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Implementation of the Least Squares Method in Determining the Parameters of Knothe's Theory**

1. Introduction

Mining causes adverse effects in the environment and infrastructure located on the surface which affects the quality of life of residents. In order to determine the adverse effects observed on the surface, caused by mining operations, predictions of mining subsidence are executed. Attitude about forecasting continuous deformation is very diverse. However, regardless of a mathematical model used to solve a problem it is always necessary to determine the correct parameters, which will be responsible for a computing process control.

The parameters reflect methods of mining and geological conditions responsible for the deformation of a rock mass. Therefore, knowledge of the correct determination of the parameters as well as its practical implementation and application is necessary.

2. Prediction of Mining Impacts

In Polish underground mines the model of Knothe [6] and its expanded versions [4], are used to predict land subsidence. This model belongs to the group of geometric-integral methods that use influence functions.

The calculation of subsidence in these models is based on the assumption that the mining of an elementary volume of deposit dV causes elementary reduction of dw on the surface. Using the principle of superposition of influence, a subsidence shall be calculated as the sum of the influences from each of the elementary part of the deposit.

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2.1. The Description of Knothe's Model

Vertical displacement of the selected area can be calculated using the formula proposed by Knothe (1):

$$w = ag \iint_P f(x,y)dxdy \tag{1}$$

where:

a – coefficient of exploitation,
g – thickness of excavation,
P – geometry of exploitation,
$f(x,y)$ – influence function, which takes the form:

$$f(x,y) = \frac{1}{r^2}\exp\left(-\pi\frac{x^2+y^2}{r^2}\right) \tag{2}$$

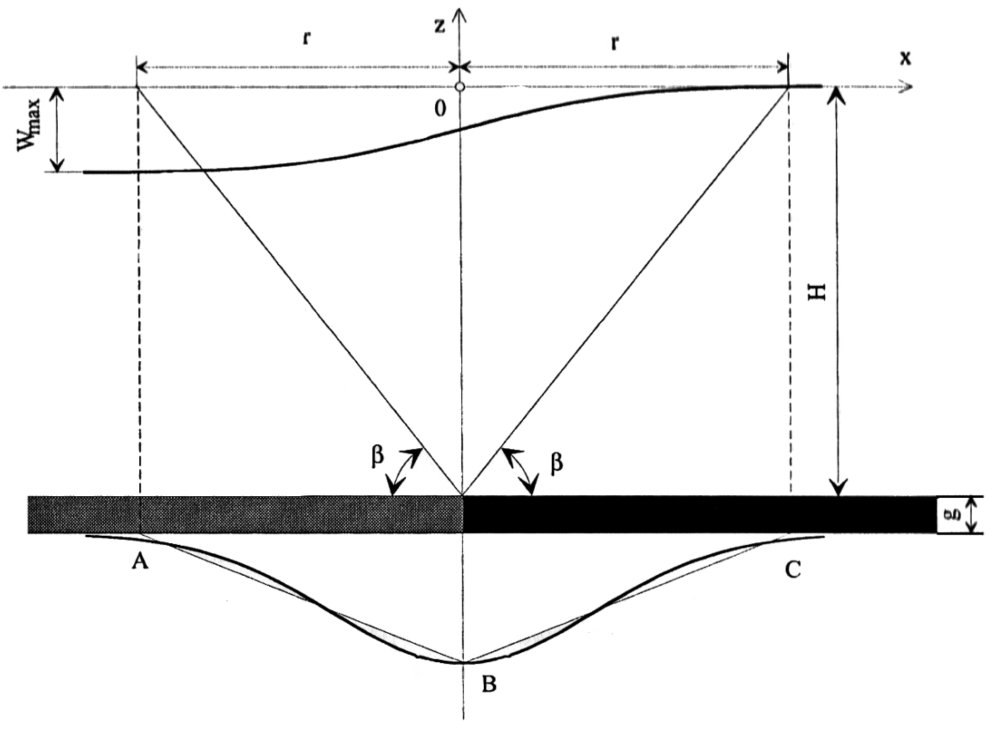

Fig. 1. Radius of influence *r* and angle of the main influences range β in Knothe's theory

Source: [4]

Radius of influence (r) in the formula (2) is related to the depth of exploitation (H) and physico-mechanical properties of the rock mass (tgβ) defined by the relation:

$$r = \frac{H}{\operatorname{tg}\beta} \tag{3}$$

where β – angle of the main influences range (according to Knothe's model).

Figure 1 shows relation between the radius of influence r and the angle β.

2.2. Parameters of the Model

The basic parameters of the Knothe's model are coefficient of exploitation a and the angle of main influences range β, the value of which is given by the tangent of the angle. The value β is responsible for visualizing mining and geological conditions prevailing in the area of mining. Therefore, there is a need to define the method of determining the parameters a and tgβ that control the computing process and at the same time are responsible for the reliability of the performed subsidence prediction.

In literature one can find various methods of determining parameters. Extensive research on the topic presents Kwinta in his scientific output [7, 8] and the others [1–3, 5, 12, 13, 15]. According to the authors the correct criterion for the designation of the parameters a and tgβ is the value of the coefficient of variation (M_w) of 5%. This value is defined as the quotient of the standard deviation and the maximum absolute value of the deformation index.

3. Observational Equations

The most common method of fitting empirical data to model data is the method of the least squares. Proposed in the article, its implementation is the use of observational equations. Necessary informations needed to create equations are the values of observed (w^{obs}) and theoretical vertical displacements (w^{theor}). Based on the theory presented in the first part of the paper, it can be calculated the vertical displacement of the set, initial values of the parameters a and tgβ. It can be determined the observed reduction by geodetic measurements and compared with the theoretical values by the equation:

$$w^{obs} - w^{theor} = \frac{\partial w^{theor}(a, \operatorname{tg}\beta)}{\partial a} da + \frac{\partial w^{theor}(a, \operatorname{tg}\beta)}{\partial \operatorname{tg}\beta} d(\operatorname{tg}\beta) + \delta \tag{4}$$

The function w^{theor} is related to the calculation of vertical displacements, and has been adopted as [6]:

$$w^{theor} = \frac{ag}{H^2} \operatorname{tg}^2\beta \iint_P \exp\left(-\pi \frac{(x-s)^2 + (y-t)^2}{H^2} \operatorname{tg}^2\beta\right) dxdy \tag{5}$$

In the observational equations we should designate derivatives of the variables a and tgβ:

$$\frac{\partial w^{\text{theor}}(a,\operatorname{tg}\beta)}{\partial a}=\frac{g}{H^2}\operatorname{tg}^2\beta\iint_P \exp\left(-\pi\frac{\left(x-s\right)^2+\left(y-t\right)^2}{H^2}\operatorname{tg}^2\beta\right) \tag{6}$$

$$\begin{gathered}\frac{\partial w^{\text{theor}}(a,\operatorname{tg}\beta)}{\partial \operatorname{tg}\beta}=2\frac{ag}{H^2}\operatorname{tg}\beta\iint_P \exp\left(-\pi\frac{\left(x-s\right)^2+\left(y-t\right)^2}{H^2}\operatorname{tg}^2\beta\right)dxdy-\\ -\frac{2\pi ag}{H^4}\operatorname{tg}^3\beta\iint_P\left(\left(x-s\right)^2+\left(y-t\right)^2\right)\exp\left(-\pi\frac{\left(x-s\right)^2+\left(y-t\right)^2}{H^2}\operatorname{tg}^2\beta\right)dxdy\end{gathered} \tag{7}$$

Therefore, the observational equations for all observations may be recorded using the matrix equation [10]:

$$A\hat{X}=L\rightarrow\hat{X}=\left(A^T A\right)^{-1}A^T L \tag{8}$$

where:

$$A=\begin{bmatrix}\dfrac{\partial w^{\text{theor}}(a,\operatorname{tg}\beta)}{\partial a} & \dfrac{\partial w^{\text{theor}}(a,\operatorname{tg}\beta)}{\partial \operatorname{tg}\beta}\end{bmatrix} \tag{9}$$

$$L=\left[w^{\text{obs}}-w^{\text{theor}}\right] \tag{10}$$

$$\hat{X}=\left[da \quad d(\operatorname{tg}\beta)\right] \tag{11}$$

The calculated matrix $\hat{X}$ is the value of corrections to be adopted for the set, initial value of a and tgβ.

4. Implementation of Calculation Algorithm

Surface integral in the formulas (5), (6) and (7) is not effective. Therefore requires the application of numerical methods for its designation. From amount of possibilities the Scilab 5.4.1 program has been chosen to implement at calculation algorithm for computing the surface integral. Created script allows to determine not only the parameters of the theory, but also the values of the standard deviations of parameters σ_a, σ_{tg} and the correlation coefficient R based on matrix equations. As with other programs [7, 12, 14] for the match criterion coefficient of variation M_w is assumed.

Main window of the program shows Figure 2.

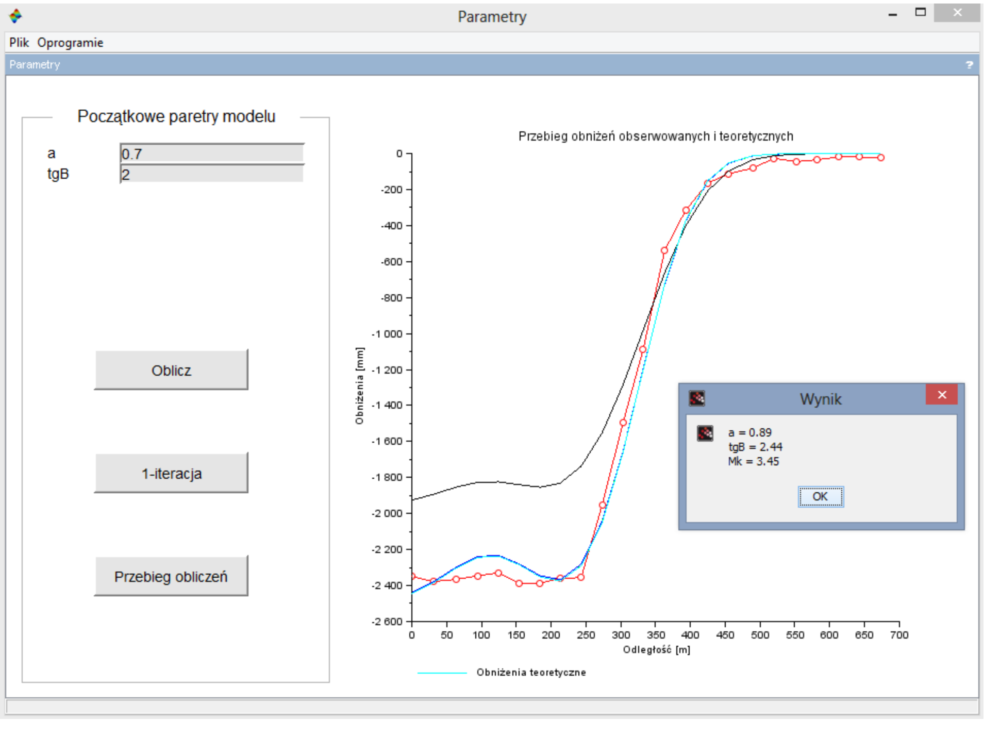

Fig. 2. The main window of the program determining parameters of the Knothe theory

4.1. Stability of Calculations

The initial values of parameters *a* and tgβ are used to determine the matrix corrections $\hat{X}$. Calculations are performed in an iterative manner until the value of da and dtgβ do not exceed assumed tolerances 0.001. The question arises whether or not the initial values affect the final values obtained? In order to verify this problem, set the parameters of theory for the accepted mine region, where there was a line of observation. In every subsequent calculation cycle a different initial values *a* and tgβ have been assumed. The second assumption by the calculation was a limited value of corrections, which cannot exceed 0.0005.

The results obtained are presented in Table 1.

Table 1. The results of calculations of parameters for various initial values in the selected area of mining

No.	Initial values (a, tgβ)	a	σ_a	tg	σ_{tg}	M_w	R	Number of iterations
1	0.70 2.00	0.52	0.01	1.33	0.03	2.4	0.89	4
2	0.30 1.00	0.52	0.01	1.33	0.03	2.4	0.89	4
3	0.30 4.00	0.52	0.01	1.33	0.03	2.4	0.89	5
4	0.90 4.00	0.52	0.01	1.33	0.03	2.4	0.89	6
5	0.90 1.00	0.52	0.01	1.33	0.03	2.4	0.89	5
6	1.20 7.00	0.52	0.01	1.33	0.03	2.4	0.89	7
7	0.30 7.00	0.52	0.01	1.33	0.03	2.4	0.89	5
8	8.00 10.00	0.52	0.01	1.33	0.03	2.4	0.89	6
9	8.00 1.00	0.52	0.01	1.33	0.03	2.4	0.89	5
10	20.00 20.00	0.52	0.01	1.33	0.03	2.4	0.89	11

The analysis proves that the initial values used in the calculation do not affect the determination of optimal solution. The final results of each case shown in Table 1 differ only in the number of iteration performed. In the case of 6 to 10 initial values assumed in the range of 0.30 to 20 in order to verify the correctness of the algorithm, omitting their interpretation. It is clear that the computational process lengthened, but the final values remain unchanged. The next stage of work was to determine the parameters a and tgβ for various operations.

4.2. Case Studies

The next stage of work was to determine parameters *a* and tgβ for various mining operations. For this purpose data on the leveling measurements has been analyzed from 10 mines located in Poland. Input to applications has been prepared and calculations were performed for the initial values:

– $a = 0.70$;
– tgβ = 2.00.

The results are presented in Table 2.

Table 2. The obtained parameter values for different mines

No.	Mine	*a*	σ_a	tgβ	σ_{tg}	M_w	*R*
1	“Bielszowice”	0.89	0.01	2.44	0.15	3.4	0.10
2	“Chwałowice”	0.99	0.02	1.53	0.09	3.2	0.75
3	“Dębieńsko”	0.90	0.01	1.97	0.05	2.5	0.62
4	“Jan Kanty”	0.52	0.01	3.07	0.17	3.5	0.61
5	“Lubin”	0.52	0.01	1.87	0.07	2.2	0.58
6	“Sieroszowice”	0.52	0.01	1.33	0.03	2.4	0.89
7	“Tadeusz”	0.55	0.01	1.81	0.11	3.1	0.30
8	“Ziemowit”	0.80	0.02	1.67	0.08	3.5	0.68

For presenting, the calculation was chosen mine “Chwałowice” and the observation line 1a located on its territory. Figure 3 shows the location of lines 1a and geometry of operation. The number of iterations and the values of the parameters *a* and tgβ in subsequent computing cycles presents Figure 4. The resulting theoretical course of the subsequent iterations is presented in Figure 5. Stabilization of the calculation process is clearly visible right in the fourth iteration. However, only in the sixth iteration the corrections were so small that the computational process could be completed. This way, the optimal parameters were obtained with the lowest coefficient of variation M_w.

In further stages of research will be carried out analysis of computational value of standard deviation of the parameters *a* and tgβ and correlation coefficients *R*, which in this publication have not been described.

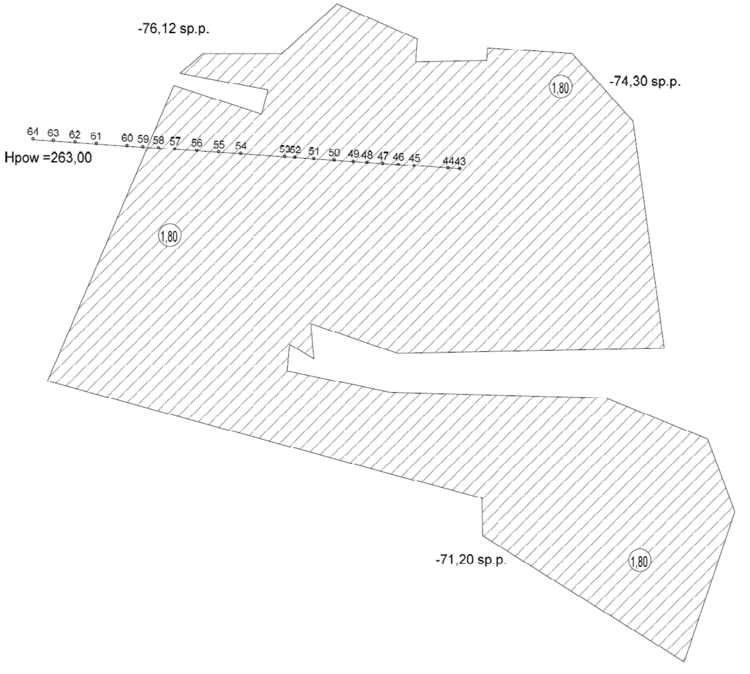

Fig. 3. The position of the benchmark line 1a to operation in coal mine "Chwałowice"

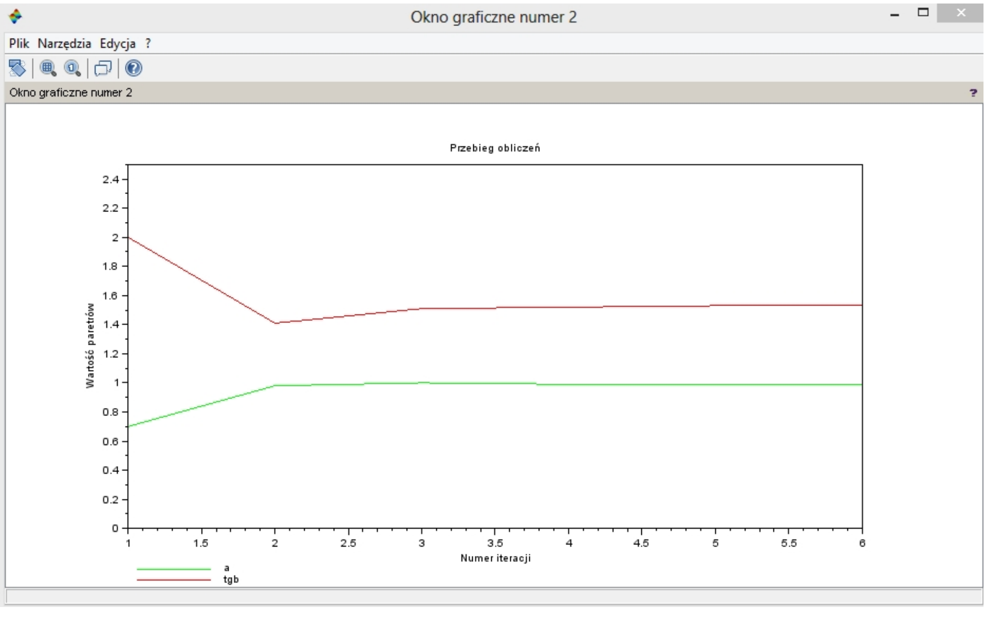

Fig. 4. The calculation and parameter values adopted in the subsequent iterations

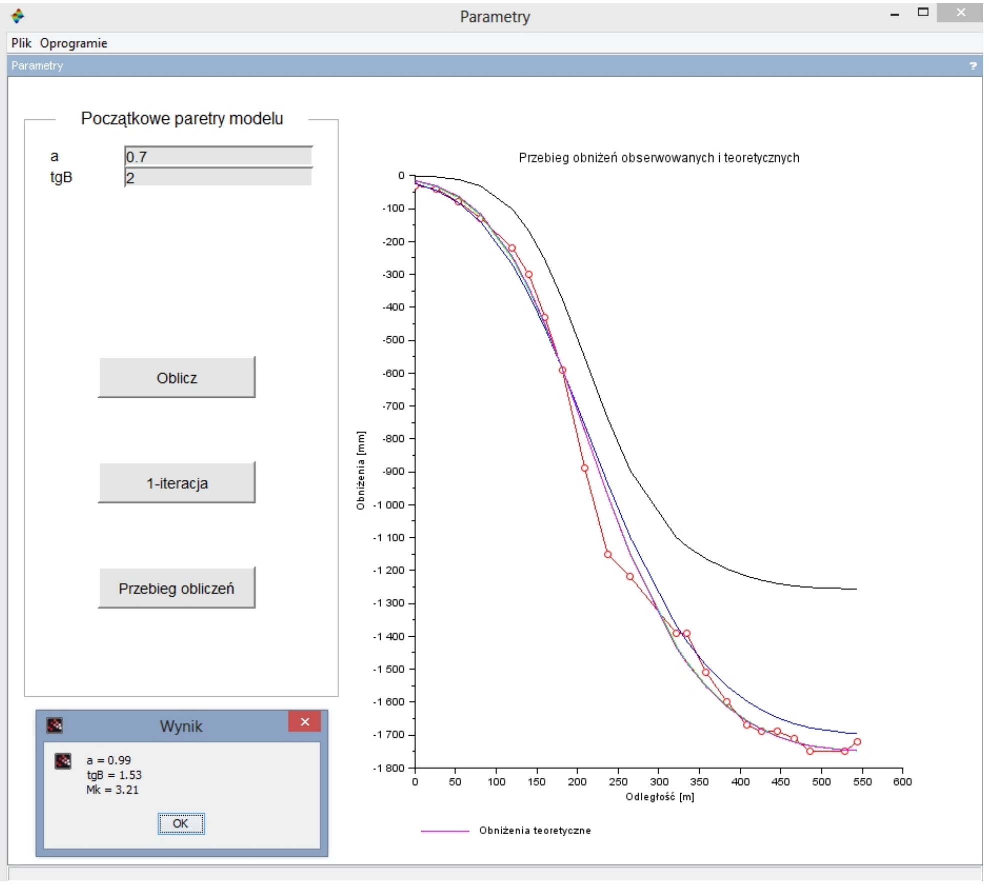

Fig. 5. Process of the next iteration and the final result for case study from the coal mine "Chwałowice"

5. Summary

In this paper the implementation of the least squares method using observational equations to determine the parameters of the Knothe theory have been proposed. The presented approach also include an analysis of the accuracy.

Based on the results of calculations following outcomes might be formulated:

- standard deviation of the parameter *a* in all instances did not exceed 0.02, while the deviation tgβ varied from 0.03 to 0.17;
- in all calculations coefficient of variation M_w did not exceed 5%, which was the criterion of correctness;
- in the process of computing even a small change in the value of the exploitation factor a significantly affect the value of the coefficient of variation;
- correlation coefficients are formed in the range of 0.10 to 0.89.

The proposed calculation algorithm can successfully be used in determining the parameters of the Knothe theory. In addition, created script can facilitate the effective and rapid implementation.

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