^2 + 2\nu\omega_{\mathrm{p}} s + \omega_p^2}, \tag{34}$$

where $K = gK_1/(J_c\omega_p^2)$ is the coefficient of proportionality of the measurement means of the dynamic torque; g is the acceleration of free decline; K_1 is the tensoresistive constant of the transducer which has a value of 489.89 N.

The transmission function of the input signal, described in expression (33), is of the form

$$X(s) = M_k / (s + \nu\omega_0). \tag{35}$$

Turning our attention to the frequency domain, and separating the real and imaginary components, as well as having conducted the relevant mathematical transformations, we obtain the following expressions for the frequency characteristics of dynamic torque measurement of the EM, and frequency characteristics in terms of equation (4), respectively:

$$S(\mathrm{j}\omega) = \frac{K}{-\omega^2 + \mathrm{j}2\nu\omega_\mathrm{p}\omega + \omega_p^2}\,; \tag{36}$$

$$\left|S(\mathrm{j}\omega)\right| = \sqrt{\frac{K^2\left(\omega^4 + 4\nu^2\omega^2\omega_p^2 - 2\omega^2\omega_p^2 + \omega_p^4\right)}{\left(\begin{array}{l}\omega^8 + 4\omega^6\omega_p^2\left(2\nu^2 - 1\right) + 2\omega^4\omega_p^4\left(3 - 8\nu^2\right) + \ldots \\ \ldots + 8\nu^2\omega^2\omega_p^4\left(2\nu^2\omega^2 + \omega_p^2\right) - 4\omega^2\omega_p^6 + \omega_\mathrm{p}^8\end{array}\right)}}\,. \tag{37}$$

The spectral function of the input signal in the frequency domain may be expressed as

$$X\left(\mathrm{j}\omega\right) = M_k / \left(\mathrm{j}\omega + \nu\omega_0\right). \tag{38}$$

The modulus of the spectral function of the input signal after the separation of the real and imaginary parts of the equation (38) and conducting the relevant mathematical transformations, takes the form

$$\left|X\left(\mathrm{j}\omega\right)\right| = \sqrt{\frac{M_k^2\left(\omega^2 + \nu^2\omega_0^2\right)}{\omega^4 + 2\omega^2\nu^2\omega_0^2 + \nu^4\omega_0^4}}\,. \tag{39}$$

Substituting in the given equation, the module of spectral functions of dynamic torque of the MI (37) and the input signal (39) in the expression (3), we obtain an equation which describes the dynamic uncertainty of the MI as dynamic torque in the frequency range from zero to infinity

$$u_D\left[y(t)\right] = \left(\frac{1}{2\pi}\int_0^\infty \frac{M_k^2\left(\omega^2 + \nu^2\omega_0^2\right)}{\omega^4 + 2\omega^2\nu^2\omega_0^2 + \nu^4\omega_0^4}\mathrm{d}\omega\right)^{\frac{1}{2}} \times$$
$$\times\left(\int_0^\infty \frac{K^2\left(\omega^4 + 4\nu^2\omega^2\omega_p^2 - 2\omega^2\omega_p^2 + \omega_p^4\right)}{\left(\begin{array}{l}\omega^8 + 4\omega^6\omega_p^2\left(2\nu^2 - 1\right) + 2\omega^4\omega_p^4\left(3 - 8\nu^2\right) + \ldots \\ \ldots + 8\nu^2\omega^2\omega_p^4\left(2\nu^2\omega^2 + \omega_p^2\right) - 4\omega^2\omega_p^6 + \omega_\mathrm{p}^8\end{array}\right)}\mathrm{d}\omega\right)^{\frac{1}{2}}. \tag{40}$$

To solve equation (40), the Maple i System mathematics package were used and when substituting nominal constants by numerical values, which are included in equation (40) (g=9.81 m/s^2); K_1=489.89 N; J_C=0.02 Nm^2; M_k=7.5 Nm; C=4000 Nm/grad; P=0.75 Nms/grad), we obtain the numerical solution of the dynamic uncertainty of the measurement of the dynamic torque of the EM, which will not exceed $u_D[y(t)]\approx 1.47\cdot 10^{-3}$ Nm when the dynamic moment after completion of the transition process t_1 (Fig. 1) from 7.5 to 0 Nm.

During the measurement of changes in the characteristics of dynamic torque of the EM shutdown, with the nominal torque value M_k=30 Nm to zero, after completing its transition process t_1, the dynamic uncertainty of measurement that is defined by the equation (40) is $u_D[y(t)]$=5.88·10^{-3} Nm.

For a graphic representation of the dynamic uncertainty of measurement of dynamic torque of an EM, we may calculate the relative dynamic using the formula

$$\widetilde{u}_D = \frac{u_D\left[y(t)\right]}{Y_\mathrm{min}}100\%\,, \tag{41}$$

where Y_min is the minimum value of the measured quantity.

Substituting our calculation using formula (40), the values of the dynamic uncertainty of the measurement of dynamic torque of the EM using a minimum torque of M_{kmin}=0.2 Nm in equation (41), we obtain the maximum relative dynamic uncertainty at the initial value of torque of the EM where M_k=7.5 Nm and M_k=30 Nm, that it does not exceed $\widetilde{u}_{D1} = 0.74\%$ and $\widetilde{u}_{D2} = 2.94\%$, respectively.

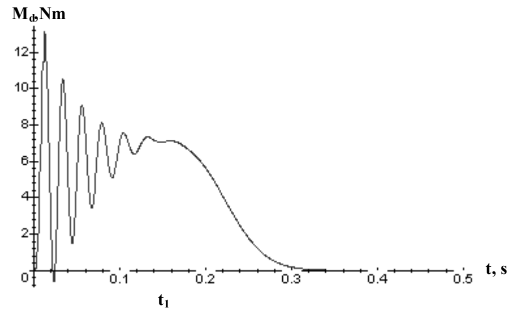

Fig. 1. Change characteristics during EM torque measurement as output of the MI
Fig. 1. Zmiana mierzonego dynamicznego momentu silnika elektrycznego

5. Conclusions

A new approach to the expression of experimental dynamic uncertainty in dynamic measurements is proposed on the basis of a priori information on the frequency characteristics of the MI and the spectral functions of the input signal. These allow us to obtain evaluations of the results of dynamic measurements to international requirements for the precision specifications. The proposed approach to estimating dynamic uncertainties can be used for measuring instruments which are characterized by dynamic circuits of any type under the operation of a stationary random input signal.

6. References

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