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HERMITE POLYNOMIALS APPLICATION FOR EXPANDING FUNCTIONS IN THE SERIES BY THESE POLYNOMIALS

Abstract

Introduction and aim: Selected elementary material about Hermite polynomials have been shown in the paper. The algorithm of expanding functions in the series by Hermite polynomials has been elaborated in the paper.
Material and methods: The selected knowledge about Hermite polynomials have been taken from the right literature. The analytical method has been used in this paper.
Results: Has been shown the theorem describing expanding functions in a series by using Hermite polynomials. It have been shown selected examples of expanding functions in a series by applying Hermite polynomials, e.g. functions exp(az), sgn(z) and z^{2p}.
Conclusion: The function f(z) can be expand in the interval $(-\infty,+\infty)$ in a series according to Hermite polynomials where the unknown coefficients can be determined from the orthogonality of Hermite polynomials

Keywords: Hermite polynomials, function of complex variable, expanding functions in a series by using Hermite polynomials.

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ZASTOSOWANIE WIELOMIANÓW HERMITE'A DO ROZWIJANIA FUNKCJI W SZEREGI WEDŁUG TYCH WIELOMIANÓW

Streszczenie

Wstęp i cel: *W pracy pokazuje się wybrane podstawowe wiadomości o wielomianach Hermite'a. W artykule opracowano algorytm rozwijania funkcji w szereg według wielomianów Hermite'a.*
Materiał i metody: *Wybrane wiadomości o wielomianach Hermite'a zaczerpnięto z literatury przedmiotu. W pracy zastosowano metodę analityczną.*
Wyniki: *W pracy pokazano twierdzenie dotyczące rozwijania funkcji w szereg według wielomianów Hermite'a. Pokazano wybrane przykłady rozwijania funkcji w szereg według wielomianów Hermite'a m.in. funkcji exp(az), sgn(z) oraz* z^{2p}.
Wniosek: *Funkcja f(z) może być w przedziale* $(-\infty,+\infty)$ *rozwinięta w szereg według wielomianów Hermite'a, gdzie nieznane współczynniki można wyznaczyć korzystając z ortogonalności wielomianów Hermite'a.*

Słowa kluczowe: *Wielomiany Hermite'a, funkcja zmiennej zespolonej, rozwijanie funkcji w szereg edług wielomianów Hermite'a.*

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1. Hermite polynomials

Definition 1.

Hermite polynomials $H_n(z)$ for complex variable z have the following form [1]-[8]:

$$H_n(z) = (-1)^n \exp(z^2) \frac{d^n}{dz^n} \exp(-z^2) \quad \text{dla} \quad n = 0,1,2,\dots. \tag{1}$$

Hermite polynomials calculated directly from definition (1) provide the system of functions:

$$H_0(z) = 1, \tag{2}$$

$$H_1(z) = 2z, \tag{3}$$

$$H_2(z) = 4z^2 - 2, \tag{4}$$

$$H_3(z) = 8z^3 - 12z, \tag{5}$$

$$H_4(z) = 16z^4 - 48z^2 - 12, \tag{6}$$

....

$$H_{n-1}(z) = (-1)^{n-1} \exp(z^2) \frac{d^{n-1}}{dz^{n-1}} \exp(-z^2), \tag{7}$$

$$H_n(z) = (-1)^n \exp(z^2) \frac{d^n}{dz^n} (-z^2), \tag{8}$$

....

Theorem 1. *(Generating function)*

Function $$w(z,t) = \exp(2zt - t^2) \tag{9}$$

is the generating function for Hermite polynomials, i.e. there is an expanding in the series [2]:

$$w(z,t) = \exp(2zt - t^2) \equiv \sum_{n=0}^{\infty} \frac{H_n(z)}{n!} t^n \quad \text{dla} \quad |t| < \infty. \tag{10}$$

2. Expanding functions in the series by using Hermite polynomials

One of the most important properties of Hermite polynomials is the ability to develop any function f (z) defined in the interval $(-\infty,+\infty)$ in a series of these polynomials [3], [7], [8]:

$$f(z) = \sum_{n=0}^{\infty} c_n H_n(z) \tag{11}$$

where $-\infty < z < +\infty$. The coefficients c_n can be determined from the orthogonality of Hermite polynomials.

Let us multiply both sides of the equation (11) by the expression $\exp(-z^2)H_m(z)$:

$$f(z)\exp(-z^2)H_m(z) = \sum_{n=0}^{\infty} c_n \exp(-z^2)H_n(z)H_m(z). \quad (12)$$

The equation (12) we integrate in the interval $(-\infty,+\infty)$ with respect to variable z:

$$\int_{-\infty}^{+\infty} \exp(-z^2) f(z)H_m(z)\,dz = \sum_{n=0}^{\infty} c_n \int_{-\infty}^{+\infty} \exp(-z^2)H_m(z)H_n(z)\,dz. \quad (13)$$

Now we calculate the integral located under the sign of the sum on the right side of the equation (13). For this purpose, in the recursive equation

$$H_{n+1}(z) - 2zH_n(z) + 2nH_{n-1}(z) = 0 \quad \text{for} \quad n = 1, 2, \ldots \quad (14)$$

we replace n by n–1 and we get:

$$H_n(z) - 2zH_{n-1}(z) + 2(n-1)H_{n-2}(z) = 0 \quad (15)$$

for $n = 2, 3, \ldots$. We multiply the obtained equation (15) by $H_n(z)$, then

$$H_n^2(z) - 2zH_{n-1}(z)H_n(z) + 2(n-1)H_{n-2}(z)H_n(z) = 0 \quad (16)$$

dla $n = 2, 3, \ldots$. Now the both sides of the equation (14) we multiply by the expression $H_{n-1}(z)$. Then we have:

$$H_{n-1}(z)H_{n+1}(z) - 2zH_{n-1}(z)H_n(z) + 2nH_{n-1}^2(z) = 0 \quad (17)$$

for $n = 1, 2, 3, \ldots$. Equation (17), we subtract by sides from the equation (15), then we have:

$$H_n^2(z) + 2(n-1)H_{n-2}(z)H_n(z) - H_{n-1}(z)H_{n+1}(z) - 2nH_{n-1}^2(z) = 0 \quad (18)$$

for $n = 2, 3, \ldots$. We multiply the abowe equation (18) by $\exp(-z^2)$, then

$$\begin{aligned} &\exp(-z^2)H_n^2(z) + 2(n-1)\exp(-z^2)H_{n-2}(z)H_n(z) + \\ &- \exp(-z^2)H_{n-1}(z)H_{n+1}(z) - 2n\exp(-z^2)H_{n-1}^2(z) = 0. \end{aligned} \quad (19)$$

Next we integrate the equation (19) in the interval $(-\infty,+\infty)$ with respect to variable z:

$$\begin{aligned} &\int_{-\infty}^{+\infty} \exp(-z^2)H_n^2(z)\,dz + 2(n-1)\int_{-\infty}^{+\infty} \exp(-z^2)H_{n-2}(z)H_n(z)\,dz + \\ &- \int_{-\infty}^{+\infty} \exp(-z^2)H_{n-1}(z)H_{n+1}(z)\,dz - 2n\int_{-\infty}^{+\infty} \exp(-z^2)H_{n-1}^2(z)\,dz = \int_{-\infty}^{+\infty} 0\,dz. \end{aligned} \quad (20)$$

With orthogonal Hermite polynomials we have that the second and third integral in the equation (20) are equal to zero. Therefore, after appropriate transformations we get:

$$\int_{-\infty}^{+\infty} \exp(-z^2)H_n^2(z)\,dz = 2n\int_{-\infty}^{+\infty} \exp(-z^2)H_{n-1}^2(z)\,dz \quad (21)$$

for $n = 2, 3, \ldots$.

By saving formula (21) successively for n = 2, 3, 4, ... we get the following sequence of equalities:

$$\int_{-\infty}^{+\infty}\exp(-z^2)H_2^2(z)\,dz = 4\int_{-\infty}^{+\infty}\exp(-z^2)H_1^2(z)\,dz = 2^{2-1}2!\int_{-\infty}^{+\infty}\exp(-z^2)H_1^2(z)\,dz, \qquad (22)$$

$$\int_{-\infty}^{+\infty}\exp(-z^2)H_3^2(z)\,dz = 6\int_{-\infty}^{+\infty}\exp(-z^2)H_2^2(z)\,dz = 24\int_{-\infty}^{+\infty}\exp(-z^2)H_1^2(z)\,dz = 2^{3-1}3!\int_{-\infty}^{+\infty}\exp(-z^2)H_1^2(z)\,dz, \qquad (23)$$

$$\int_{-\infty}^{+\infty}\exp(-z^2)H_4^2(z)\,dz = 8\int_{-\infty}^{+\infty}\exp(-z^2)H_3^2(z)\,dz = 192\int_{-\infty}^{+\infty}\exp(-z^2)H_1^2(z)\,dz = 2^{4-1}4!\int_{-\infty}^{+\infty}\exp(-z^2)H_1^2(z)\,dz, \qquad (24)$$

. . . .

$$\int_{-\infty}^{+\infty}\exp(-z^2)H_4^2(z)\,dz = 2^{n-1}n!\int_{-\infty}^{+\infty}\exp(-z^2)H_1^2(z)\,dz. \qquad (25)$$

Because

$$H_1(z) = 2z\,, \qquad (26)$$

then

$$\int_{-\infty}^{+\infty}\exp(-z^2)H_1^2(z)\,dz = 4\int_{-\infty}^{+\infty}\exp(-z^2)z^2\,dz. \qquad (27)$$

If we take into account the formula:

$$\int_{-\infty}^{+\infty}\exp(-z^2)z^2\,dz = \frac{1}{2}\sqrt{\pi} \qquad (28)$$

then we can write:

$$\int_{-\infty}^{+\infty}\exp(-z^2)H_1^2(z)\,dz = 2\sqrt{\pi}. \qquad (29)$$

Therefore:

$$\int_{-\infty}^{+\infty}f(z)\exp(-z^2)H_n(z)\,dz = 2^n n!\sqrt{\pi}\cdot c_n \qquad (30)$$

for $n = 0, 1, \dots$. From the equality (30) we obtain:

$$c_n = \frac{1}{2^n n!\sqrt{\pi}}\int_{-\infty}^{+\infty}f(z)\exp(-z^2)H_n(z)\,dz \qquad (31)$$

for $n = 0, 1, \dots$.

<u>Theorem 2</u>.

Let f (z) be an arbitrary function of the variable z defined in the interval $(-\infty,+\infty)$ and satisfying the conditions:

- f(z) in the intervals is a smooth function (i.e., has a continuous derivative in the intervals) in the finite interval $(-a,+a)$

- the integral $\int_{-\infty}^{+\infty} |z| \exp(-z^2) f^2(z)\,dz$ has a finite value,

  then the series

$$f(z) = \sum_{n=0}^{\infty} c_n H_n(z) \quad \text{for} \quad -\infty < z < +\infty \tag{32}$$

where

$$c_n = (n + \tfrac{1}{2}) \int_{-1}^{+1} f(z) P_n(z)\,dz \quad \text{for} \quad n = 0, 1, 2, \dots \tag{33}$$

is convergent and its sum is the function f(z) at each point of the inner interval (–∞,+∞) which is a point of continuity of this function [2].

Therefore by virtue of theorem 1 the series of the form (1) is convergent and its sum is a function f (z).

3. Examples of expanding functions in the series by using Hermite polynomials

Example 1.

Let us consider the following function [2]:

$$f(z) = \begin{cases} 0 & \text{for} \quad -a \le z < a, \\ 1 & \text{for} \quad z < -a \ \text{ or } \ z > a. \end{cases} \tag{34}$$

Let us note that the function (34) is even. According to the theorem 1, that function can be expand in following series:

$$f(z) = \sum_{n=0}^{\infty} c_{2n} H_{2n}(z) \tag{35}$$

where

$$c_{2n} = \frac{1}{2^{2n}(2n)!\sqrt{\pi}} \cdot \int_{-a}^{a} \exp(-z^2) H_{2n}(z)\,dz. \tag{36}$$

Let us calculate the expression $\exp(-z^2)H_{2n}(z)$. Then we multiply both sides of equations:

$$\frac{dH_n(z)}{dz} = 2n\,H_{n-1}(z) \quad \text{for} \quad n = 1, 2, \dots \tag{37}$$

and

$$\frac{d^2H_n(z)}{dz^2} - 2z\frac{dH_n(z)}{dz} + 2n\,H_n(z) = 0 \quad \text{for} \quad n = 0, 1, 2, \dots \tag{38}$$

by expression $\exp(-z^2)$. Hence, we get:

$$\exp(-z^2)\frac{dH_n(z)}{dz} = 2n\exp(-z^2)H_{n-1}(z), \tag{39}$$

$$\exp(-z^2)\frac{d^2H_n(z)}{dz^2} - 2z\exp(-z^2)\frac{dH_n(z)}{dz} + 2n\exp(-z^2)H_n(z) = 0. \tag{40}$$

Equation (39) we differentiate with respect to variable z:

$$-2z\exp(-z^2)\frac{dH_n(z)}{dz}+\exp(-z^2)\frac{d^2H_n(z)}{dz^2}+ \\ +4nz\exp(-z^2)H_{n-1}(z)-2n\exp(-z^2)\frac{dH_{n-1}(z)}{dz}=0. \tag{41}$$

From the equation (41) then subtract by sides the equation (40).

Then we have:

$$4nz\exp(-z^2)H_{n-1}(z)-2n\exp(-z^2)\frac{dH_{n-1}(z)}{dz}-2n\exp(-z^2)H_n(z)=0. \tag{42}$$

From which it follows that

$$-\left[-2z\exp(-z^2)H_{n-1}(z)+\exp(-z^2)\frac{dH_{n-1}(z)}{dz}\right]=\exp(-z^2)H_n(z). \tag{43}$$

Which means, that

$$-\frac{d}{dz}\left[\exp(-z^2)H_{n-1}(z)\right]=\exp(-z^2)H_n(z). \tag{44}$$

For the even index we have:

$$-\frac{d}{dz}\left[\exp(-z^2)H_{2n-1}(z)\right]=\exp(-z^2)H_{2n}(z). \tag{45}$$

Therefore we have:

$$c_{2n}=\frac{1}{2^{2n}(2n)!\sqrt{\pi}}\cdot\int_{-a}^{a}\left\{-\frac{d}{dz}\left[\exp(-z^2)H_{2n-1}(z)\right]\right\}dz= \\ =\frac{1}{2^{2n}(2n)!\sqrt{\pi}}\cdot\left[\exp(-z^2)H_{2n-1}(z)\right]\Big|_{-a}^{+a}=\frac{2\cdot\exp(-a^2)H_{2n-1}(a)}{2^{2n}(2n)!\sqrt{\pi}} \tag{46}$$

where n = 1, 2, … . We calculate the coefficient c_0 taking n = 0 and $H_0(z) = 1$:

$$c_0=\frac{1}{2^0\cdot 0!\sqrt{\pi}}\cdot\int_{-a}^{a}\exp(-z^2)H_0(z)\,dz=\frac{1}{\sqrt{\pi}}\cdot\int_{-a}^{a}\exp(-z^2)\,dz. \tag{47}$$

In this way we get the following expansion of the function (34) in series:

$$f(z)=\frac{1}{\sqrt{\pi}}\int_{-a}^{a}\exp(-z^2)\,dz=\frac{2}{\sqrt{\pi}}\exp(-a^2)\sum_{n=1}^{\infty}\frac{H_{2n-1}(a)}{2^{2n}(2n)!}\cdot H_{2n}(z) \tag{48}$$

where $-\infty < z < +\infty$.

Example 2.

We have to expand in series according to Hermite polynomials the function [2]:

$$f(z) = \exp(az) \tag{49}$$

where a is any real or complex number. The function (49) we expand in the series:

$$f(z) = \sum_{n=1}^{+\infty} c_n H_n(z). \tag{50}$$

where c_n coefficients are calculated from the formula:

$$c_0 = \frac{1}{2^n \cdot n! \sqrt{\pi}} \cdot \int_{-\infty}^{+\infty} \exp(-z^2 + az) H_n(z)\, dz, \tag{51}$$

or by using the generating function (9)-(10) for Hermite polynomials, then we get:

$$\exp\left(2z\frac{a}{2} - \frac{a^2}{4}\right) = \sum_{n=0}^{\infty} \frac{H_n(z)}{n!} \cdot \left(\frac{a}{2}\right)^n \tag{52}$$

for $-\infty < z < +\infty$. Whence

$$\exp(az) = \exp\left(\frac{a^2}{4}\right) \cdot \sum_{n=0}^{\infty} \frac{a^n}{2^n n!} \cdot H_n(z) \tag{53}$$

for $-\infty < z < +\infty$. We obtained the following expression in the series of the function (49):

$$f(z) = \exp\left(\frac{a^2}{4}\right) \cdot \sum_{n=0}^{\infty} \frac{a^n}{2^n n!} \cdot H_n(z) \tag{54}$$

for $-\infty < z < +\infty$.

Example 3. Let us consider the following function [2]:

$$f(z) = \mathrm{sgn}(z) = \begin{cases} +1 & \text{for} \quad z > 0, \\ -1 & \text{for} \quad z < 0. \end{cases} \tag{55}$$

We expand in series according to Hermite polynomials the function (55):

$$\mathrm{sgn}(z) = \sum_{n=0}^{\infty} c_{2n+1} H_{2n+1}(z) \tag{56}$$

in which the coefficients c are defined as follows:

$$c_{2n+1} = \frac{1}{2^{2n+1}(2n+1)!\sqrt{\pi}} \cdot \int_{-\infty}^{+\infty} \mathrm{sgn}(z) \exp(-z^2) H_{2n+1}(z)\, dz. \tag{57}$$

Taking sgn = 1, where $z > 0$ we get:

$$c_{2n+1} = \frac{1}{2^{2n+1}(2n+1)!\sqrt{\pi}} \cdot \int_{0}^{+\infty} \exp(-z^2) H_{2n+1}(z)\, dz. \tag{58}$$

Whence:

$$c_{2n+1} = \frac{1}{2^{2n+1}(2n+1)!\sqrt{\pi}} \cdot \int_{-\infty}^{+\infty} \operatorname{sgn}(z)\exp(-z^2)H_{2n+1}(z)\,dz = \\ = \frac{1}{2^{2n+1}(2n+1)!\sqrt{\pi}} \cdot \int_{0}^{+\infty} \exp(-z^2)H_{2n+1}(z)\,dz\,. \tag{59}$$

The expression $\exp(-z^2)H_{2n+1}(z)$ we get taking into account the formula (45). Then:

$$\exp(-z^2)H_{2n+1}(z) = -\frac{d}{dz}\left[\exp(-z^2)H_{2n}(z)\right]. \tag{60}$$

So what

$$c_{2n+1} = \frac{1}{2^{2n+1}(2n+1)!\sqrt{\pi}} \cdot \int_{0}^{+\infty} \left\{-\frac{d}{dz}\left[\exp(-z^2)H_{2n}(z)\right]\right\} dz = \\ = \frac{-1}{2^{2n}(2n+1)!\sqrt{\pi}} \cdot \left[\exp(-z^2)H_{2n}(z)\right]\Big|_{0}^{+\infty} = \frac{(-1)H_{2n}(0)}{2^{2n}(2n+1)!\sqrt{\pi}}\,. \tag{61}$$

Using the fact:

$$H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}\,, \tag{62}$$

then we will get:

$$c_{2n+1} = \frac{(-1)^n(2n)!}{2^{2n}(2n)!(2n+1)\sqrt{\pi}} = \frac{(-1)^n}{2^{2n}(2n+1)\sqrt{\pi}}\,. \tag{63}$$

Hence, the expansion of the function (49) in the order according to Hermite polynomials has the following form:

$$\operatorname{sgn}(z) = \frac{1}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}(2n+1)\sqrt{\pi}} \cdot H_{2n+1}(z) \tag{64}$$

for $-\infty < z < +\infty$.

<u>Example 4</u>. Let us consider the following function [4]:

$$f(z) = z^{2p} \tag{65}$$

for p = 0, 1, 2, … .

The function (65) we can expand in the following form:

$$z^{2p} = \sum_{n=0}^{p} c_{2n} H_{2n}(z) \tag{66}$$

where the coefficients c_{2n} can be defined in the formula:

$$c_{2n} = \frac{1}{2^{2n}(2n)!\sqrt{\pi}} \cdot \int_{-\infty}^{+\infty} z^{2p} \exp(-z^2) H_{2n}(z)\, dz\,. \tag{67}$$

We use the definition of Hermite polynomials to decompose the function (67) in series according to these polynomials. So we have:

$$H_{2n}(z) = (-1)^{2n} \exp(z^2) \cdot \frac{d^{2n}}{dz^{2n}} \exp(-z^2)\,, \tag{68}$$

for n = 0, 1, 2, … . Therefore:

$$c_{2n} = \frac{1}{2^{2n}(2n)!\sqrt{\pi}} \cdot \int_{-\infty}^{+\infty} \left[z^{2p} \exp(-z^2)(-1)^{2n} \exp(z^2) \cdot \frac{d^{2n}}{dz^{2n}} \exp(-z^2) \right] dz\,. \tag{69}$$

for n = 0, 1, 2, … . What follows is that

$$c_{2n} = \frac{1}{2^{2n}(2n)!\sqrt{\pi}} \cdot \int_{-\infty}^{+\infty} \left[z^{2p} \cdot \frac{d^{2n}}{dz^{2n}} \exp(-z^2) \right] dz\,, \tag{70}$$

for n = 0, 1, 2, … . Now we will use the following formula:

$$\int \left[z^{2p} \cdot \frac{d^{2n}}{dz^{2n}} \exp(-z^2) \right] dz = \frac{(2p)!}{(2p-2n)!} \int \left[z^{2p-2n} \cdot \exp(-z^2) \right] dz\,. \tag{71}$$

So we continue:

$$c_{2n} = \frac{1}{2^{2n}(2n)!\sqrt{\pi}} \cdot \frac{(2p)!}{(2p-2n)!} \cdot \int_{-\infty}^{+\infty} \left[z^{2p-2n} \cdot \exp(-z^2) \right] dz\,. \tag{72}$$

Using the definition of gamma function [1], [4]-[6]:

$$\Gamma(z) = \int_{0}^{+\infty} e^{-t} t^{z-t} dt\,, \quad \mathrm{Re}(z) > 0 \tag{73}$$

where Re (z) denotes the integer part of the complex number z, we have:

$$c_{2n} = \frac{1}{2^{2n}(2n)!\sqrt{\pi}} \cdot \frac{(2p)!}{(2p-2n)!} \cdot \Gamma(p - n + \tfrac{1}{2})\,. \tag{74}$$

Using the formula

$$2^{2p-2n} \Gamma(p - n + \tfrac{1}{2})(p-n)! = \sqrt{\pi} \cdot (2p-2n)! \tag{75}$$

we have:

$$\Gamma(p - n + \tfrac{1}{2}) = \frac{\sqrt{\pi} \cdot (2p-2n)! \cdot 2^{2n}}{2^{2p} \cdot (p-n)!}\,. \tag{76}$$

Therefore:

$$c_{2n} = \frac{1}{2^{2n}(2n)!\sqrt{\pi}} \cdot \frac{(2p)!}{(2p-2n)!} \cdot \frac{\sqrt{\pi}\cdot(2p-2n)!\cdot 2^{2n}}{2^{2p}\cdot(p-n)!}. \tag{77}$$

Finally

$$c_{2n} = \frac{(2p)!}{2^{2p}(2n)!(p-n)!}. \tag{78}$$

Thus we have the following expansion of the function f (z) in series according to Hermite polynomials:

$$z^{2p} = \frac{(2p)!}{2^{2p}} \cdot \sum_{n=0}^{\infty} \frac{H_{2n}(z)}{(2n)!\cdot(p-n)!}. \tag{79}$$

4. Conclusion

The function f(z) can be expand in the interval $(-\infty,+\infty)$ in a series according to Hermite polynomials, i.e. $f(z) = \sum_{n=0}^{\infty} c_n H_n(z)$ where the unknown coefficients c_n can be determined from the orthogonality of Hermite polynomials.

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