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## **Optimization of critical infrastructure accident consequences related to climate-weather change process influence – losses minimizing**

### Keywords

critical infrastructure, sea accident, environment degradation, cost of accident consequences

### Abstract

The methods based on the results of the General Model of Critical Infrastructure Accident Consequences and the linear programming are proposed to the optimization of critical infrastructures accident consequences with considering the climate-weather change process influence. The critical infrastructure accident consequences determining the optimal values of limit transient probabilities at the process of environment degradation states that minimize the critical infrastructure accident expected value of the total environment losses impacted by the climate-weather change for the fixed time interval are proposed.

### 1. Introduction

The critical infrastructure accident is understand as an event that causes changing the critical infrastructure safety state into the safety state worse than the critical safety state that is dangerous for the critical infrastructure itself and its operating environment as well. Each critical infrastructure accident can generate the initiating event causing dangerous situations in the critical infrastructure operating surroundings. The process of initiating events can result in this environment threats and lead to the environment dangerous degradations.

Thus, the probabilistic General Model of Critical Infrastructure Accident Consequences (GMCIAC) was proposed in [Bogalecka & Kołowrocki, 2017] that includes the process of initiating events generated either by its accident or by its loss of safety critical level, the process of environment threats and the process of environment degradation models.

### 2. Critical infrastructure accident losses

We denote by

$$L_{(k)}^{i}(t), \ i = 1, 2, ..., \ell_{k}, \ k = 1, 2, ..., n_{3},$$
 (1)

the losses associated with the process of the environment degradation

$$R_{(k)}(t), t \in (0, +\infty), k = 1, 2, ..., n_3,$$

in the sub-region  $D_k$ ,  $k = 1, 2, ..., n_3$ , at the environment degradation state  $r_{(k)}^i$ ,  $i = 1, 2, ..., \ell_k$ ,  $k = 1, 2, ..., n_3$ , in the time interval  $\langle 0, t \rangle$ .

Thus, the approximate expected value of the losses in the time interval  $\langle 0, t \rangle$ , associated with the process of the environment degradation  $R_{(k)}(t)$  of the sub-region  $D_k$  can be defined by

$$L_{(k)}(t) \cong \sum_{i=1}^{\ell_k} q_{(k)}^i \cdot L_{(k)}^i(t) \text{ for } k = 1, 2, \dots, n_3, \qquad (2)$$

where  $q_{(k)}^{i}$ ,  $i = 1, 2, ..., \ell_{k}$ , mean the boundary probabilities of the process of the environment degradation at its particular states and are given by (17) in [Bogalecka & Kołowrocki, 2017], and  $L_{(k)}^{i}(t)$ ,

 $i = 1, 2, ..., \ell_k, k = 1, 2, ..., n_3$  are defined by (1).

The losses associated with particular environment degradation states are involved with negative consequences in the accident area. The types of consequences are various for different kinds of accident and accident area. For instance, in the shipping, the closure of port, closure of fishery area and people death can be considered as the negative consequences. The losses can be expressed by the cost of the negative consequences in case like the closure of port, closure of fishery area. In the case of negative consequences like people death, the losses can be expressed as the number of loss of life. In the paper we only consider the accident consequences that can be expressed by cost.

Under these assumption, if we fix the number of kinds of accident consequences by  $\xi$  and the cost function of this consequence lasting *t* 

$$[K_{(k)}^{i}(t)]^{(j)}, (3)$$

for  $j = 1, 2, ..., \xi$ ,  $i = 1, 2, ..., \ell_k$ ,  $k = 1, 2, ..., n_3$ , than the loss for the sub-region  $D_k$ ,  $k = 1, 2, ..., n_3$ , is expressed by the total cost of all consequences lasting *t* in the sub-region  $D_k$ ,  $k = 1, 2, ..., n_3$ , is given by

$$L_{(k)}^{i}(t) \cong \sum_{j=1}^{\xi} [K_{(k)}^{i}(t)]^{(j)}, \ t \in \langle 0, +\infty \rangle,$$
(4)

for  $i = 1, 2, ..., \ell_k$ ,  $k = 1, 2, ..., n_3$ .

Hence, according to (2), losses associated with the process of the environment degradation  $R_{(k)}(t)$  of the sub-region  $D_k$ ,  $k = 1, 2, ..., n_3$ , are given by

$$L_{(k)}(t) \cong \sum_{i=1}^{\ell_k} q_{(k)}^i \left[ \sum_{j=1}^{\xi} [K_{(k)}^i(t)]^{(j)} \right], \ t \in \langle 0, +\infty \rangle, \quad (5)$$

for  $k = 1, 2, ..., n_3$ .

Furthermore, the total expected value of the losses for the fixed time  $\varphi$ ,  $\varphi \ge 0$ , associated with the process of the environment degradation R(t), in all sub-regions of the considered critical infrastructure operating environment region *D*, can be evaluated by

$$L(\varphi) \cong \sum_{k=1}^{n_3} L_{(k)}(\varphi), \tag{6}$$

where  $L_{(k)}(\varphi)$  are given by (5) for  $t = \varphi$ .

# **3.** Critical infrastructure accident losses with considering climate-weather change process impact

### **3.1.** Critical infrastructure accident area climate-weather change process

Critical infrastructure accident area climate-weather change process parameters (either identified statistically or evaluated by experts) are [Kołowrocki & Soszyńska-Budny, 2016, 2017]:

- the number of climate-weather states *w*;
- the vector  $[q_b(0)]_{1xw}$  of the initial probabilities

$$q_b(0) = P(C(0) = c_b), b = 1, 2, \dots, w,$$

of the climate-weather change process C(t) staying at particular climate-weather states  $c_b$  at the moment t = 0;

- the matrix  $[q_{bl}]_{wxw}$  of the probabilities of transitions

$$q_{bl}, b, l = 1, 2, \dots, w, b \neq l,$$

of the climate-weather change process C(t) from the climate-weather states  $c_b$  to  $c_l$ ;

- the matrix  $[N_{bl}]_{wxw}$  of the mean values

$$N_{bl} = E[C_{bl}], b, l = 1, 2, \dots, w, b \neq l,$$

of the climate-weather change process C(t) conditional sojourn times  $C_{bl}$  at the climate-weather states  $c_b$  when its next climate-weather state is  $c_l$ .

Critical infrastructure operating area climate-weather change process characteristic (either calculated analytically or evaluated by experts) is [Kołowrocki & Soszyńska-Budny, 2016, 2017] the vector

$$[q_b]_{1xw} = [q_1, q_2, \dots, q_w]$$
(7)

of the limit values of transient probabilities

$$q_b(t) = P(C(t) = c_b), \ t \in <0,+\infty), \ b = 1,2,...,w,$$

of the climate-weather change process C(t) at the particular operation states  $c_b$ .

(In the case of a periodic climate-weather change process, the limit transient probabilities  $q_b$ , b = 1, 2, ..., w, at the climate-weather states defined by (7), are the long term proportions of the climate-weather change process C(t) sojourn times at the particular climate-weather states  $c_b$ , b = 1, 2, ..., w).

### **3.2.** Critical infrastructure accident losses related to climate-weather impact

We denote the losses associated with the process of the environment degradation

$$R_{(k)}(t), t \in (0, +\infty), k = 1, 2, \dots, n_3$$

in the sub-region  $D_k$ ,  $k = 1, 2, ..., n_3$ , at the environment degradation state  $r_{(k)}^i$ ,  $i = 1, 2, ..., \ell_k$ ,  $k = 1, 2, ..., n_3$ , in

the time interval  $\langle 0, t \rangle$ , while the climate-weather change process C(t) at the critical infrastructure accident area is at the climate-weather state  $c_b$ , b = 1, 2, ..., w, by

$$[L_{(k)}^{i}(t)]^{(b)},$$

$$t \in <0,+\infty), \quad i = 1,2,...,\ell_{k}, \quad k = 1,2,...,n_{3},$$

$$b = 1,2,...,w.$$
(8)

The losses  $[L_{(k)}^{i}(t)]^{(b)}$  are the conditional losses, while the climate-weather change process C(t) is at the climate-weather state  $c_b$ , b = 1, 2, ..., w, defined by

$$[L_{(k)}^{i}(t)]^{(b)} = [\rho_{(k)}^{i}]^{(b)} \cdot L_{(k)}^{i}(t), \qquad (9)$$
  
$$t \in <0,+\infty), \quad i = 1,2,...,\ell_{k}, \quad k = 1,2,...,n_{3},$$

b = 1, 2, ..., w,

where

$$[\rho_{(k)}^{i}]^{(b)}, (10)$$

$$i = 1, 2, \dots, \ell_k, \ k = 1, 2, \dots, n_3, \ b = 1, 2, \dots, w,$$

are the coefficients of the climate-weather change process impact on the losses associated with the process of the environment degradation in the subregion  $D_k$ ,  $k = 1, 2, ..., n_3$ , at the environment degradation state  $r_{(k)}^i$ ,  $i = 1, 2, ..., \ell_k$ ,  $k = 1, 2, ..., n_3$ , in the time interval  $\langle 0, t \rangle$ , while the climate-weather change process C(t) at the critical infrastructure accident area is at the climate-weather state  $c_b$ , b = 1, 2, ..., w.

Thus, by (5) and (9) the conditional approximate expected value of the losses in the time interval  $\langle 0, t \rangle$ , associated with the process of the environment degradation  $R_{(k)}(t)$  of the sub-region  $D_k$ ,  $k = 1, 2, ..., n_3$ , while the climate-weather change process C(t) is at the climate-weather state  $c_b$ , b = 1, 2, ..., w, can be defined by

$$[L_{(k)}(t)]^{(b)} \cong \sum_{i=1}^{\ell_k} q^i_{(k)} \cdot [L^i_{(k)}(t)]^{(b)}$$
(11)

for  $k = 1, 2, ..., n_3$ , b = 1, 2, ..., w, where  $q_{(k)}^i$ ,  $i = 1, 2, ..., \ell_k$ ,  $k = 1, 2, ..., n_3$ , are given by (17) in [Bogalecka & Kołowrocki, 2017], and  $[L_{(k)}^{i}(t)]^{(b)}$ ,  $t \in < 0,+\infty$ ),  $i = 1,2,..., \ell_{k}, k = 1,2,...,n_{3}, b = 1,2,...,w$ ,

are defined by (9)-(10). Further, applying the formula for total probability, the unconditional approximate expected value of the environment losses, impacted by the climate-weather change process C(t), in the time interval  $\langle 0, t \rangle$ , associated with the process of the environment degradation  $R_{(k)}(t)$  of the sub-region  $D_k$ ,  $k = 1, 2, ..., n_3$ , can be expressed by

$$\overline{L}_{(k)}(t) \cong \sum_{b=1}^{w} q_b \cdot [L_{(k)}(t)]^{(b)}, \ k = 1, 2, \dots, n_3,$$
(12)

where  $q_b$ , b = 1,2,...,w, are given in [Kołowrocki & Soszyńska-Budny, 2016, 2017], and  $[L_{(k)}(t)]^{(b)}$ ,  $t \in < 0,+\infty$ ),  $k = 1,2,...,n_3$ , b = 1,2,...,w, are determined by (11).

Hence, according to (11), we have

$$\overline{L}_{(k)}(t) \cong \sum_{b=1}^{w} \sum_{i=1}^{\ell_{k}} q_{b} \cdot q_{(k)}^{i} \cdot [L_{(k)}^{i}(t)]^{(b)}, \qquad (13)$$

 $k = 1, 2, ..., n_3.$ 

Finally, the total expected value of the losses, impacted by the climate-weather change process C(t), in the fixed time interval  $\langle 0, \varphi \rangle$ , associated with the process of the environment degradation R(t), in all sub-regions of the considered critical infrastructure operating environment region *D*, can be evaluated by

$$\overline{L}(\varphi) \cong \sum_{k=1}^{n_3} \overline{L}_{(k)}(\varphi), \tag{14}$$

where  $\overline{L}_{(k)}(\varphi)$ ,  $k = 1, 2, ..., n_3$ , are given by (12) for  $t = \varphi$ .

Thus, considering (9), the coefficient of the climateweather change process impact on the losses associated with the process of the environment degradation in the sub-region  $D_k$ ,  $k = 1, 2, ..., n_3$ , in the time interval  $\langle 0, \varphi \rangle$ , may be defined as

$$\rho_{(k)} = \overline{L}_{(k)}(\varphi) / L_{(k)}(\varphi), \qquad (15)$$

where  $\overline{L}_{(k)}(\varphi)$ ,  $\varphi \in <0,+\infty)$ ,  $k = 1,2,...,n_3$ , are the losses related to the climate-weather impact

 $\varphi \in < 0, +\infty), k = 1, 2, \dots, n_3,$ 

determined by (12) and  $L_{(k)}(\varphi)$ ,  $\varphi \in <0,+\infty)$ ,  $k = 1,2,...,n_3$ , are the losses without considering climate-weather impact determined by (2).

Similarly, the coefficient of the climate-weather change process impact on the total losses associated with the process of the environment degradation in the entire considered region D, in the time interval  $\langle 0, \varphi \rangle$ , may be defined as

$$\rho(\varphi) = \overline{L}(\varphi) / L(\varphi), \quad \varphi \in <0, +\infty), \tag{16}$$

where  $\overline{L}(\varphi)$ ,  $\varphi \in < 0, +\infty$ ), are the total losses related to the climate-weather impact determined by (14) and  $L(\varphi)$ ,  $\varphi \in < 0, +\infty$ ), are the total losses without considering climate-weather impact determined by (6).

Other practically interesting characteristics of the environment degradation caused by critical infrastructure accident consequences related to the climate-weather are the indicators of the environment of the sub-regions  $D_k$ ,  $k = 1, 2, ..., n_3$ , resilience to the losses associated with the critical infrastructure accident related to the climate-weather change that are proposed to be defined by

$$RI_{(k)}(\varphi) = 1/\rho_{(k)}, \ \varphi \in <0,+\infty), \ k = 1,2,...,n_3,(17)$$

where  $\rho_{(k)}$ ,  $k = 1, 2, ..., n_3$ , are determined by (15) and the indicator of the environment of the entire region *D* resilience to the total losses associated with the critical infrastructure accident consequences related to the climate-weather change that are proposed to be defined by

$$RI(\varphi) = 1/\rho, \ \varphi \in <0, +\infty), \tag{18}$$

where  $\rho$  is determined by (16).

### 4. Critical infrastructure accident losses minimization

From the linear equation (13), we can see that the mean value of expected critical infrastructure accident losses  $\overline{L}_{(k)}(t)$ ,  $t \in < 0, +\infty$ ),  $k = 1, 2, ..., n_3$ , associated with the process of the environment degradation  $R_{(k)}(t)$  of the sub-region  $D_k$ ,  $k = 1, 2, ..., n_3$ , impacted by the climate-weather change process C(t) is determined by the limit value of transient probabilities  $q_b$ , b = 1, 2, ..., w of the climate-weather change process C(t) at the particular climate-weather state  $c_b$ , b = 1, 2, ..., w, the limit value of transient probabilities

 $q_{(k)}^{i}$ ,  $i = 1, 2, ..., \ell_{k}$ ,  $k = 1, 2, ..., n_{3}$  of the process of the environment degradation at the state  $r_{(k)}^{\prime}$ ,  $i = 1, 2, ..., \ell_k$ ,  $k = 1, 2, ..., n_3$ , and by the mean value of the critical infrastructure accident losses  $[L^i_{(k)}(t)]^b$ ,  $t \in < 0, +\infty$ ),  $i = 1, 2, ..., \ell_k$ ,  $k = 1, 2, ..., n_3$ , b = 1, 2, ..., w, associated with the process of the environment degradation  $R_{(k)}(t)$  of the sub-region  $D_k$ ,  $k = 1, 2, ..., n_3$ , impacted by the climate-weather change process C(t). Therefore, the optimization based on the linear [Klabjan Adelman, programming & 2006], [Kołowrocki & Soszyńska-Budny, 2011] of the critical infrastructure accident losses associated with the process of the environment degradation  $R_{(k)}(t)$  of the sub-region  $D_k$ ,  $k = 1, 2, ..., n_3$ , with considering the climate-weather change process C(t) can be proposed. Namely, we may look for the corresponding optimal  $q_b \dot{q}^i_{(k)}, \qquad b = 1, 2, \dots, w,$  $i = 1, 2, \dots, \ell_{k}$ values  $k = 1, 2, ..., n_3$  of the transient probabilities  $q_b q_{(k)}^i$ ,  $b = 1, 2, \dots, w, \quad i = 1, 2, \dots, \ell_k, \quad k = 1, 2, \dots, n_3$  of the process of the environment degradation at the state  $r_{(k)}^{i}$ ,  $i = 1, 2, ..., \ell_{k}$ ,  $k = 1, 2, ..., n_{3}$ , to minimize the mean value of critical infrastructure accident losses  $\overline{L}_{(k)}(t)$  impacted by the climate-weather change process C(t), in the sub-region  $D_k$ ,  $k = 1, 2, ..., n_3$ .

Thus, we may formulate the optimization problem as a linear programming model with the objective function of the following form

$$\overline{L}_{(k)}(t) \cong \sum_{b=1}^{w} \sum_{i=1}^{\ell_{k}} q_{b} \cdot q_{(k)}^{i} \cdot [L_{(k)}^{i}(t)]^{(b)}, \qquad (19)$$
$$t \in <0, +\infty), \ k = 1, 2, \dots, n_{3},$$

with the following bound constraints

$$q_{b} \overline{q}_{(k)}^{i} \leq q_{b} q_{(k)}^{i} \leq q_{b} \widehat{q}_{(k)}^{i}, \quad \sum_{b=1}^{w} \sum_{i=1}^{\ell_{k}} q_{b} q_{(k)}^{i} = 1, \quad (20)$$
  
$$k = 1, 2, \dots, n_{3},$$

where  $[L_{(k)}^{i}(t)]^{(b)}$ ,  $[L_{(k)}^{i}(t)]^{(b)} \ge 0$ , b = 1,2,...,w $i = 1,2,..., \ell_{k}$ ,  $k = 1,2,...,n_{3}$ , are fixed mean values of the losses associated with the process of the environment degradation  $R_{(k)}(t)$  of the sub-region  $D_{k}$ ,  $k = 1,2,...,n_{3}$ , impacted by the climate-weather change process C(t), for a fixed  $t, t \in < 0, +\infty$ ), and  $q_b \breve{q}_{(k)}^i, \ 0 \le q_b \breve{q}_{(k)}^i \le 1, \text{ and } q_b \widehat{q}_{(k)}^i, \ 0 \le q_b \widehat{q}_{(k)}^i \le 1,$ 

$$q_b \breve{q}_{(k)}^i \le q_b \widehat{q}_{(k)}^i, \tag{21}$$

$$b = 1, 2, \dots, w, i = 1, 2, \dots, \ell_k, k = 1, 2, \dots, n_3,$$

are lower and upper bounds of the unknown transient probabilities  $q_b q_{(k)}^i$ , b = 1, 2, ..., w,  $i = 1, 2, ..., \ell_k$ ,  $k = 1, 2, ..., n_3$ , respectively. Now, we can obtain the optimal solution of the

formulated by (19)-(21) the linear programming problem, i.e. we can find the optimal values  $q_b \dot{q}_{(k)}^i$ b = 1,2,...,w,  $i = 1,2,...,\ell_k$ ,  $k = 1,2,...,n_3$  of the transient probabilities  $q_b q_{(k)}^i$ , b = 1,2,...,w,  $i = 1,2,...,\ell_k$ ,  $k = 1,2,...,n_3$ , that minimize the objective functions given by (19). First, we arrange the mean values of the losses  $[L_{(k)}^i(t)]^{(b)}$ ,  $t \in \langle 0, +\infty \rangle$ , b = 1,2,...,w,  $i = 1,2,...,\ell_k$ ,  $k = 1,2,...,n_3$ , associated with the process of the environment degradation  $R_{(k)}(t)$  of the sub-region  $D_k$ ,  $k = 1,2,...,n_3$ , impacted by the climate-weather change

process C(t) in increasing order

$$[L_{(k)}^{i_1}(t)]^{(b)} \le [L_{(k)}^{i_2}(t)]^{(b)} \le \dots \le [L_{(k)}^{i_j}(t)]^{(b)},$$
  
$$k = 1, 2, \dots, n_3, \ t \in < 0, +\infty),$$

where  $i_j \in \{1, 2, ..., \ell_k\}$  for  $j = 1, 2, ..., \ell_k \cdot w$ . Next, we substitute

$$x_{(k)}^{j} = q_{b}q_{(k)}^{i_{j}}, \quad \breve{x}_{(k)}^{j} = q_{b}\breve{q}_{(k)}^{i_{j}}, \quad \widetilde{x}_{(k)}^{j} = q_{b}\widetilde{q}_{(k)}^{i_{j}}$$
(22)

for  $j = 1, 2, ..., \ell_k \cdot w$ , and we minimize with respect to  $x_{(k)}^j$ ,  $j = 1, 2, ..., \ell_k \cdot w$ ,  $k = 1, 2, ..., n_3$ , the linear form (19) that after this transformation takes the form

$$\overline{L}_{(k)}(t) = \sum_{j=1}^{\ell_k \cdot w} x_{(k)}^j \cdot [L_{(k)}^{i_j}(t)]^{(b)}$$
(23)

for a fixed t,  $t \in < 0, +\infty$ ), with the following bound constraints

 $\begin{aligned} \breve{x}_{(k)}^{j} &\leq x_{(k)}^{j} \leq \widehat{x}_{(k)}^{j}, \quad \sum_{j=1}^{\ell_{k} \cdot w} x_{(k)}^{j} = 1, \ k = 1, 2, \dots, n_{3}, \ (24) \end{aligned}$ where  $[L_{(k)}^{i_{j}}(t)]^{(b)}, \quad [L_{(k)}^{i_{j}}(t)]^{(b)} \geq 0, \quad b = 1, 2, \dots, w,$   $k = 1, 2, \dots, n_{3}, \ i_{j} \in \{1, 2, \dots, \ell_{k}\} \text{ for } j = 1, 2, \dots, \ell_{k} \cdot w, \text{ are }$ 

fixed mean values of the losses associated with the process of the environment degradation  $R_{(k)}(t)$  of the sub-region  $D_k$ ,  $k = 1, 2, ..., n_3$ , impacted by the climate-weather change process C(t) for a fixed t,  $t \in < 0, +\infty$ ), arranged in increasing order and

$$\vec{x}_{(k)}^{j}, \ 0 \le \vec{x}_{(k)}^{j} \le 1, \text{ and } \hat{x}_{(k)}^{j}, \ 0 \le \hat{x}_{(k)}^{j} \le 1,$$

$$\vec{x}_{(k)}^{j} \le \hat{x}_{(k)}^{j}, \ j = 1, 2, ..., \ell_{k} \cdot w, \ k = 1, 2, ..., n_{3}, \quad (25)$$

are lower and upper bounds of the unknown probabilities  $x_{(k)}^{j}$ ,  $j = 1, 2, ..., \ell_{k} \cdot w$ ,  $k = 1, 2, ..., n_{3}$ , respectively.

To find the optimal values of  $x_{(k)}^{j}$ ,  $j = 1, 2, ..., \ell_k \cdot w$ ,  $k = 1, 2, ..., n_3$ , we define

$$\breve{x}_{(k)} = \sum_{j=1}^{\ell_k \cdot w} \breve{x}_{(k)}^j, \quad \widehat{y}_{(k)} = 1 - \breve{x}_{(k)}$$
(26)

and

$$\widetilde{x}_{(k),0} = 0, \ \widehat{x}_{(k),0} = 0 \text{ and}$$

$$\widetilde{x}_{(k),J} = \sum_{i=1}^{J} \widetilde{x}_{(k)}^{j}, \ \widehat{x}_{(k),J} = \sum_{i=1}^{J} \widehat{x}_{(k)}^{j}$$
(27)

for

$$J=1,2,\ldots,\ell_k\cdot w.$$

Next, we find the largest value  $J \in \{0, 1, ..., \ell_k \cdot w\}$  such that

$$\widehat{x}_{(k),J} - \widecheck{x}_{(k),J} < \widehat{y}_{(k)}$$
(28)

and we fix the optimal solution that minimize (23) in the following way:

i) if J = 0, the optimal solution is  $\dot{x}_{(k)}^{1} = \hat{y}_{(k)} + \breve{x}_{(k)}^{1}$  and  $\dot{x}_{(k)}^{j} = \breve{x}_{(k)}^{j}$ for  $j = 2, 3, ..., \ell_{k} \cdot w$ ; (29)

ii) if  $0 < J < \ell_k \cdot w$ , the optimal solution is

$$\dot{x}_{(k)}^{j} = \hat{x}_{(k)}^{j}$$
 for  $j = 1, 2, ..., J$ ,

$$\dot{x}_{(k)}^{J+1} = \hat{y}_{(k)} - \hat{x}_{(k),J} + \breve{x}_{(k),J} + \breve{x}_{(k)}^{J+1}$$

and 
$$\dot{x}_{(k)}^{j} = \breve{x}_{(k)}^{j}$$
 for  $j = J + 2, J + 3, ..., \ell_{k} \cdot w$ ; (30)

iii) if  $J = \ell_k \cdot w$ , the optimal solution is

$$\dot{x}_{(k)}^{j} = \hat{x}_{(k)}^{j}$$
 for  $j = 1, 2, ..., \ell_{k} \cdot w.$  (31)

Finally, after making the inverse to (22) substitution, we get the optimal limit transient probabilities

$$q_b \dot{q}_{(k)}^{i_j} = \dot{x}_{(k)}^j \text{ for } j = 1, 2, ..., \ell_k \cdot w,$$
 (32)

that minimize the mean values of the losses associated with the process of the environment degradation  $R_{(k)}(t)$ of the sub-region  $D_k$ ,  $k = 1, 2, ..., n_3$ , impacted by the climate-weather change process C(t), defined by the linear form (19), giving its minimum value in the following form

$$\dot{\overline{L}}_{(k)}(t) = \sum_{b=1}^{w} \sum_{i=1}^{\ell_k} q_b \dot{q}_{(k)}^i [L_{(k)}^i(t)]^{(b)}, \qquad (33)$$

for  $k = 1, 2, \dots, n_3$ , and a fixed *t*.

### 5. Conclusion

The procedure [Kołowrocki & Soszyńska-Budny, 2011] and the liner programing [Klabjan & Adelman, 2006] was presented to the optimization of the critical infrastructure accident losses. Presented in the paper tools can be useful in accident consequences cost optimization through the accident losses minimizing. Therefore the results can be interesting for emergency rescuers and services as well as other government, administrative and technical services bearing costs to remove and/or reduce the critical infrastructure accident consequences.

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