

CONTINUOUS WAVELET AND HILBERT-HUANG TRANSFORMS APPLIED FOR ANALYSIS OF ACTIVE AND REACTIVE POWER CONSUMPTION

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Abstract

Analysis of power consumption presents a very important issue for power distribution system operators. Some power system processes such as planning, demand forecasting, development, etc., require a complete understanding of behaviour of power consumption for observed area, which requires appropriate techniques for analysis of available data. In this paper, two different time-frequency techniques are applied for analysis of hourly values of active and reactive power consumption from one real power distribution transformer substation in urban part of Sarajevo city. Using the continuous wavelet transform (CWT) with wavelet power spectrum and global wavelet spectrum some properties of analysed time series are determined. Then, empirical mode decomposition (EMD) and Hilbert-Huang Transform (HHT) are applied for the analyses of the same time series and the results showed that both applied approaches can provide very useful information about the behaviour of power consumption for observed time interval and different period (frequency) bands. Also it can be noticed that the results obtained by global wavelet spectrum and marginal Hilbert spectrum are very similar, thus confirming that both approaches could be used for identification of main properties of active and reactive power consumption time series.

Keywords: active and reactive power consumption, continuous wavelet transform, empirical mode decomposition, Hilbert-Huang transform.

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1. Introduction

Analysis of power consumption behaviour presents a very important issue for power distribution system operators [1]. Activities like operation planning, demand forecast, integration of distributed generators, connection of new customers, electricity trading etc. require among others the complete understanding of power consumption behaviour over time [2]. In the literature one can find different techniques for analysing the time series. In the context of analysing power consumption data, different methodological approaches are being applied in order to determine the interrelationships between power consumption and some factors such as air temperature variations, changes in Gross Domestic Product (GDP), etc. One of the most commonly used techniques for the time-frequency analysis of time series is continuous wavelet transform (CWT) [3], and its practical application on active power consumption time series can be found in [4]. On the other hand, Hilbert-Huang transform (HHT) is a somewhat younger technique and today one of the most popular approaches for analysing non-stationary and non-linear signals and time series [9–12]. HHT is being applied in almost all fields of science [12–17]. HHT is composed of two parts: (i) Empirical Mode Decomposition (EMD) and (ii) Hilbert spectral analysis. By using EMD, signal or time series is decomposed on intrinsic mode functions (IMFs), where each IMF represents a simple harmonic function whose amplitude and frequency can be functions of time. Final presentation of signal through Hilbert spectrum is the

result of time-frequency-energy distribution which enables better understanding of analysed phenomenon.

In this paper, two different time-frequency techniques (CWT and HHT) are applied for analysis of hourly values of active and reactive power consumption from a real power distribution transformer substation in urban part of Sarajevo city. Using the CWT (wavelet power spectrum and global wavelet spectrum) and HHT approaches some properties of analysed time series were determined, providing better understanding of local power consumption behaviour during observed time interval. As a general rule, it can be stated that CWT and HHT approaches are often applied for the analysis of nonstationary time series and study of different phenomena. According to the knowledge of the authors and available literature, this is the first paper that presents the application of HHT approach in the analysis of active and reactive power consumption, which are also nonstationary time series. HHT approach provides the study of active and reactive power consumption variability (diurnal, weekly, monthly cycle, etc.) over observed time horizon, which provides useful information about the behaviour of analysed electricity consumption. By comparison with the CWT approach it has been shown that both approaches give similar results and thus can be used for the analysis of electricity consumption. This paper is organised as follows. The time series used for practical analyses are introduced in Section 2. They represent the real hourly values of active and reactive power consumption measured on 110/x kV/kV transformer station situated in the urban part of Sarajevo city. In Section 3, CWT and HHT approaches are explained in short. The results and discussions are presented in Section 4, while Section 5 brings out the conclusions of this paper.

2. Active and reactive power consumption time series

The properties of power consumption on some area depend on several factors such as ambient temperature and space heating technology, customs and standard of living, economic development, types of industrial consumers etc. Consumption of active and reactive power varies considerably for different seasons, which can largely be due to temperature variations. The peaks of active power consumption are in winter season while in recent years peaks of reactive power consumption are measured in summer days as a consequence of widespread use of air conditioning devices. However, in some areas such as touristic settlements, situated on the coast of Adriatic Sea, peaks of both active and reactive power consumption can be identified during the summer season. This indicates that each area has its own specificities regarding power consumption with which engineers working in power distribution companies need to be familiar with. In this paper, for the practical analysis of active and reactive power consumption time series the hourly measurements from 110/x kV/kV Sarajevo 5 transformer substation, for year 2011, were selected (Fig. 1).

From this transformer station a part of old city of Sarajevo is being supplied, which includes mostly households and small industry (services). From the bigger customers supplied by this transformer substation the Clinical Centre of University of Sarajevo can be singled out, with number of specific consumer devices. From the Fig. 1 it can be clearly seen that consumption of both active and reactive power was the highest during the winter season, after which consumption of active power gradually decreases. However, the consumption of reactive power stays relatively high during the summer months because of the intensive use of air conditioning devices. Consumption of reactive power should however be given special attention because it significantly impacts the voltage profiles in the network and can cause increased losses of power. Basic information about these time series is shown in Table 1, and both of these time series have 8.760 samples.

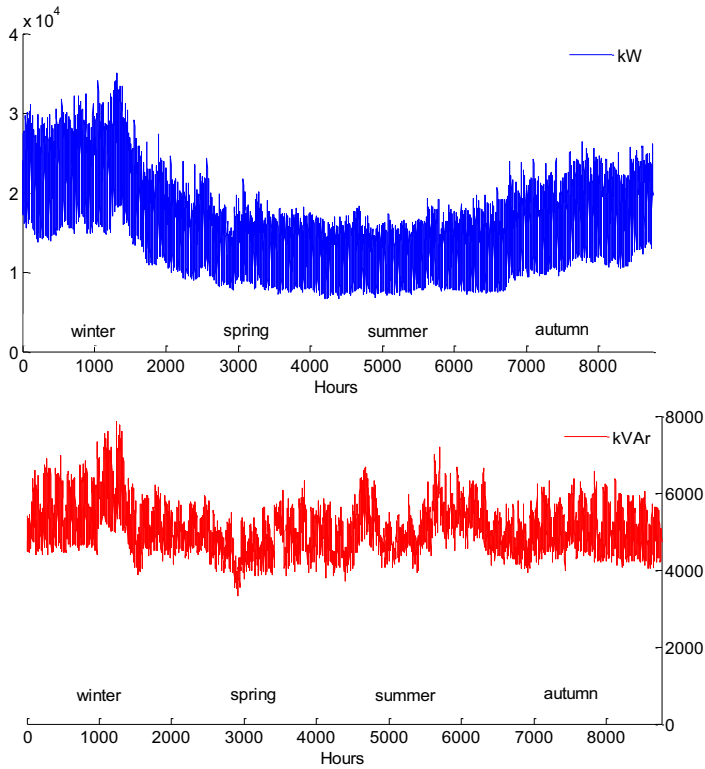


Fig. 1. Hourly values of active and reactive power consumption measured in TS 110/x kV/kV Sarajevo 5.

Table 1. Basic data about the time series from Fig. 1.

	Active power [kW]	Reactive power [kVAr]
Minimum value	6.593,7	3.320,0
Average	16.774,2	5.119,2
Maximum value	35.048,2	7.840,0

3. A brief description of applied approaches

As previously noted, CWT and HHT approaches are relatively young mathematical techniques which have found its application in almost all areas of science. CWT is a very successful approach for the analysis of time series [3-4], and is defined as:

$$W_{x,\psi}(\tau, s) = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{s}} \psi^* \left(\frac{t - \tau}{s} \right) dt, \quad (1)$$

where $x(t)$ represents a signal or time series, asterisk denotes the conjugate complex value, τ and s correspond to the time dimension and the scale dimension, respectively, and $\psi(t)$ represents the Morlet wavelet function [3-4].

Equation $WPS_{x,\psi}(\tau, s) = |W_{x,\psi}(\tau, s)|$ represents the wavelet power spectrum (WPS) or local variance of signal or time series x , while time-averaged wavelet spectrum represents global wavelet power spectrum (GWPS) [3-4]. The WPS and GWPS were both used for gaining insight into properties of active and reactive power consumption time series from Fig. 1.

On the other hand, HHT approach is composed of two parts: empirical mode decomposition (EMD) and Hilbert spectral analysis. Based on [9–17], this approach is described in short as follows:

Using the EMD method, signal or time series is decomposed into IMFs. The signal can be composed of several IMF components, where each one has the following properties:

- (i) in the whole data set the number of extrema (minima and maxima) and the number of zero crossings must be equal or differ by at most one,
- (ii) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

IMFs represent simple functions whose amplitude and frequency changes over time. After determining the local extrema of analysed signal, connecting local maxima and local minima with cubic spline functions and thus obtaining the upper and lower envelope ($x_{up}(t)$ and $x_{low}(t)$), the mean value of these envelopes is then defined as $m_1(t) = (x_{up}(t) + x_{low}(t))/2$. The difference between the original signal $x(t)$ and the mean value of envelopes m_1 gives $h_1 = x(t) - m_1$. In case h_1 does not satisfy the criteria for IMF functions noted earlier, the procedure is repeated k times: $h_{11} = h_1 - m_{11}$, i.e. by repeating the iterations we get an IMF: $h_{1k} = h_{1(k-1)} - m_{1k}$; $c_1 = h_{1k}$. The isolated IMF function is then subtracted from the original signal $x(t)$ and then from the residue $r_1 = x(t) - c_1$ other IMF functions are found using the same procedure. At the end of the process the sum of all the IMFs and residue should be equal to the original signal:

$$x(t) = \sum_{j=1}^n c_j + r_n. \tag{2}$$

After obtaining the IMF functions, the instantaneous frequencies are determined by using the Hilbert transform:

$$y(t) = H[x(t)] = \frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{x(\tau)}{t - \tau} d\tau, \tag{3}$$

with $p.v.$ as the Cauchy principle value of the integral [9-17]. The final analytical signal after the application of Hilbert transform is:

$$z(t) = x(t) + iy(t), \tag{4}$$

where:

$$a(t) = \sqrt{x(t)^2 + y(t)^2} \quad \text{and} \quad \theta(t) = \arctan\left(\frac{y(t)}{x(t)}\right),$$

represent the instantaneous amplitude ($a(t)$) and the instantaneous phase angle ($\theta(t)$), and the instantaneous frequency is calculated as:

$$\omega(t) = \frac{d\theta(t)}{dt}. \tag{5}$$

After the Hilbert transform is applied on every IMF component, the original signal can be expressed as the real component of expression:

$$x(t) = \Re\{\sum_{j=1}^n a_j(t) \cdot \exp(\int \omega_j(t) dt)\}. \tag{6}$$

Detailed mathematical description of HHT approach can be found in Refs. [9-17], based on which this short overview was done.

4. Results and discussion

This section presents the results of the analysis of time series from Fig. 1, applying above mentioned approaches, with appropriate discussion. Based on (1) and [3-4], main properties of active power consumption time series identified by CWT approach can be seen in Fig. 2 (top), where the colour codes for the power ranges are between blue (low power) and red (high power). Active power consumption time series represents a dominantly daily time series (around 0.04 cycles per hour or 24 hours period). The variations of active power consumption in 24 hour period are more intensive during the autumn and winter in comparison to spring and summer seasons (Fig. 2 (top)). Significant variations of active power can also be identified around 0.08 cycles per hour or 12-hour period, especially for the seasons: end of winter, spring and autumn. In other words, 0.08 cycles per hour or 12-hour variations of active power consumption in these seasons are much higher than the variations in summer season. This can be a consequence of higher daily temperature variations in these seasons in comparison to summer and winter seasons but also a consequence of length of day and the prolonged use of lighting. Significant variations (wavelet power spectrum and global wavelet spectrum- Fig. 2 (top)) are identified around 0,004 cycles per hour or for the period of 256 hours or 10 days during the winter season, which is a consequence of extremely low temperatures and higher consumption during this period. This can also be concluded from Fig 1.

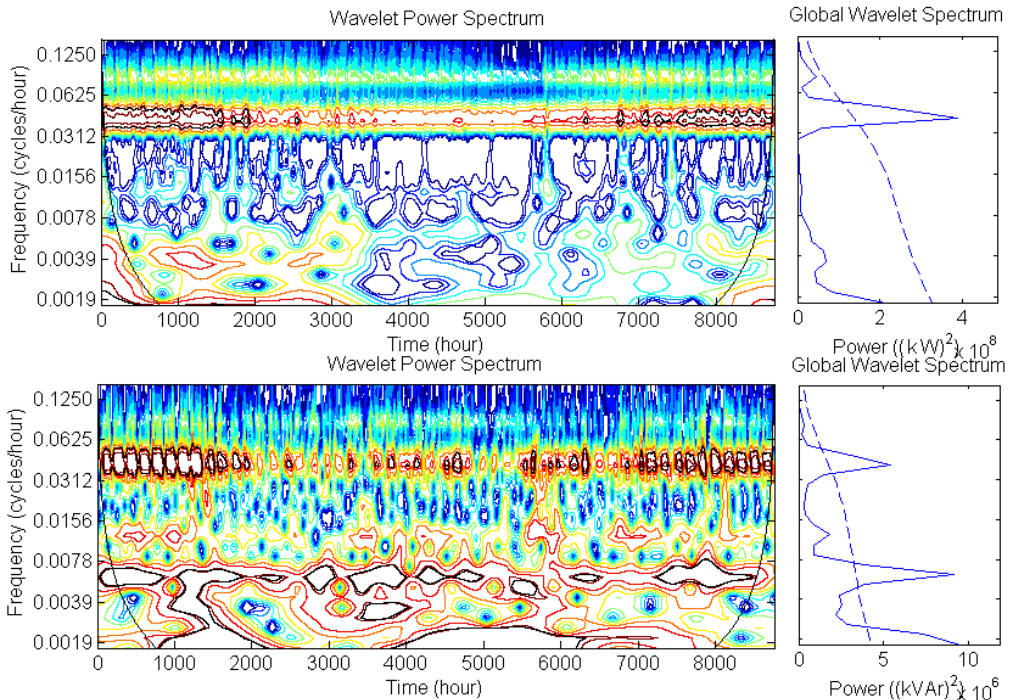


Fig. 2. Wavelet power spectrum and Global Wavelet spectrum for time series of active power consumption (top) and reactive power consumption (bottom).

The Wavelet power spectrum and Global wavelet spectrum of reactive power consumption time series are shown in Fig. 2 (bottom). Several conclusions can be drawn regarding the main properties of this time series. The properties of reactive power consumption time series are significantly different from the properties of active power consumption time series. Reactive power consumption time series is dominantly weekly time series (global wavelet spectrum maximum is around 0.006 cycles per hour or for the period of 168 hours or 7 days). This is a consequence of significantly different reactive power consumption during weekends or holidays, which gives this characteristic to the time series (lower intensity of use of devices in industry and service sectors, which have higher consumption of reactive energy). Similar to the active power consumption, reactive power consumption also has significant daily variations (around 0.04 cycles per hour or 24 hour time band), with more significant variations during winter and second half of autumn. Some variations in the time band around 0.04 cycles per hour or around 70 hours or three days were also identified for the analysed period, but with significantly lower value of global wavelet spectrum. On the other hand, unlike for the active power consumption time series, significant variations of around 0,08 cycles per hour or for the period of 12 hours for reactive power consumption time series were not identified.

Following the procedure explained in Section 2 [9–17], the EMD and HHT approach was applied on active power consumption time series from Fig. 1 and the results are shown on Figs. 3 and 4. From the IMF functions, we can draw several very interesting conclusions about the behaviour of power consumption over time and for different periods, where the first IMFs represent the higher frequencies. Because of the large number of pictures, instantaneous frequencies of individual IMFs will not be shown separately. The last IMF from the Fig. 3 represents the residuum and it describes the trend of the time series. The amplitudes of other IMFs (c1-c9) from Fig. 3 vary between ± 1000 kW and ± 5000 kW, which points out to the different variations of consumption during the course of the year in different frequencies/periods (cycles per hour). First IMF (c1) describes active power consumption variations around 0.25 cycles per hour (4 hours), and it is evident that these variations are the highest in the winter period (Fig. 3). Somewhat larger amplitudes of variations are identified during the summer and autumn seasons for 0.155 cycles per hour (6.5 hours), which is clearly evident from IMF2 function (Fig. 3). It is evident that IMF 5 (c5) has the highest amplitude of variations, with more significant changes identified during the autumn and winter seasons. Frequency or cycles per hour of this IMF is around 0.04 Hz or 25 hours which points out to the daily variations of active power consumption. Third IMF (c3), with amplitude of variations around ± 4000 kW and frequency around 0.1 Hz or 10 hours, points out to the variations in this frequency band, with the most significant variations during the winter, spring and autumn seasons. The same has also been identified from the results of CWT approach shown in Fig. 2. HHT results of active power consumption time series plotted in time-frequency plane and Hilbert marginal spectrum are presented in Fig. 4. Hilbert marginal spectrum is created as temporal sum of instantaneous amplitudes for given frequencies (see [14]). Because for the higher frequencies significant amplitudes of variations were not identified, for the reason of better visibility Fig. 4 is zoomed up to 0.1 cycles per hour (10 hours). It is clear that the highest amplitude can be identified for the last IMF (residue), which represents the trend of the time series. However, amplitudes around 0.04 cycles per hour or 25 hours and amplitudes around 0.09 cycles per hour or 11 hours confirm the results obtained by CWT approach shown in Fig. 2 and point out the fact that the active power consumption time series has the properties of dominantly daily time series.

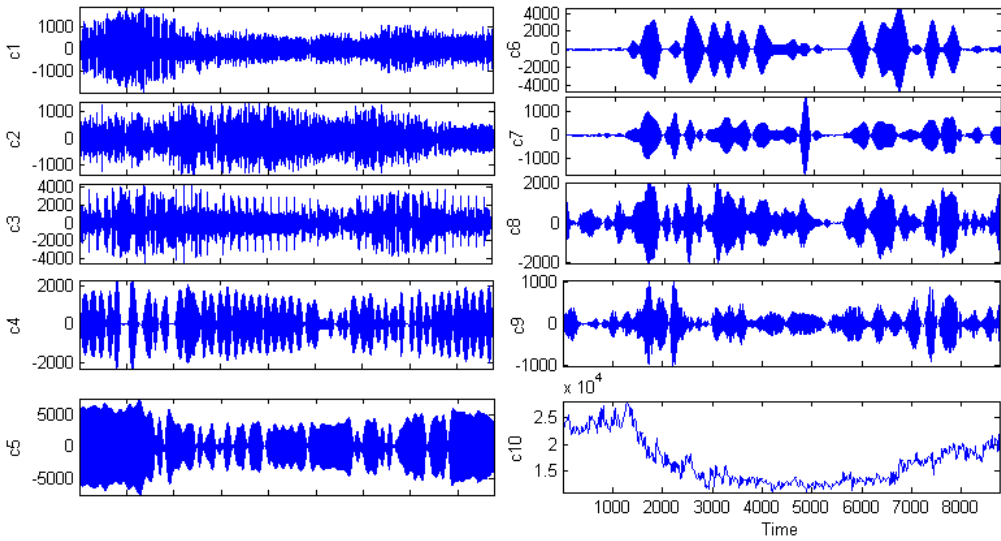


Fig. 3. IMF components of active power consumption time series.

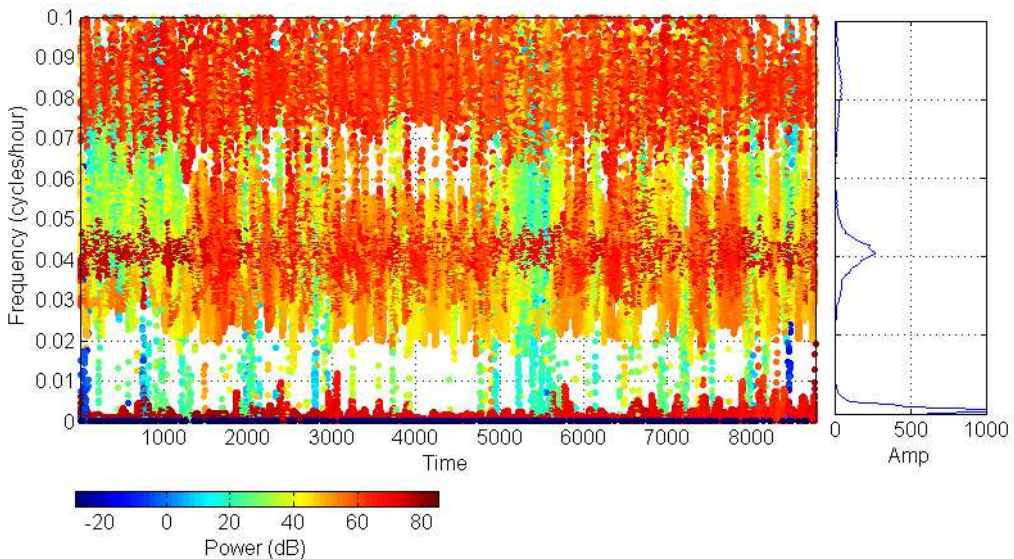


Fig. 4. HHT results of active power consumption time series plotted in time-frequency plane and Hilbert marginal spectrum.

In the same way as for the time series of active power consumption, the results for the reactive power consumption time series are presented in Figs. 5 and 6. Amplitude variations of individual IMF functions are shown in Fig. 5 and for some IMFs their values vary in the range from ± 100 kVar to ± 1000 kVar. HHT results of reactive power consumption time series plotted in time-frequency plane and Hilbert marginal spectrum are presented in Fig. 6. From the identified peaks it is clear that for this time series also the results of the HHT approach confirm the results of the CWT approach shown in Fig. 2. Dominant component is identified for the 0.005 cycles per hour or around 200 hours which represents weekly variations of reactive power consumption. It can be said that they are constant during the whole observed period.

Apart from this frequency, frequencies around 0.013 cycles per hour or 77 hours (around 3 days) and frequencies around 0.045 cycles per hour which represent daily variations of analysed time series (intensity is significantly higher in winter season), can also be identified. In this way the results of the CWT analysis shown in Fig. 2 are confirmed.

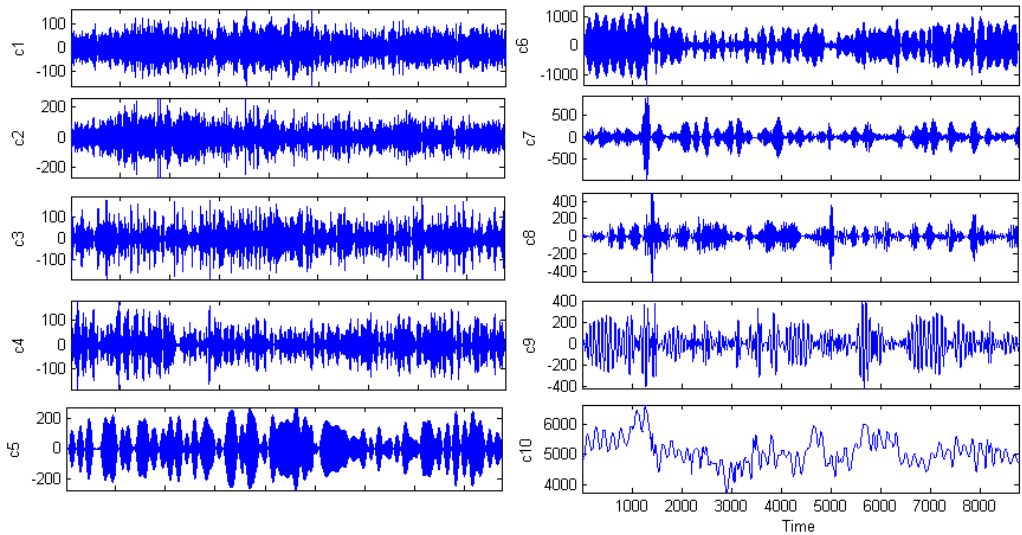


Fig. 5. IMF components of reactive power consumption time series.

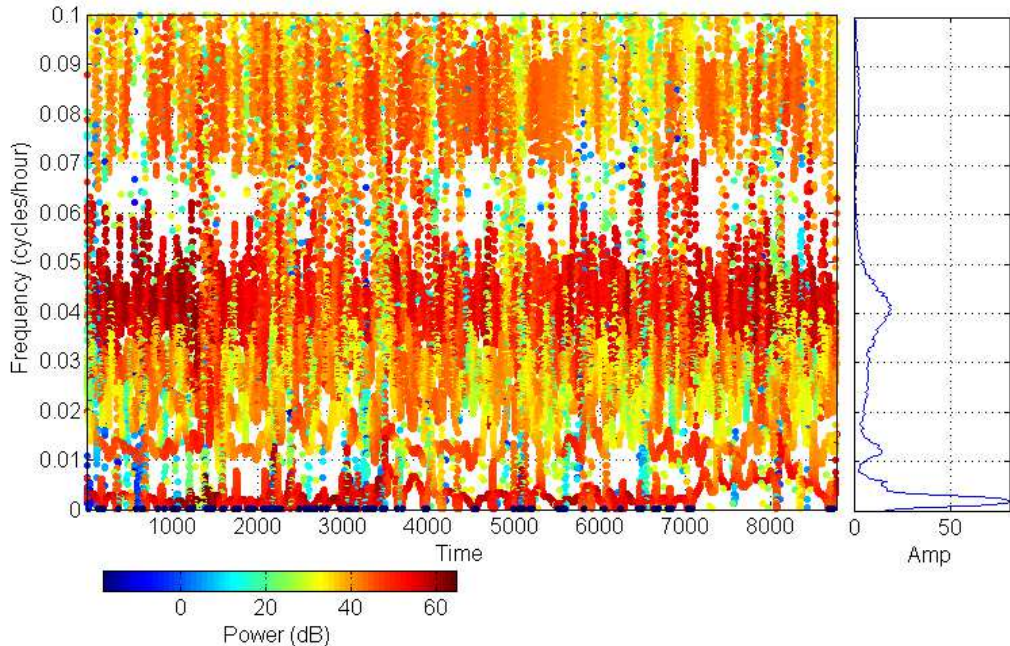


Fig. 6. HHT results of reactive power consumption time series plotted in time-frequency plane and Hilbert marginal spectrum.

There is also the possibility of further analysis of individual IMF functions which in practical terms leaves room for engineers to obtain additional information on behaviour of time series i.e. power consumption, for different periods throughout the year.

The knowledge about the variability of active and reactive power consumption in terms of dominant oscillatory components and their intensities during different period bands (days, weeks, months or seasons) can be very useful for the distribution system operator (DSO) in different operation processes and can lead to more accurate operation planning or more favourable electricity trading. In other words, by obtaining the information about the variability of electricity consumption in different periods during the analysed time period, DSO can have useful information about the behaviour of consumption on certain area that can be practically used in usual business processes. On the analysed example it has been shown that certain variations (daily, weekly, monthly) are more intensive during certain months or seasons, which ultimately has a significant impact on the electricity distribution network (higher demand, higher electricity losses, bigger voltage drops etc.).

3. Conclusion

In this paper CWT and HHT approaches have been applied for practical analysis of active and reactive power consumption time series measured in the real power distribution network of city of Sarajevo. The results showed that these approaches can provide lots of useful data when used for practical analysis of power consumption. The CWT approach is very useful for time series analyses and provides (on one picture) large number of information about studied phenomenon. Also, EMD and HHT approach are very useful techniques for time series analyses. In the context of studied phenomenon in this paper, IMF functions can enable insight into variations of power consumption and providing overview on amplitude variations over time. On the other hand, HHT time-frequency approach can provide information about the characteristics of time series. In analysed case it has been concluded that active power consumption time series is dominantly daily time series (daily variations are dominant), while weekly variations are dominant for reactive power consumption time series. The results obtained with HHT approach were compared with CWT approach and they confirmed the results obtained with CWT approach.

4. References

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