Control and Cybernetics

vol. **45** (2016) No. 3

Extending Lyapunov redesign method for robust stabilization of non-affine quadratic polynomial systems*

by

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Abstract: The Lyapunov redesign method is basically used for robust stabilization of nonlinear systems with an affine structure. In this paper, for the first time, by suggestion of a simple but effective idea, this approach is developed for robust stabilization of non-affine quadratic polynomial systems in the presence of uncertainties and external disturbances. In the proposed method, according to the upper bound of an uncertain term, a quadratic polynomial is constructed and with respect to the position of the roots of this polynomial, the additional feedback law is designed for robustness of the quadratic polynomial system. The proposed technique is also used for robust stabilizing of a magnetic ball levitation system. When the coil current is the control input of the magnetic ball levitation system, equations of this system are increasingly nonlinear with respect to control input and have quadratic polynomial structure. The effectiveness of the proposed control law is also demonstrated through computer simulations.

Keywords: Lyapunov redesign, non-affine, quadratic polynomial systems, magnetic ball levitation system

1. Introduction

Recently, stabilization of non-affine systems has attracted increasing attention. Indeed, the state space equations of many of physical systems (like active magnetic bearings systems, flight control etc.) have non-affine structure (Shiriaev and Fradkov, 2000; Gutierrez and Ro, 2005; Young et al., 2006; Tombul et al., 2009; Yurkevich, 2011). These systems display nonlinearities with respect to the control inputs. If the control input appears linearly in the state-space equations, the system is called affine and the Lyapunov-based stabilization methods (like sliding mode, backstepping, control Lyapunov function and etc.) are proposed

*Submitted: May 2016; Accepted: November 2016

exclusively for nonlinear affine systems. The controller design for non-affine systems is a more complicated task.

There are three general approaches in designing a controller for non-affine systems. The first approach is to transform the non-affine system into an affine structure by a nonlinear transformation and to use the stabilization methods for nonlinear affine systems (Tombul et al., 2009; Meng et al., 2014). However, for obtaining a nonlinear transformation, an accurate model of the system is needed. Therefore, the main problem of this approach is leakage of the robustness. In addition to that, such an approach has some problems that may lead to weak performance and even system instability (Tombul et al., 2009). The second approach consists in application of the intelligent control methods like fuzzy control, neural networks, etc. Since these methods are not basically model-based and they do not need any accurate system model, they are extensively used in numerous studies (Ge and Zhang, 2003; Labiod and Guerra, 2007; Liu and Wang, 2007; Chien et al., 2011; Boulkroune et al., 2012; Wang et al., 2013; Yang et al., 2015; Dai et al., 2014). The third approach is to control the system in its non-affine structure and to use the nonlinear control techniques such as CLF, passivity based control, etc. Because of the complex structure of the non-affine systems, only few papers were published with consideration of this approach (Shiriaev and Fradkov, 2000; Moulay and Perruquetti, 2005; Binazadeh et al., 2015). Although the design of robust controllers following this approach is also an important task, there are very few papers in this regard.

Among the robust stabilizing methods, Lyapunov redesign method is an effective and useful method for robust stabilization of nonlinear systems in the presence of matched uncertainties and external disturbances. This method has been basically presented for nonlinear systems with affine structure. In this method, after stabilization of the nominal system (with nominal controller), by adding an additional term to the nominal control law, robust stabilization of the uncertain system is guaranteed (Khalil, 2002). To the best knowledge of the authors, there is no work, reported in the literature, on extending this method for non-affine quadratic polynomial systems.

In this paper, an effective idea is suggested for extending the Lyapunov redesign method for robust stabilization of non-affine quadratic polynomial systems. In the proposed method, according to the upper bound of uncertainties, a quadratic polynomial is constructed and with respect to the position of the roots of this polynomial, the additional term is designed. Moreover, in order to show the applicability of the proposed approach, it is used in designing a robust stabilization control law for the magnetic ball levitation system.

Magnetic levitation systems have many applications, like super-fast magnetic train, high accuracy positioning systems etc. These systems are inherently unstable and nonlinear and have noticeable uncertainty. Magnetic ball levitation system is one of such systems. In this system, a magnetic ball is suspended by the electromagnetic force from a coil to a specific reference. When the coil voltage is considered as the control input, state-space equations of this system are in an affine form. Most of the presented control algorithms for the magnetic levitation system are based on the voltage mode control (Bonivento et al., 2003; Shen, 2002). However, some limitations of system's performance are imposed in the voltage mode control, which can be removed by considering the coil current as the control input (Gutierrez and Ro, 2005). In this case, state equations of the system have a non-affine quadratic polynomial structure and controller design is more difficult. Gutierrez and Ro (2005) proposed an SM controller for a non-affine magnetic servo levitation system based on a modified sliding condition. However, this approach is applicable in special cases and also the problem is not discussed constructively for quadratic polynomial systems. In Binazadeh et al. (2015), a new version of sliding mode controller was suggested for a class of non-affine quadratic polynomial systems and the proposed method applied for robust stabilization of the magnetic ball levitation system. In the present paper, this problem is solved by extending the Lyapunov redesign method. Moreover, computer simulations are performed to verify the effectiveness of the proposed method. Briefly, the main contributions of this paper are as follows:

(1) Extending the Lyapunov redesign method for non-affine quadratic polynomial systems.

(2) Considering the effect of external disturbances and model uncertainties.

(3) Asymptotic stabilization of magnetic ball levitation system in the current mode.

2. Problem statement

Consider the nonlinear quadratic polynomial system (1) which is a non-affine system:

$$\dot{x} = f_0(x) + f_1(x)u + f_2(x)u^2 + \omega(x, u, t) \tag{1}$$

where $x \in D \subset R$ $(0 \in D)$ is the state vector, $u \in R$ is control input, $f_i : R^n \to R$ (for i=0,1,2) are continuous vector functions, and $f_0(0) = 0$. Moreover, $\omega(x, u, t)$ is a nonlinear unknown vector function that may arise from model reduction, inaccurate modelling, external disturbances or parameter uncertainties that exist in all practical systems. It is assumed that the upper bound of $\omega(x, u, t)$ is known:

$$\|\omega(x, u, t)\| \leq \eta$$

where η is a positive constant. Because of the term u^2, this system has nonlinearity with respect to its control input and therefore is a non-affine system.

The task is to design a robust asymptotic stabilizing control law for system (1). For this purpose, first the controller for the nominal systems (i.e., system (1) with $\omega = 0$) is designed (which guarantees the asymptotic stability of the closed-loop nominal system). Then, by considering the uncertainties and external disturbances (i.e., $\omega \neq 0$), the additional term is designed to achieve the asymptotic stability for the closed-loop system (1).

Considering $\omega = 0$, the nominal system is as follows:

$$\dot{x} = f_0(x) + f_1(x)u + f_2(x)u^2. \tag{2}$$

Definition 1 *Considering the continuous positive definite Lyapunov function ($V(x) : D \to R^+$, $V(0) = 0$), the functions $\bar{a}(x)$, $\bar{b}(x)$ and $\bar{c}(x)$ are defined as follows ($\partial V/\partial x$ is a row vector):*

$$\bar{a}(x) = \frac{\partial V}{\partial x} f_2\ ,\ \ \bar{b}(x) = \frac{\partial V}{\partial x} f_1\ ,\ \ \bar{c}(x) = \frac{\partial V}{\partial x} f_0.$$

Lemma 1 *System* (2) *is asymptotically stabilizable if there exists a control law $u = \varphi(x) : D \to R$ such that $u(0) = 0$ and according to the converse Lyapunov theorem (Khalil, 2002) there exists a continuous positive definite Lyapunov function ($V(x) : D \to R^+, V(0) = 0$) so that for all $(x \neq 0) \in D$*

$$\dot{V} = \frac{\partial V}{\partial x}(f_0 + f_1\varphi + f_2\varphi^2) = \bar{a}(x)\varphi^2 + \bar{b}(x)\varphi + \bar{c}(x) < 0.$$

The following control law, which guarantees asymptotic stabilization (i.e., $\dot{V}(x) = \partial V/\partial x(f_0 + f_1\varphi + f_2\varphi^2) < 0$) of the nominal system (2) was proposed in Moulay and Perruquetti (2005):

$$u_{\text{nominal}} = \varphi(x) = \begin{cases} \frac{(-\bar{b}(x) + \sqrt{\bar{b}(x)^2 - 4\bar{a}(x)(\bar{c}(x) + \phi(x))})}{2\bar{a}(x)} & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases} \tag{3}$$

where $\phi(x)$ is a positive definite function and satisfies $\bar{b}^2(x) - 4\bar{a}(x)\bar{c}(x) \geq 4\bar{a}(x)\phi(x)$.

Now the task is to design the feedback law $\upsilon(x)$ in such a way that the robust control law $u = \varphi(x) + \upsilon(x)$ guarantees stabilization of the non-affine system (1) in the presence of $\omega \neq 0$.

3. Extending the Lyapunov redesign method for non-affine quadratic polynomial system

In this section, a novel robust approach is proposed for non-affine quadratic polynomial systems based on the Lyapunov redesign method. The additional control component $\upsilon(x)$ may be designed in such a way that the new control law $u = \varphi(x) + \upsilon(x)$ leads to robust stabilization of the nonlinear uncertain system (1). For this purpose, consider the system (1) and apply the control law $u = \varphi(x) + \upsilon(x)$. Therefore:

$$\begin{aligned} \dot{x} &= f_0(x) + f_1(x)(\varphi + \upsilon) + f_2(x)(\varphi + \upsilon)^2 + \omega(x, u, t) \\ &= \underbrace{f_0 + f_1\varphi + f_2\varphi^2}_{k_1(x)} + \underbrace{f_1\upsilon + f_2\upsilon^2 + 2f_2\varphi\upsilon + \omega(x, u, t)}_{k_2(x)}\ . \end{aligned} \tag{4}$$

Considering the Lyapunov function of the nominal system for the uncertain system, we have

$$\dot{V}(x) = \frac{\partial V}{\partial x}\dot{x} = \frac{\partial V}{\partial x}k_1(x) + \frac{\partial V}{\partial x}k_2(x)$$

whereas as shown in Moulay and Perruquetti (2005),

$$\frac{\partial V}{\partial x}k_1(x) < -\gamma\left(\|x\|\right)$$

(where $\gamma\left(\|x\|\right)$ is a class K function). The purpose is to choose $\upsilon(x)$ such that

$$\frac{\partial V}{\partial x}k_2(x) \leq 0,$$

which leads to

$$\frac{\partial V}{\partial x}k_1(x) \ + \ \frac{\partial V}{\partial x}k_2(x) < -\gamma\left(\|x\|\right)$$

and guarantees the asymptotic stability of the closed-loop uncertain system (4). Since, $\frac{\partial V}{\partial x}k_2(x)$ is a quadratic polynomial expression in terms of υ, according to the upper bound of ω, one has:

$$\frac{\partial V}{\partial x}(f_1\upsilon + f_2\upsilon^2 + 2f_2\varphi\upsilon + \omega(x,u,t)) \leq \frac{\partial V}{\partial x}(f_1\upsilon + f_2\upsilon^2 + 2f_2\varphi\upsilon) + \left\|\frac{\partial V}{\partial x}\right\|\eta. \quad (5)$$

In the above inequality, the expression in the right-hand side is a quadratic polynomial, with respect to $\upsilon(x)$. According to the position of the root of this polynomial, the additional controller $\upsilon(x)$ may be designed such that the signum of the resulting parabola for the proposed $\upsilon(x)$ be non-positive. This approach guarantees $\frac{\partial V}{\partial x}k_2(x) \leq 0$. In the following section, this idea is explained in detail by its application in the magnetic ball levitation system.

4. Robust stabilization of magnetic ball levitation system

The following figure (Fig.1) shows the scheme of the magnetic ball levitation system. In this system, magnetic ball is suspended by electromagnetic force from a coil toward a specific reference. When the coil voltage (e) is considered as the control input, state equations of this system are in an affine form. However, if the coil current (i) is considered as the control input, state equations of the system have a non-affine quadratic polynomial structure and controller design is more difficult. There are some limitations of system's performance, which are imposed in the voltage mode control, and which can be removed by considering the coil current as the control input.

State equations of the magnetic ball levitation system are as follows (Binazadeh et al., 2015):

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = g - \frac{k_0}{(l_0 + x_1 + x_{ref})^2}u^2 + \omega_1(x,u,t) \end{cases} \quad (6)$$

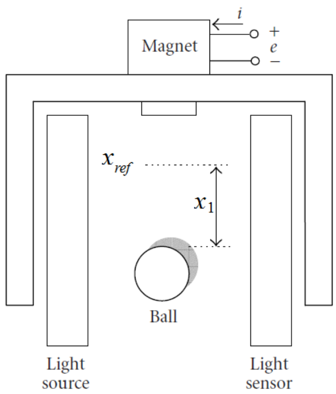

Figure 1. Schematic diagram of the magnetic ball levitation system

where x_1 is the distance between the magnetic ball and a reference position x_{ref}, x_2 is the velocity of the ball, u is the coil current, g is the earth gravity and k_0 , l_0 are nominal values of the physical parameters of the coil. Also, $\omega_1(x, u, t)$ is a nonlinear function, resulting from inaccurate modelling, parameter uncertainties or external disturbances. Suppose that $|\omega_1| \leq \eta_1$ where η_1 is a known positive constant.

Considering the equations (6) with respect to the structure of (1), one has:

$$f_0 = \begin{bmatrix} x_2 \\ g \end{bmatrix}, \quad f_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad f_2 = \begin{bmatrix} 0 \\ -\frac{k_0}{(l_0+x_1+x_{ref})^2} \end{bmatrix}, \quad \omega = \begin{bmatrix} 0 \\ \omega_1 \end{bmatrix}.$$

The Lyapunov function $V(x) = \alpha x_1^2 + x_2^2$, $\alpha > 0$ was considered in Moulay and Perruquetti (2005) for the nominal system (6) in the case when $\omega_1 = 0$ and it was shown that

$$\frac{\partial V(x)}{\partial x} k_1(x) < -\gamma\left(\|x\|\right)$$

with the following controller:

$$\begin{aligned} u_{\text{nominal}} &= \varphi(x) \\ &= -\left|l_0 + x_1 + x_{ref}\right| sgn(x_2)\sqrt{(\alpha x_1 + g)(1 + \beta sgn(x_2))/k_0} \end{aligned} \tag{7}$$

where $\beta \in \begin{pmatrix} 0 & 1 \end{pmatrix}$. Now, for $\omega_1 \neq 0$, using the Lyapunov function of the nominal system ($V(x) = \alpha x_1^2 + x_2^2$), one has:

$$\dot{V} = \frac{\partial V}{\partial x} \underbrace{(f_0 + f_2\varphi^2)}_{k_1(x)} + \frac{\partial V}{\partial x} \underbrace{(f_2\upsilon^2 + (2f_2\varphi)\upsilon + \omega\ )}_{k_2(x)}.$$

The term of the control law $\upsilon(x)$ should be chosen such that $\frac{\partial V}{\partial x}k_2(x) \le 0$:

$$\frac{\partial V}{\partial x}k_2(x) = \begin{bmatrix} 2\alpha x_1 & 2x_2 \end{bmatrix} \left(\underbrace{\begin{bmatrix} 0 \\ \frac{-k_0}{(l_0+x_1+x_{ref})^2} \end{bmatrix}}_{f_2} \upsilon(\upsilon + 2\varphi) + \underbrace{\begin{bmatrix} 0 \\ \omega_1 \end{bmatrix}}_{\omega} \right) \quad (8)$$

$$= \frac{-2k_0 x_2}{(l_0+x_1+x_{ref})^2}\upsilon^2 - \frac{4x_2\varphi k_0}{|l_0+x_1+x_{ref}|^2}\upsilon + 2x_2\omega_1.$$

By inserting φ from (7) into (8) and considering $|\omega_1| \le \eta_1$, one has:

$$\frac{\partial V}{\partial x}k_2(x) \le \frac{-2k_0 x_2}{(l_0+x_1+x_{ref})^2}\upsilon^2 + 2\eta_1 |x_2|$$
$$+\frac{4k_0|x_2|}{|l_0+x_1+x_{ref}|}\sqrt{\frac{(\alpha x_1+g)(1+\beta sgn(x_2))}{k_0}}\upsilon$$
$$= |x_2|\, k_3(x) \quad (9)$$

where

$$k_3(x) =$$
$$\frac{-2k_0 sgn(x_2)}{(l_0+x_1+x_{ref})^2}\upsilon^2 + 2\eta_1 + \frac{4k_0}{|l_0+x_1+x_{ref}|}\sqrt{\frac{(\alpha x_1+g)(1+\beta sgn(x_2))}{k_0}}\upsilon.$$

Now, according to the sign of x_2, two cases may be considered.
Case (1): Assume $x_2 > 0$, thus $k_3 = k_{31}$, where:

$$k_{31} = \underbrace{\frac{-2k_0}{(l_0+x_1+x_{ref})^2}}_{a_1(x)}\upsilon^2 + \underbrace{\frac{4k_0}{|l_0+x_1+x_{ref}|}\sqrt{\frac{(\alpha x_1+g)(1+\beta)}{k_0}}}_{b_1(x)}\upsilon + \underbrace{2\eta_1}_{c_1(x)} \quad (10)$$

and k_{31} is a quadratic polynomial in terms of υ and its coefficients are $a_1(x)$, $b_1(x)$ and $c_1(x)$. For this polynomial $\Delta_1 = b_1^2 - 4a_1c_1$ is always positive, thus k_{31} has two real roots $\alpha_1(x)$ and $\alpha_2(x)$(assume $\alpha_1(x) < \alpha_2(x)$). Choosing $\upsilon(x) > \alpha_2(x)$ or $\upsilon(x) < \alpha_1(x)$ makes k_{31} negative. One selection of $\upsilon(x)$ is as $\upsilon(x) = \alpha_2(x) + \rho$ where ρ is a positive function constant and $\alpha_2(x)$ is as follows:

$$\alpha_2(x) =$$
$$|l_0+x_1+x_{ref}|\,(\sqrt{(\alpha x_1+g)(1+\beta)/k_0} + \sqrt{[(\alpha x_1+g)(1+\beta)+\eta_1]/k_0}). \quad (11)$$

Case (2): Assume $x_2 < 0$, thus $k_3 = k_{32}$, where:

$$k_{32} = \underbrace{\frac{+2k_0}{(l_0+x_1+x_{ref})^2}}_{a_2(x)}\upsilon^2 + \underbrace{\frac{4k_0}{|l_0+x_1+x_{ref}|}\sqrt{\frac{(\alpha x_1+g)(1-\beta)}{k_0}}}_{b_2(x)}\upsilon + \underbrace{2\eta_1}_{c_2(x)}. \quad (12)$$

In this case

$$\Delta_2 = 16k_0\left[(\alpha x_1 + g)(1-\beta) - \eta_1\right]/(l_0 + x_1 + x_{ref})^2.$$

In order to make Δ_2 positive, the condition $x_1 > \left(\eta_1 k_0^2/(1-\beta) - g\right)/\alpha$ should be satisfied. Since the region of study should include the origin, thus the term $\left(\eta_1 k_0^2/(1-\beta) - g\right)$ should be negative. For small β (i.e., $\beta \approx 0$), the uncertainties with the condition $\eta_1 < g/k_0^2$ are acceptable and practical considerations show that this assumption is reasonable. In this situation, k_{32} has two real roots $\bar{\alpha}_1(x)$ and $\bar{\alpha}_2(x)$, therefore choosing $\bar{\alpha}_1(x) < \upsilon(x) < \bar{\alpha}_2(x)$ makes it negative. Hence, in this case $\upsilon(x)$ may be chosen as $\upsilon(x) = \bar{\alpha}_2(x) - \rho$. If ρ is a small enough positive constant, it can guarantee that the inequality $\bar{\alpha}_1(x) < \upsilon < \bar{\alpha}_2(x)$ is satisfied:

$$\bar{\alpha}_2(x) = \\ |l_0 + x_1 + x_{ref}|\left(-\sqrt{(\alpha x_1 + g)(1-\beta)/k_0} + \sqrt{\left[(\alpha x_1 + g)(1-\beta) - \eta_1\right]/k_0}\right). \tag{13}$$

Therefore, the robust control law $u(x)$ is obtained as;

$$u(x) = \begin{cases} \varphi(x) + \alpha_2(x) + \rho & x_2 > 0 \\ \varphi(x) + \bar{\alpha}_2(x) - \rho & x_2 < 0 \end{cases} \tag{14}$$

which guarantees robust asymptotic stability of the closed-loop system (6).

Remark 1 *In the control law* (14)*, ρ is a small enough positive constant, $\alpha_2(x)$ and $\bar{\alpha}_2(x)$ are functions of x_1, which are independent of the value of x_2. Moreover, $\varphi(x)$ depends on x_1 and the $sgn(x_2)$. Therefore, the proposed controller is not dependent on the value of x_2 and only needs to know whether the ball velocity is positive or negative, which can be established according to the changes in the value of ball displacement measurement. Therefore, for practical implementation, there is no need to dispose of the ball velocity measurement, which is a crucial point, and it is not assumed that the state vector of the system* (2) *is measurable and only the measurability of the first state is sufficient.*

5. Computer simulations

In this section, computer simulations are reported, in order to show the performance of the proposed controller. In this regard, the proposed controller is compared with the controller given in Binazadeh et al.(2015). For simulations, $k_0 = 1$, $l_0 = 0.01$, $x_{ref} = 0m, \omega_1 = 0.5\sin(10t)$, $\rho = 0.01$ and $\eta_1 = 0.5$ are chosen. Time histories of the state variables are shown in Figs. 2 and 3. These figures illustrate the ability of the proposed controller of asymptotic stabilization of the state variables in the presence of uncertainties and also the better characteristics of time response of the state variables, displayed by the proposed controller (in terms of settling time and steady state error) in comparison with the reference controller from Binazadeh et al. (2015). Moreover, in Fig. 4, the time response of the proposed control input ($u = \varphi(x) + \upsilon(x)$) and of the controller given in Binazadeh et al. (2015) are demonstrated. As it is seen, the designed control law deals effectively with the uncertainties and guarantes the asymptotic stability of the state variables with lower chattering.

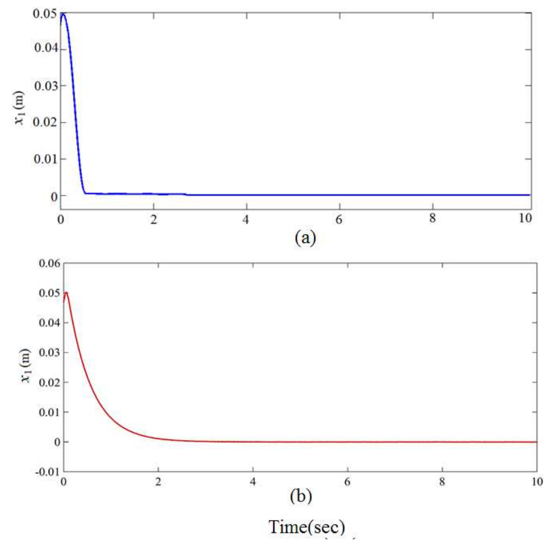

Figure 2. Time history of $x_1(t)$ in the closed-loop system: (a) with the proposed controller (b) with the controller given in Binazadeh et al. (2015)

6. Conclusions

In this paper, a robust controller was designed based on the Lyapunov redesign technique for non-affine quadratic polynomial systems. The Lyapunov redesign technique is basically considered for systems with affine structure. In this paper this method was developed for non-affine quadratic polynomial systems and practically illustrated for the magnetic ball levitation system. The simulation results reveal that the proposed controller can robustly stabilize the magnetic ball levitation system.

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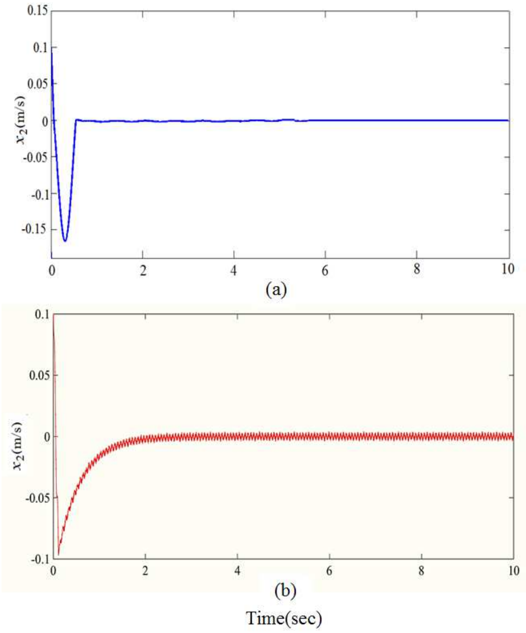

Figure 3. Time history of $x_2(t)$ in the closed-loop system: (a) with the proposed controller (b) with the controller given in Binazadeh et al.(2015)

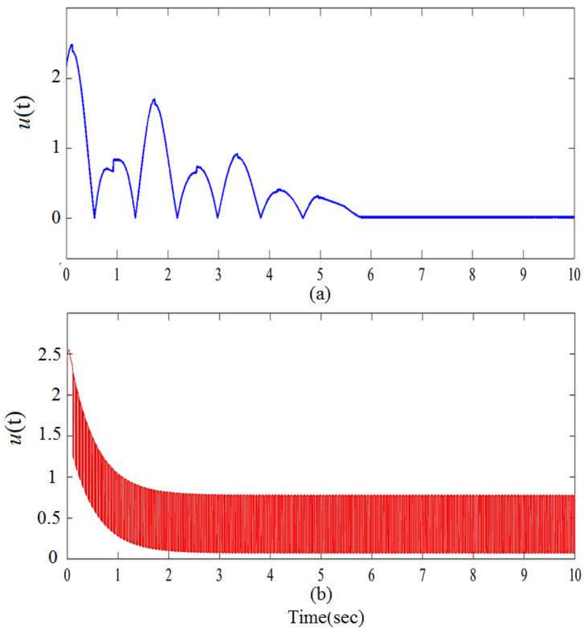

Figure 4. Time history of the control signal $u(t)$ for (a) the proposed controller (b) the controller given in Binazadeh et al.(2015)

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