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RESONANT VIBRATIONS OF THE MOVING BAND SAW BLADE WITH VARIABLE TENSION

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A b s t r a c t

Resonant transverse bending vibrations of the band saw blade have been investigated taking into consideration variable in the time tension force of the blade and the external periodic perturbation. Amplitude of resonant vibrations of the band saw blade piece has been determined by the Bubnov-Galerkin method and basing on the idea of perturbation theory.

Introduction

Significant dynamic loads arise in the elastic system of horizontal cutting mechanism of the band saw machine when it is turned on. This is caused by the variable electromagnetic engine torque and springy elements of this mechanism (REBEZNYUK 2009, DZYUBA et al. 2012). The vibrations and dynamic loads that occur during transition process and steady-state mode of the band saw machine adversely affect durability of a band saw and quality of received timber. In particular, the transverse vibrations of band saw blade, i.e. vibration in blade plane of the least stiffness, what causes cyclic tension, which reduces durability of a band saw. These vibrations are especially dangerous when their amplitude increases significantly and resonance occurs.

The known researches of oscillating occurrences in cutting mechanisms in general and of band saw machines vibration sin particular did not take into account movement of the saw blade and variability of tension force (MOTE

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1965, 1966, SUGIHARA 1977, ISUPOVA 1981). Therefore it is advisable to study transverse vibrations of band saw blade, taking into account the speed of its longitudinal movement (cutting speed) and the variation of tension force.

Theoretical studies

A band saw is a component of a cutting mechanism elastic system, which is set and pulled on the saw pulleys. A working branch of a band saw is directed by leading rollers. A small eccentricity of saw pulleys and leading rollers and variable angular speed of attraction saw pulley during the transitional mode (turning on the machine) leads to change of the initial tension of band saw blade. In general, we shall assume that tension force is variable in time.

Vibrations of the working branch section between the leading rollers and of the non-working branch section between saw pulleys may occur due to external perturbation. In both cases, the design scheme of the saw blade sections is considered to be a bar on two hinged beams (Fig. 1). We assume that contact of a band saw blade with saw pulleys and leading rollers is constantly unseparated. Therefore, we suggest that there are no transverse displacement in the points of contact of a saw blade with saw pulleys and leading rollers. Considering the longitudinal movement of a saw we calculate speed v and study transverse displacement $u(x,t)$ of a saw blade section.

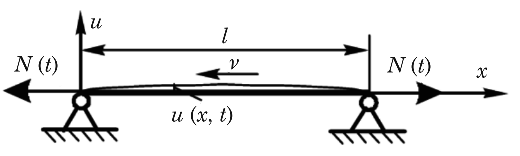

Fig. 1. The design scheme of a band saw blade

Forced transverse vibrations of a band saw blade as a moving bar with constant cross-section, can be described in Euler's variables by the differential equation with partial derivatives (CHEN et. al. 2004, MOTE 1965, MOTE, NAGULESWARAN 1966, SZE et. al. 2005, SUGIHARA 1977):

$$\frac{\partial^2 u}{\partial t^2} + 2\nu \frac{\partial^2 u}{\partial x \partial t} + \nu^2 \frac{\partial^2 u}{\partial x^2} - \frac{N(t)}{m} \frac{\partial^2 u}{\partial x^2} + \beta^2 \frac{\partial^4 u}{\partial x^4} = \varepsilon f(x,t) \qquad (1)$$

where:

$u(x,t)$ – transverse displacement of band saw blade with the coordinate x of an arbitrary point in time,

ε – a small parameter, which in the right side of the equation means small value of perturbation force $f(x,t)$ versus regenerative force,

$\beta^2 = \frac{EI}{m}$,

m – mass per unit length of a saw blade,

E – modulus of elasticity of the material (steel),

$I = \frac{B \cdot s^2}{12}$ – moment of inertia of a rectangular cross-section of a saw blade with respect to the neutral axis,

B, s – width and height of the cross-section of a saw blade,

$N(t)$ – variable tension force.

According to the design scheme (Fig. 1) boundary conditions needed to solve the differential equation (1) are as follows

$$u\big|_{x=0} = u\big|_{x=1} = 0; \quad \frac{\partial^2 u}{\partial x^2}\bigg|_{x=0} = \frac{\partial^2 u}{\partial x^2}\bigg|_{x=1} = 0 \tag{2}$$

where l – length of the respective section of a band saw blade.

The effect of external perturbation force that may arise from eccentricity of a saw pulley or leading rollers is set as follows:

$$f(x,t) = H \sin(\Omega t) \tag{3}$$

where:

H, Ω – amplitude and frequency of the external perturbation.

Tension force variable is set inharmonic law (HASCHUK, NAZAR 2008, SOKIL, LISCHINSKA 2008)

$$N(t) = N_0 + \varepsilon N_1 \cos \mu t \tag{4}$$

where:

N_0 – a constant component of tension force,

μ – frequency of tension force variation,

N_1 – the amplitude of a variable component of tension force.

We assume that the static component conveys initial tension in the band saw blade (ISUPOVA 1981). When there is no cutting process the dynamic component, defined by oscillating occurrences and dynamic loads in the elastic cutting mechanism system, is negligible compared with static. This reflects the small parameter *4* in relationship (4).

Given (3), (4), the differential equation (1) becomes:

$$\frac{\partial^2 u}{\partial t^2} + 2v\frac{\partial^2 u}{\partial x \partial t} - \left(\frac{N_0}{m} - v^2\right)\frac{\partial^2 u}{\partial x^2} + \beta^2 \frac{\partial^4 u}{\partial x^4} = \varepsilon \left(H \sin \Omega t + \frac{N_1}{m} \cos \mu t \frac{\partial^2 u}{\partial x^2}\right) \quad (5)$$

Therefore, the challenge is to find the solution of equation (5) under boundary conditions (2). Depending on the ratio between frequency of free vibrations of a band saw blade ω, frequency of the external perturbation Ω and frequency of tension change μ different cases of transverse vibrations can be considered: nonresonant, approximate to resonant and resonant. As noted above, it is the most important to study the vibration of a band saw blade under the influence of periodic perturbation and variable tension force in the resonant case. In this case, a significant increase of the amplitude fluctuations results from periodic forces, frequencies of which are in rational relationship to the main frequency of the frequency spectrum $\Omega \approx \frac{p}{q}\,\omega$ where p and q are relatively prime integers. In case $p = q = 1$, (i.e. $\Omega \approx \omega$) then resonance is called main or ordinary. In case $q = 1$ (i.e. $\Omega \approx p\omega$) the overtone resonance at the eigenfrequency (parametric resonance) occurs. Regarding the parametric resonance it should be noted that if we consider the dynamic component of tension force as the perturbation force and take into account the average effect of external force for a long period of time, then in the first approximation there is only one kind of resonance $2\omega \approx \mu$ (VASILENKO 1992).

We will study main and parametric resonance in a complex. This allows for a small harmonic perturbation to confine only to the first approximation. The analytical solution of this task considering the speed of the longitudinal movement poses considerable mathematical difficulties. We therefore suggest a way of solving that is based on the use of the basic ideas of the Bubnov-Galerkin methods (VASILENKO 1992) and perturbation theory (NAIFE 1976). Accordingly, the solution of (5) satisfying the boundary conditions (2) can be represented as:

$$u(x,t) = \sum_{k=1}^{\infty} X_k(x)\, T_k(t) \quad (6)$$

where:

$X_k(x)$ – functions that satisfy the boundary conditions (2), i.e. $X_k(0) = X_k(l) = 0$ and $X_k''(0) = X_k''(l) = 0$.

It is easy to check that the function system $\{X_k(x)\} = \left\{\sin \frac{k\pi}{l} x\right\}$ is acceptable.

Single solution of equation (5) under homogeneous boundary conditions (2) can be represented in the form

$$u(x,t) = \sin\left(\frac{k\pi x}{l}\right) T(t),\ k = 1, 2, ... \tag{7}$$

where the function $T(t)$ is determined depending on the initial conditions. Hereinafter index k that indicates the shape oscillations is omitted.

Substituting (7) into (5), we obtain:

$$\begin{gathered}\ddot{T}(t) \sin\left(\frac{k\pi x}{l}\right) - 2v\frac{k\pi}{l}\dot{T}(t) \cos\left(\frac{k\pi x}{l}\right) + \left(\frac{N_0}{m} - v^2\right)\left(\frac{k\pi}{l}\right)^2 T(t) \sin\left(\frac{k\pi x}{l}\right) + \\ + \beta^2\left(\frac{k\pi}{l}\right)^4 \sin\left(\frac{k\pi x}{l}\right) = \varepsilon\left(H \sin \Omega t - \frac{N_1}{m}\left(\frac{k\pi}{l}\right)^2 T(t) \cos \mu t \cdot \sin\left(\frac{k\pi x}{l}\right)\right)\end{gathered} \tag{8}$$

Multiplying both sides of the differential equation (8) by $\sin \frac{k\pi}{l} x$ and integrating the expressions in the range of 0 to l, we obtain the differential equation to find the unknown $T_k(t)$:

$$\ddot{T}(t) + \omega^2 T(t) = \varepsilon\left(\omega^2 H_2 T(t) \cos \mu t + H_3 \sin \Omega t\right) \tag{9}$$

By definition, put, $\omega^2 = \left(\frac{k\pi}{l}\right)^2\left[\left(\frac{k\pi}{l}\right)^2 \beta^2 + \frac{N_0}{m} - v^2\right]$, $\quad H_2 = \frac{2H(1 - \cos k\pi)}{k\pi}$,

$$H_3 = \frac{-N_1}{S_0 - mv^2 + m\left(\frac{k\pi}{l}\right)^2 \beta^2}.$$

Equation (9) belongs to the class of non-homogeneous linear differential equations with quasiperiodic coefficients. Their research is much complicated compared to the homogeneous linear differential equations with periodic coefficients, but the presence of the small parameter ε greatly simplifies the task. The above mentioned fact allows using asymptotic methods of Krylov-

-Bogolyubov to build solution of equation (9) and basing on it to study amplitude of resonant vibrations.

It is known (NAIFE 1976) that at resonance the phase difference between free vibrations and the external perturbation significantly influences amplitude and frequency of resonant vibrations. Therefore, to build a differential equation describing the change of amplitude and phase fluctuations, it is convenient to introduce the phase difference in the form:

$$\gamma = \psi - \theta$$

where $\theta = \frac{\mu t}{2}$, ψ denotes free vibrations phase.

Using a general idea of the method of Krylov-Bogolyubov-Mitropolsky (BOGOLIUBOV, MITROPOLSKY 1961), one-frequency solution of (9) in the first approximation we get:

$$T(t) = \alpha \cos (\theta + \gamma) + \varepsilon T_1 (\alpha, \gamma \theta) \tag{10}$$

where the amplitude α and the phase difference between the free vibrations and the external perturbation γ are values variable in time. They are determined basing on the differential equations:

$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} = \varepsilon \Lambda(\alpha, \gamma);$$
$$\frac{\mathrm{d}\gamma}{\mathrm{d}t} = \omega - \frac{\mu}{2} + \varepsilon \Xi(\alpha, \gamma) \tag{11}$$

where $\Lambda(\alpha, \gamma)$, $\Xi(\alpha, \gamma)$ are unknown functions that need to be found so that the relation (10), when $\alpha = \alpha(t)$, $\gamma = \gamma(t)$ which is solution of system (11) satisfies equation (9). It should be noted that the function $\Lambda(\alpha, \gamma)$, $\Xi(\alpha, \gamma)$ must be periodic with variable γ to the period 2π. To find these functions let's differentiate (10) in time:

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{\mathrm{d}\alpha}{\mathrm{d}t}\cos(\theta + \gamma) - \alpha \sin(\theta + \gamma)\left\{\frac{\mu}{2} + \frac{\mathrm{d}\gamma}{\mathrm{d}t}\right\} + \varepsilon\left\{\frac{\mathrm{d}\alpha}{\mathrm{d}t}\frac{\partial T_1}{\partial \alpha} + \frac{\mathrm{d}\gamma}{\mathrm{d}t}\frac{\partial T_1}{\partial \gamma} + \frac{\partial T_1}{\partial t}\right\};$$

$$\frac{\mathrm{d}^2T}{\mathrm{d}t^2} = \frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2}\cos(\theta + \gamma) - 2\frac{\mathrm{d}\alpha}{\mathrm{d}t}\sin(\theta + \gamma)\left\{\frac{\mu}{2} + \frac{\mathrm{d}\gamma}{\mathrm{d}t}\right\} -$$

$$-\alpha \cos(\theta + \gamma) \left\{ \frac{\mu}{2} + \frac{d\gamma}{dt} \right\}^2 - \alpha \sin(\theta + \gamma) \frac{d^2\gamma}{dt^2} + \tag{12}$$

$$+ \varepsilon \left\{ \frac{d^2\alpha}{dt^2} \frac{\partial^2 T_1}{\partial \alpha} + \left(\frac{d\alpha}{dt}\right)^2 \frac{\partial^2 T_1}{\partial \alpha^2} + \frac{d^2\gamma}{dt^2} \frac{\partial T_1}{\partial \gamma} + \left(\frac{d\gamma}{dt}\right)^2 \frac{\partial^2 T_1}{\partial \gamma^2} + \right.$$

$$\left. + 2 \frac{d\alpha}{dt} \frac{d\gamma}{dt} \frac{\partial^2 T_1}{\partial \alpha \partial \gamma} + 2 \frac{d\alpha}{dt} \frac{\partial^2 T_1}{\partial t \, \partial \alpha} + 2 \frac{d\gamma}{dt} \frac{\partial^2 T_1}{\partial t \, \partial \gamma} + \frac{\partial^2 T_1}{\partial t^2} \right\}$$

From the relations (11) it follows that:

$$\frac{d^2\alpha}{dt^2} = \varepsilon^2 \frac{\partial \Lambda(\alpha,\gamma)}{\partial \alpha} \Lambda(\alpha,\gamma) + \varepsilon \left(\omega - \frac{\mu}{2}\right) \frac{\partial \Lambda(\alpha,\gamma)}{\partial \gamma} + \varepsilon^2 \frac{\partial \Xi(\alpha,\gamma)}{\partial \gamma} \Xi(\alpha,\gamma);$$

$$\frac{d^2\gamma}{dt^2} = \varepsilon^2 \frac{\partial \Xi(\alpha,\gamma)}{\partial \alpha} \Lambda(\alpha,\gamma) + \varepsilon \left(\omega - \frac{\mu}{2}\right) \frac{\partial \Xi(\alpha,\gamma)}{\partial \gamma} + \varepsilon^2 \frac{\partial \Xi(\alpha,\gamma)}{\partial \gamma} \Xi(\alpha,\gamma); \tag{13}$$

$$\left(\frac{d\alpha}{dt}\right)^2 = \varepsilon^2 \Lambda^2(\alpha,\gamma); \ \frac{d\alpha}{dt} \frac{d\gamma}{dt} = \varepsilon \left(\omega - \frac{\mu}{2}\right) \Lambda(\alpha,\gamma) + \varepsilon^2 \Lambda(\alpha,\gamma) \Xi(\alpha,\gamma);$$

$$\left(\frac{d\gamma}{dt}\right)^2 = \left(\omega - \frac{\mu}{2}\right)^2 + 2\varepsilon \left(\omega - \frac{\mu}{2}\right) \Xi(\alpha,\gamma) + \varepsilon^2 \Xi(\alpha,\gamma)^2$$

Substituting (10) into equation (9) and taking into account (12) and (13), after equating coefficients of the same ε powers, we obtain the differential equation of the first approximation, which combines functions $t_1(\alpha, \gamma, \theta)$, $\Lambda(\alpha, \gamma)$, $\Xi(\alpha, \gamma)$:

$$\left(\omega - \frac{\mu}{2}\right) \frac{\partial \Lambda(\alpha,\gamma)}{\partial \gamma} \cos(\theta + \gamma) - \left(\omega - \frac{\mu}{2}\right) \alpha \frac{\partial \Xi(\alpha,\gamma)}{\partial \gamma} \sin(\theta + \gamma) -$$

$$- 2\omega \Lambda(\alpha,\gamma) \sin(\theta + \gamma) - 2\omega \alpha \Xi(\alpha,\gamma) \cos(\theta + \gamma) + \tag{14}$$

$$+ \left(\omega - \frac{\mu}{2}\right)^2 \frac{\partial^2 T_1}{\partial \gamma^2} + 2\left(\omega - \frac{\mu}{2}\right) \frac{\partial^2 T_1}{\partial t \partial \gamma} + \frac{\partial^2 T_1}{\partial t^2} + \omega^2 T_1 =$$

$$= \omega^2 H_2 \alpha \cos(\theta + \alpha) \cos 2\theta + H_3 \sin \theta$$

The resonance (14) takes the form:

$$\frac{\partial T_1}{\partial t^2} + \omega^2 T_1 \ \omega^2 H_2 \alpha \cos(\theta + \gamma) \cos 2\theta + \\ + H_3 \sin \theta + 2\omega \Lambda(\alpha,\gamma) \sin(\theta + \gamma) + 2\omega\alpha \Xi(\alpha,\gamma) \cos(\theta + \gamma) \tag{15}$$

For unambiguous determination of $\Lambda(\alpha,\gamma)$, $\Xi(\alpha,\gamma)$ additional conditions are applied to function $T_1(\alpha, \gamma, \theta)$ (BOGOLIUBOV, MITROPOLSKY, 1961). It is believed that the function $T_1(\alpha, \gamma, \theta)$ and its partial derivatives up to the second order including are 2π periodic for $\psi = \theta + \gamma$ and do not include in expansion items proportional to $\sin \psi$ and $\cos \psi$.

Thus functions $\Lambda(\alpha,\gamma)$, $\Xi(\alpha,\gamma)$ are defined in such way that the following equations are satisfied.

$$\int_0^{2\pi} T_1(\alpha,\psi) \cos \psi d\psi = 0; \quad \int_0^{2\pi} T_1(\alpha,\psi) \sin d\psi = 0$$

From physical point of view application of the mentioned above conditions corresponds to choosing a as full amplitude of the first fundamental harmonic of oscillations.

The mentioned conditions provide absence of terms with first harmonics in the right part of equations for determining the desired functions $\Lambda(\alpha, \gamma)$, $\Xi(\alpha, \gamma)$, which makes it possible to avoid secular terms in solutions.

The differential equation (15) allows to get an equation (16) to find functions $\Lambda(\alpha, \gamma)$, $\Xi(\alpha, \gamma)$:

$$\omega^2 H_2 \alpha \int_0^{2\pi} \cos \psi \cos 2(\psi - \gamma) \sin \psi d\psi + \int_0^{2\pi} H_3 \sin(\psi - \gamma) \sin \psi d\psi + \\ + 2\omega \Lambda(\alpha,\gamma) \int_0^{2\pi} \sin \psi \sin \psi d\psi + 2\omega\alpha \Xi(\alpha,\gamma) \int_0^{2\pi} \cos \psi \sin \psi d\psi = 0$$

$$\omega^2 H_2 \alpha \int_0^{2\pi} \cos \psi \cos 2(\psi - \gamma) \cos \psi d\psi + \int_0^{2\pi} H_3 \sin(\psi - \gamma) \cos \psi d\psi + \\ + 2\omega \Lambda(\alpha,\gamma) \int_0^{2\pi} \sin \psi \cos \psi d\psi + 2\omega\alpha \Xi(\alpha,\gamma) \int_0^{2\pi} \cos \psi \cos \psi d\psi = 0$$

Calculating integrals, we have:

$$\frac{\omega^2 H_2\alpha\pi}{2}\sin\gamma + H_3\pi\cos\gamma + 2\omega\pi\Lambda(\alpha,\gamma) = 0$$
$$\frac{\omega^2 H_2\alpha\pi}{2}\cos 2\gamma - H_3\pi\sin\gamma + 2\omega\alpha\pi\Xi(\alpha,\gamma) = 0 \quad (16)$$

Defining from (16) functions $\Lambda(\alpha, \gamma)$, $\Xi(\alpha, \gamma)$ based on (11), we obtain system of differential equations which describes the basic parameters of vibrations:

$$\frac{d\alpha}{dt} = -\frac{\varepsilon}{(\omega + 0.5\mu)}\left(\frac{\alpha H_2\omega^2}{2}\sin 2\gamma + H_3\cos\gamma\right)$$
$$\frac{d\gamma}{dt} = \omega - 0.5\mu - \frac{\varepsilon}{(\omega + 0.5\mu)}\left(\frac{H_2\omega^2}{2}\cos 2\gamma - \frac{H_3}{\alpha}\sin\gamma\right) \quad (17)$$

Results and discussion

Graphic dependences of the amplitude of resonance oscillations for a band saw blade, which is $s = 1$ mm thick, $B = 26$ mm wide, and $l = 0.5$ m long on the working section (provided that: band saw material is steel, $E = 2.1\times10^5$ MPa, density $\rho = 7850$ kg/m^3, speed $\rho = 30$ m/s), were built basing on solution to equation (17) for the static component of tension force, which corresponds to different tensions of a saw blade (Fig. 2).

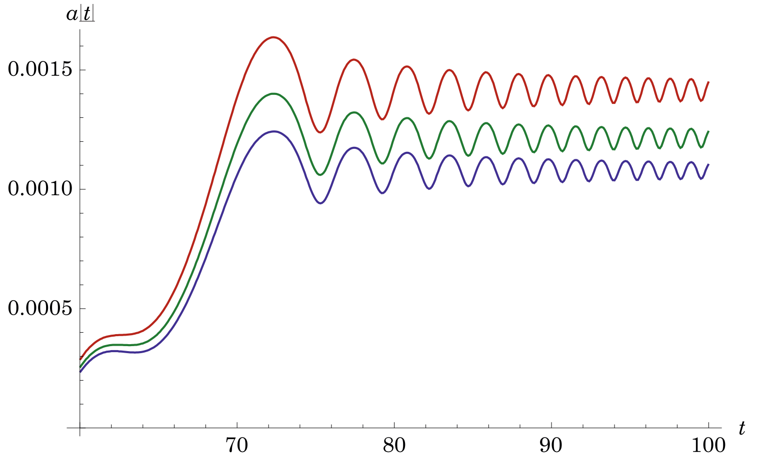

Fig. 2. Amplitude of resonant vibrations of working area for a band saw blade under initial tension: curve 1 – 200 MPa; curve 2 – 160 MPa, curve 3 – 120 MPa

As shown on Figure 2, on the area of a band saw blade between the leading rollers, when tension increases from of 120 MPa (curve 3) to 200 MPa (curve 1), the amplitude of resonance vibrations decreases by 0.4 mm. Thus a band saw blade under greater initial tension smoothly enters into saw cut under the condition of minor fluctuations of pretension force when the variable component of this force is 0.1% of the static one.

Resonant vibrations amplitudes of a band saw blade non-working branch under different pretension stresses are received under the same geometrical parameters of saw blade cross-section as for a working branch and a section $l_1 = 1.3$ m. When the length of the blade increases, the resonance vibrations amplitude grows to 4.4 mm in the middle section at the slightest pretension of 120 MPa and decreases to 3.3 mm for the pretension stress of 200 MPa (Fig. 3).

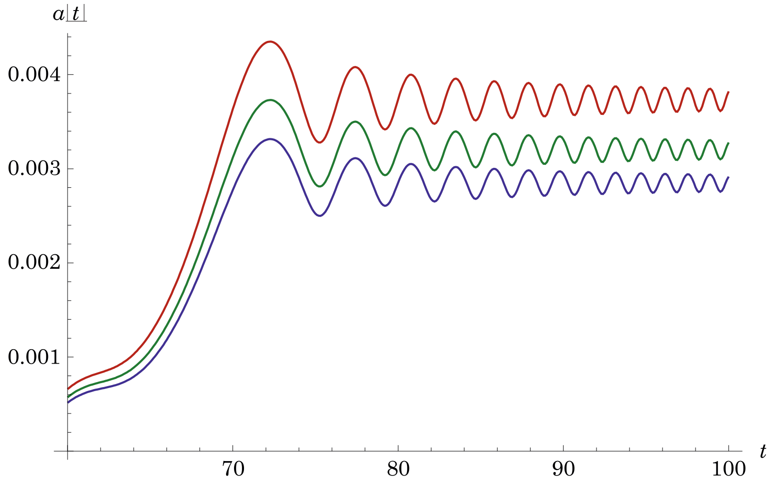

Fig. 3. Resonant vibrations amplitude of a band saw blade non-working branch under pretension stress: curve 1 – 200 MPa; curve 2 – 160 MPa, curve 3 – 120 MPa

Conclusions

Amplitude and frequency features of band saw blade sections were obtained in the result of solving the differential equation of forced transverse vibrations of a band saw blade as a stretched moving rod with stable cross-section, under condition of variable tension force. It was theoretically determined that section of a working branch of a saw blade between the leading rollers and an area of a drawn non-working branch can vary in resonant mode, if there is even a small perturbation force that results from the eccentricity of the leading rollers or saw pulleys. However, varying tension forces determined by oscillating occurrences and dynamic loads in the elastic cutting mechanism system were taken into account.

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