A COMPARISON BETWEEN SOME APPROXIMATE AND NUMERICAL SOLUTION OF DYNAMICAL EULER-LIOUVILLE EQUATION FOR RIGID NON-SYMETRICAL FREE BODY WITH FAST OSCILATING INTERACTION IN GRAVITATIONAL FIELD

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Abstract

Tumbling NPA (non-principal axis) asteroids are discovered by the investigation of photometric data and asteroids lightcurves. A particular solution of Euler-Liouville equation was found for an elongated symmetrical body. It may be useful for investigations of dual periods of rotational motion NPA tumbling asteroids. Physical formula of the gravity force moment acting at a rigid body was derived. A numerical solution of the same accurate equation was found and compared with the analytical ones. A stable region of the numerical solution was investigated and numerical algorithms for fast variables were examined. Very long time period of the numerical solution of Euler-Liouville equation for the body in gravitational field was found.

Key words: rotation of asteroids, gravitational force moment, Euler-Liouville equations, numerical computing, systems of non-linear differential equations

1 Introduction

Astrophysics of asteroids in recent years accomplished appreciable development thanks to a new observatory and computational technology. Observed asteroids are divided into groups consisting of objects with comparable properties [2, 7, 11, 12]. By photometric and radar measurements of rotation periods, diameters of many asteroids found. One of most distinctive property is a self-rotational motion of asteroids. The knowledge about its origin is still unsatisfactory and there are many hypothesis clarifying that problem. The base of the numerical simulation is gravitational interaction [1, 7, 14], YORP effect [13] and collisions of asteroids [3, 4, 9]. Generally, all rotation periods are asteroids diameter dependant and are in a range of a few hours. Lower limit 2.2h is clearly evident for all diameters [15]. Higher periods in range of 10-1000hr were established for tumbling asteroids. The most of known asteroids rotate fast (rotation period ≈2÷5hr) around principal inertia tensor axis and are almost symmetrical in other axes.

Tumbling asteroids discovered in nineties of 20th century are characterized by slow rotation (rotation period 20÷200hr) around non-principal axis (NPA asteroids) and significant elongation. Rotation of these asteroids was named tumbling. From photometric data two different periods of rotation tumbling asteroids have been discovered. Lightcurve analysis of tumbling asteroids is based on the solution of dynamical Euler-Liouville equation. This paper concentrates on the gravitational base of rotational motion and discounts YORP effect.

Dynamical Euler-Liouville (E-L) equation:

$$\hat{I}\frac{d\vec{\omega}(t)}{dt} + \vec{\omega}(t) \times \hat{I}\vec{\omega}(t) = \vec{M}(t) \tag{1}$$

is a fundamental formula, describing rotational motion of bodies under action of external torques. $\hat{I}$ is a diagonal inertia tensor ($\hat{I} = diag(I_x, I_y, I_z)$), $\vec{\omega}(t)$ is angular velocity in the center mass reference frame and $\vec{M}(t)$ is an external torque. This equation is a part of a problem of the body movement in the gravitational field. A full rigid body movement description needs coordinates of mass body center and Euler angle and if feasible, spatial density of a rigid body (a three-dimensional spatial body model). In the gravity field, spatial and angular variables may be separated and an equation of center mass motions are Euler angle independent. We may resolve it separately and in (1) this coordinates are known as a function of time. A standard method of solving this is the transformation to the mass body center reference system fixed to principal axes of body inertia momentum tensor. Let vector $\vec{R}'(t) = [X'(t); Y'(t); Z'(t)]$ give a position of mass center of the rigid body in inertial coordinate solar system (orbital system). In this study coordinates of vector $\vec{R}(t) = [X(t); Y(t); Z(t)]$ are determined in the coordinate system fixed to the body (non-inertial system). Euler angles are defined in a usual manner, see fig.1.

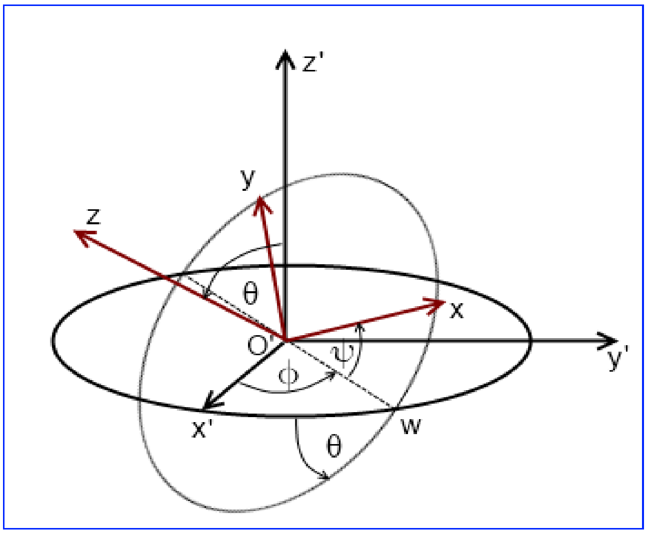

Figure 1.

In fig. 1, the coordinate system X'Y'Z' means the inertial system with center O' in center mass of the body and axes parallel to orbital system axes. In general, the body rotates in inertial system X'Y'Z' and Euler angle is a function of time connected to angular velocity $\vec{\omega}(t) = [\omega_x(t); \omega_y(t); \omega_z(t)]$ of a body in non-inertial system XYZ:

$$\begin{aligned} \omega_x &= \dot{\varphi}\sin\psi\sin\vartheta + \dot{\vartheta}\cos\psi; \\ \omega_y &= \dot{\varphi}\cos\psi\sin\vartheta - \dot{\vartheta}\sin\psi; \\ \omega_z &= \dot{\psi} + \dot{\varphi}\cos\theta; \end{aligned} \tag{2}$$

where dot above φ, ψ, θ denotes time derivatives. Reversing equations, we find kinematical Euler equations:

$$\begin{aligned} \dot{\varphi} &= \frac{\omega_x \sin\psi + \omega_y \cos\psi}{\sin\vartheta}; \\ \dot{\psi} &= \omega_z - (\omega_x \sin\psi + \omega_y \cos\psi)\, c\,tg\theta; \\ \dot{\vartheta} &= \omega_x \cos\psi - \omega_y \sin\psi; \end{aligned} \tag{3}$$

Some authors (1) mention that orbital motion is not quite separated from rotational one and accurate (3) equations will contain terms dependent on the orbital motion. In this work these terms are neglected.

Equation (3) and earlier (1) are full systems describing the rotational motion of the rigid body. The property of the rigid body describes mass, density, volume and tensor of inertia Î. There exists an analytical solution of E-L equation (9) in case of free nonsymmetrical body ($I_x \neq I_y \neq I_z$) and some solutions of this equation for the symmetrical ($I_x = I_y$) body in the homogenous gravitational field. The first solution is given by elliptic integrals and its discussion needs numerical methods for its complexity. E-L equation in general case of the nonsymmetrical rigid body at the non-uniform gravitational field was never resolved before by anybody.

Now we are going to find expressions for the components of the solar gravitational force moment acting on the asteroid.

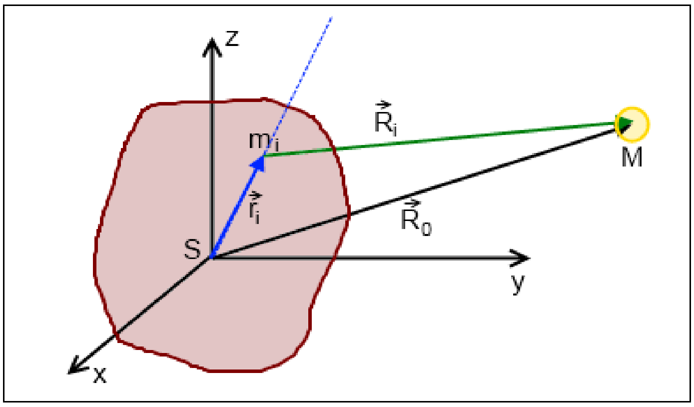

Figure 2.

Let vector $\overrightarrow{R_0}$ denote the center of the gravitational force (the Sun - fig. 2) in the inertial coordinate system with origin S in the body mass center. The mass of Sun M is the source of the gravitational field strength $\vec{\gamma}_0(\vec{r} = 0) = \overrightarrow{\gamma_0}(\vec{R}_0) = \frac{GM\vec{R}_0}{R_0^3}$ in point S and strength $\vec{\gamma}_i(\vec{r}_i) = \vec{\gamma}_i(\vec{R}_i) = \frac{GM\vec{R}_i}{R_i^3} = \frac{GM(\vec{R}_0 - \vec{r_i})}{R_i^3}$ in any point $\vec{r_i}$ of the body. Field strength in point $\vec{r_i}$ may be written in form:

$$\vec{\gamma}_i(\vec{r}_i) = \vec{\gamma}_0(\vec{R}_0) + [\vec{\gamma}_i(\vec{r}_i) - \vec{\gamma}_0(\vec{R}_0)] = \vec{\gamma}_0(\vec{R}_0) + \Delta\vec{\gamma}_i(\vec{r}_i) \quad (4)$$

Gravitational force moment acting on the body is:

$$\vec{M} = \sum_i \vec{r}_i \times m_i\, \vec{\gamma}_i(\vec{r}_i) = \sum_i \vec{r}_i \times m_i\, [\, \vec{\gamma}_0(0) + \Delta\vec{\gamma}_i(\vec{r}_i)] = \\ = \sum_i \vec{r}_i \times m_i\, \vec{\gamma}_0(0) + \sum_i \vec{r}_i \times m_i\, \Delta\vec{\gamma}_i(\vec{r}_i); \qquad (5)$$

The first term in the mass center coordinates is zero. The field strength variation in the second term is the sum of perpendicular and parallel to $\vec{r}_i$ components and because field strength is parallel to $\vec{R}_i$ we obtain:

$$\vec{\gamma}_{i\perp}(\vec{r}_i) = \vec{\gamma}_{i\perp}(\vec{R}_i) = \frac{GM\vec{R}_{i\perp}}{R_i^3} = \frac{GM(\vec{R}_{i\perp} - \frac{\vec{r}_i(\vec{r}_i\, g\vec{R}_i)}{r^2})}{R_i^3} \qquad (6)$$

Similarly, for strength in point S and moreover:

$$\vec{R}_{i\perp} = \left(\vec{R}_i - \frac{\vec{r}_i(\vec{r}_i\, g\vec{R}_i)}{r^2}\right) = \left(\vec{R}_0 - \vec{r}_i - \frac{\vec{r}_i\left[\vec{r}_i\, g(\vec{R}_0 - \vec{r}_i)\right]}{r^2}\right) = \vec{R}_{0\perp} \qquad (7)$$

hence we obtain :

$$\Delta\vec{\gamma}_{i\perp}(\vec{r}_i) = \frac{GM\vec{R}_{i\perp}}{R_i^3} - \frac{GM\vec{R}_{0\perp}}{R_0^3} = GM\vec{R}_{0\perp}\left(\frac{1}{R_i^3} - \frac{1}{R_0^3}\right) \cong \\ \cong \frac{3GM(\vec{r}_i g\vec{R}_0\,)}{R_0^5}(\vec{R}_0 - \frac{\vec{r}_i(\vec{r}_i\, g\vec{R}_0)}{r_i^2}) \qquad (8)$$

The last results assuming that the body is small sufficiently or far away from the field center, it is $r_i g R_0$ and then we may extend function $\Delta\vec{\gamma}_{i\perp}(\vec{r}_i)$ into Taylor series, where 0 means point S:

$$\Delta\vec{\gamma}_{i\perp}(\vec{r}_i) = \frac{GM\vec{R}_{i\perp}}{R_i^3} - \frac{GM\vec{R}_{0\perp}}{R_0^3} = GM\vec{R}_{0\perp}\left(\frac{1}{|\vec{r}_i - \vec{R}_0|^3} - \frac{1}{R_0^3}\right); \\ \Delta\vec{\gamma}_{i\perp}(\vec{r}_i) = \sum_{k=0}^{\infty} \frac{\left(\vec{r}_i\, g\vec{\nabla}\right)^k}{k!}\Delta\vec{\gamma}_{i\perp}(0) \qquad (9)$$

Because $r_i/R_0 << 1$ we may neglected terms of the second and higher-order.
Introducing (9) to (5) we get:

$$\vec{M} = \sum_i \vec{r}_i \times m_i\left[\Delta\vec{\gamma}_{i\perp}(\vec{r}_i) + \Delta\vec{\gamma}_{ig}(\vec{r}_i)\right] = \sum_i \vec{r}_i \times m_i \Delta\vec{\gamma}_{i\perp}(\vec{r}_i) = \\ = \sum_i \vec{r}_i \times m_i \frac{3GM(\vec{r}_i\, g\, \vec{R}_0)}{R_0^5}(\vec{R}_0 - \frac{\vec{r}_i(\vec{r}_i\, g\, \vec{R}_0)}{r_i^2}) \qquad (10)$$

Because the second term is zero ($\vec{r}_i \times \vec{r}_i = 0$), we obtain:

$$\vec{M} = \frac{3GM}{R_0^5} \sum_i \vec{r}_i \,(\vec{r}_i g \vec{R}_0) m_i \times \vec{R}_0 \tag{11}$$

To find components of the gravitational force momentum, we write it in the explicit form in the mass center coordinate system:

$$M_x : \left[\sum_i \vec{r}_i \left(\vec{r}_i g \vec{R}_0\right) m_i \times \vec{R}_0 \right]_X =$$
$$= \sum_i [Z_0 y_i (x_i X_0 + y_i Y_0 + z_i Z_0) - Y_0 z_i (x_0 X_0 + y_i Y_i + z_i Z_0)] \cdot m_i$$

Sum over *i* in barrycentric coordinate system fixed to principal axes of the body inertia tensor gives zero for moments of deviation and diagonal non-zero elements of inertia tensor, hence:

$$\begin{aligned} M_x &= \frac{3GMY_0Z_0}{R_0^5}(J_z - J_y); \\ M_y &= \frac{3GMX_0Z_0}{R_0^5}(J_x - J_z); \\ M_z &= \frac{3GMX_0Y_0}{R_0^5}(J_y - J_x); \end{aligned} \tag{12}$$

or in convenient symmetrical form:

$$\begin{aligned} M_x &= \frac{\partial \gamma_y(0)}{\partial z}(J_z - J_y) = -\frac{\partial^2 V(0)}{\partial y \partial z}(J_z - J_y); \\ M_y &= \frac{\partial \gamma_z(0)}{\partial x}(J_x - J_z) = -\frac{\partial^2 V(0)}{\partial z \partial x}(J_x - J_z); \\ M_z &= \frac{\partial \gamma_x(0)}{\partial y}(J_y - J_x) = -\frac{\partial^2 V(0)}{\partial x \partial y}(J_y - J_x); \end{aligned} \tag{13}$$

where differentiation is done in the center of the mass coordinate system and V is gravity potential in the center of body mass. In general, this expression corresponds with formulas given by other authors [1]. This result bonds momentum of gravitational force and orientation body in the inertial coordinate system by Euler angle.

Transform matrix from X'Y'Z' to XYX system defined earlier has a standard form $\hat{R} = \hat{R}_z(\psi)\hat{R}_w(\Theta)\hat{R}_{z'}(\varphi)$, where 'w' denotes nodal line and $(\varphi, \psi, \vartheta)$ are Euler angle. Full matrix $\hat{R}$ image is:

$$\hat{R}(\varphi,\psi,\vartheta) = \begin{vmatrix} \cos\varphi\cos\psi - \sin\varphi\sin\psi\cos\theta & \sin\varphi\cos\psi + \cos\varphi\sin\psi\cos\theta & \sin\psi\sin\vartheta \\ -\cos\varphi\sin\psi - \sin\varphi\cos\psi\cos\theta & -\sin\varphi\sin\psi + \cos\varphi\cos\psi\cos\theta & \cos\psi\sin\vartheta \\ \sin\varphi\sin\vartheta & -\cos\varphi\sin\vartheta & \cos\vartheta \end{vmatrix}$$

For asteroids Euler angle are fast changing periodically variables with short periods comparable to period of orbital motion range of a few years. In this work, we concern the solution perturbed by gravity E-L equation, where $\overrightarrow{R_0}(t)$ is slow periodical function of time and asteroids orbit nearly circular (eccentricity e≈0) , so we may take $\overrightarrow{R_0}(t) = R_0[sin\omega_s t; cos\omega_s t; 0]$. Z' axis is perpendicular to the asteroid orbit plane and ω_s is an angular velocity orbital motion of the asteroid. Taking into the account only the Sun potential, we may choose X'Y' as the orbit plane and neglect the inclination angle. This significantly simplifies the equation while Z component of $\overrightarrow{R_0}(t)$ is zero.

2 Approximate analytical solution of Euler – Liouville equation in some particular case

Now we find solution of E-L equation in a particular case of the elongated symmetrical body with inertia tensor $\hat{I} = diag(I_X, I_X, I_Z)$, and $I_Z > I_X$. For such asteroids tumbling NPA rotations were observed. This is a non-linear system of differential equations of the first order. Let $Y(t) = \omega_x(t); \omega_y(t); \omega_z(t); \varphi(t); \psi(t); \Theta(t)$ denote a matrix vector and $\dot{Y} = f(Y)$is the sixth equation:

$$\begin{aligned} \dot{\omega}_x &= \frac{(J_y - J_z)\omega_y\omega_z + M_x}{J_x}; \\ \dot{\omega}_y &= \frac{(J_z - J_x)\omega_z\omega_x + M_y}{J_y}; \\ \dot{\omega}_z &= \frac{(J_x - J_y)\omega_y\omega_z + M_z}{J_z}; \end{aligned} \tag{14}$$

$$\dot{\varphi} = \frac{\omega_x \sin\psi + \omega_y \cos\psi}{\sin\vartheta};$$
$$\dot{\psi} = \omega_z - (\omega_x \sin\psi + \omega_y \cos\psi)\cot\theta; \qquad (15)$$
$$\dot{\vartheta} = \omega_x \cos\psi - \omega_y \sin\psi;$$

If $I_x = I_y$ and M=0, then the third equation in (14) has a simple solution $\omega_Z = \omega_{\mathbf{0}} =$ const. Introducing $a=(I_z-I_x)/I_x$ and $\omega_E = a\omega_0$ we have a well known rotational movement around Z' axis with angular velocity $\dot{\psi}$ and simultaneously around Z axis with angular velocity $\dot{\varphi}$, while θ angle remains constant.

If M≠0, then in the rotating body-fixed system for vector $\overrightarrow{R_0}(t) = R_0[sin\omega_s t; cos\omega_s t; 0]$ we obtain:

$$\overrightarrow{R_0}(t) = \hat{R}(\varphi, \psi, \vartheta)\,\overrightarrow{R_0}(t);$$
$$\dot{\omega}_x = -\omega_E \omega_y + b f_x(t); \qquad (16)$$
$$\dot{\omega}_y = -\omega_E \omega_x + b f_y(t);$$

where $b = 3aGM_S / R_0^{\,3} = 3a\omega_S^2$ (G – gravitational constant, M_S – mass of the Sun) and:

$$f_x(t) = [\cos(\omega_S t + \varphi)\cos\psi\cos\theta - \sin(\omega_S t + \varphi)\sin\psi]\cos(\omega_S t + \varphi)\sin\theta;$$
$$f_y(t) = -[\cos(\omega_S t + \varphi)\sin\psi\cos\theta + \sin(\omega_S t + \varphi)\cos\psi]\cos(\omega_S t + \varphi)\sin\theta; \qquad (17)$$

This formula is exact. Variables φ and ψ are bounded with the rotation of the body incomparably faster than the orbital motion, so usual treatment provides an averaging procedure against the fastest variables. From observational data (6,11,15) it follows b/ω_E ratio of range 10-3 ÷ 10-2, and for the second elements in (16) don't exceed 2b in first approximation we may take $\theta = const$. Thus, accordingly (16) $\psi(t) = \omega_E t + \psi_0$ and $\varphi(t) \cong \frac{\omega_0 - \omega_E}{cos\theta} t + \psi_0$ Now we may, by usual procedure, simply integrate (16). After some complex calculation the result is:

$$\omega_x(t) = -A\sin(\omega_E t + \alpha) + b[\frac{\sin 2\vartheta}{4\omega_A}\sin(\omega_A t + 2\varphi_0)\cos(\omega_E t + \psi_0) + \\ + \frac{\sin 2\vartheta}{4} t\cos(\omega_E t + \psi_0) + \frac{\sin\theta}{2\omega_A}\cos(\omega_A t + 2\varphi_0)\sin(\omega_E t + \psi_0)];$$

$$\omega_y(t) = A\cos(\omega_E t + \alpha) - b[\frac{\sin 2\vartheta}{4\omega_A}\sin(\omega_A t + 2\varphi_0)\sin(\omega_E t + \psi_0) + \\ + \frac{\sin 2\vartheta}{4} t\sin(\omega_E t + \psi_0) - \frac{\sin\theta}{2\omega_A}\cos(\omega_A t + 2\varphi_0)\cos(\omega_E t + \psi_0)]; \quad (18)$$

where $\omega_A = 2(\omega_S + \frac{\omega_E - \omega_0}{cos\theta})$. If initial conditions are $\omega_x(t=0) = \omega_{x0}$ and $\omega_y(t=0) = \omega_{y0}$, then:

$$A = \sqrt{(\omega_{x0} + c)^2 + (\omega_{y0} + d)^2}; \alpha = \arctan\frac{\omega_{x0} + c}{\omega_{y0} + d}; \quad (19)$$

where c and d are constants:

$$c = b\sin\vartheta(\cos\theta\sin 2\varphi_0\cos\psi_0 + \cos 2\varphi_0\sin\psi_0)/2\omega_A;$$
$$d = b\sin\vartheta(\cos\theta\sin 2\varphi_0\sin\psi_0 - \cos 2\varphi_0\cos\psi_0)/2\omega_A.$$

This result provides a linear time depending on angular velocity. The first terms in solution (18) mean an unperturbed motion of free body without the gravity influence. We see that finally angular velocity consists of two periodical terms of period $\omega_A \pm \omega_E$. The third component ω_Z remains constant.

3 Numerical solution of E-L equations

Generally, it does not exist analytical solution of E-L equation with external torque given by formula (12) so we need a numerical algorithm to solve it. There are many different numerical methods, but all have the same small defect – a lack of possibility of comparison of numerical and accurate solution if it does not exist. On the other hand, without the numerical solution of the accurate equation, we may not estimate an approximate solution. There are only few analytical solutions of the equation above with M≠0 and this completely new, nowhere to find.

The main problem in solving E-L equation is a long period of solution needed because of astronomical time scale of the problem and a total error of the solution after all steps of the solving procedure. The numerical method integration systems of differential ordinary equations are very well known and described in many positions [9,10] and there are many convergent and stable algorithms and the problem is which of them to choose. We may use "ode" procedure Matlab or nearly equivalent Scilab platform and Adams or BDF algorithms with a variable step or Runge-Kutta method of order 4 or more. "Ode" procedures are very quick and effective, but in some cases the solution may be incorrect.

Because some parameters function f(Y) changes very fast and "ode" automatically choose the best algorithm and step, so "ode" method may fail. Runge-Kutta (R-K) method needs a very short step to achieve sufficient accuracy and take more time but for its simplicity is most popular. In this work, we used all these procedures adequate to problems. "Ode" procedure based on Adams and BDF algorithms is absolutely stable and convergent for any time step. R-K method is not absolutely convergent and procedure step has an upper limit.

Let $Y(t) = \omega_x(t); \omega_y(t); \omega_z(t); \varphi(t); \psi(t); \Theta(t)$ denote the matrix vector and $\dot{Y} = f(Y)$ is the sixth equation (14) and (15). Because we have found an approximate solution in particular case $J_x=J_y$, so we will be looking for a numerical solution upon the same condition. All parameters in equations are dimensionless or in rd/y. For most simulations following data were used (from typical astronomical data): ws=1rd/y; wz0=30rd/y(slowly rotated asteroid) and ω_{x0}, ω_{y0} range of 0÷1rd/y (initial value of transversal to ω_z angular velocity), a range of 0.1÷10 and time in years.

Many numerical simulations were made and it was pointed out that the numerical and analytical solutions are only for a short time almost identical in a tight area of parameters $a=(I_z-I_x)/I_x$ <1 and θ<0.1 angle. It means, that our assumption constant θ is true, if only the main body axis is nearly perpendicular to the orbital plane and body elongation is small (see fig. 3 and 4).

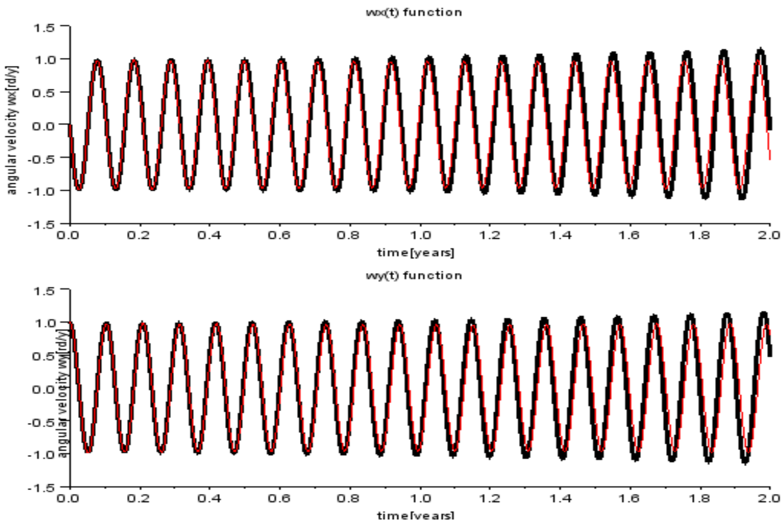

Figure 3. Numerical and analytical solution E-L equation. Parameters: a=2; θ=0.1; φ=0.5; ω_{x0}=0, ω_{y0}=1;ω_{z0}=30;ω_S=1; tk=2years, n=1000.

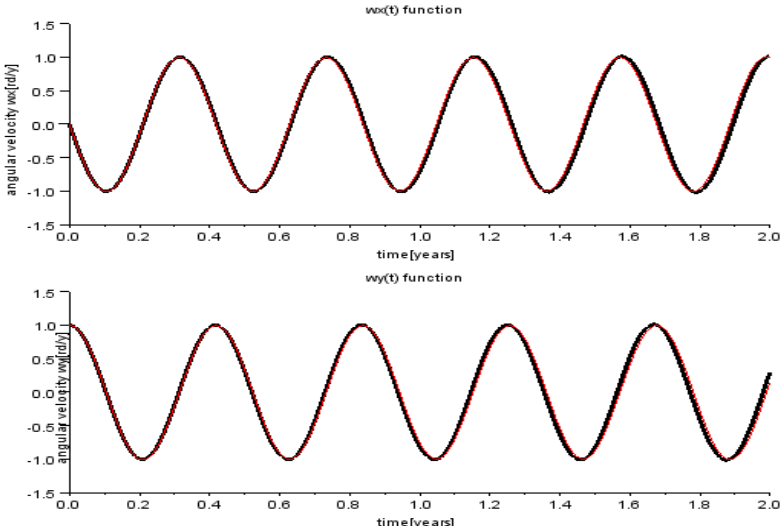

Figure 4. Numerical and analytical(bold line) solution E-L equation. Parameters: a=0.5; θ=0.1; φ=0.5; ω_{x0}=0, ω_{y0}=1;ω_{z0}=30;ω_S=1; t_k=2years, n=1000.

It is an evident major effect of θ angle and a as it is in fig. 5 and 6.

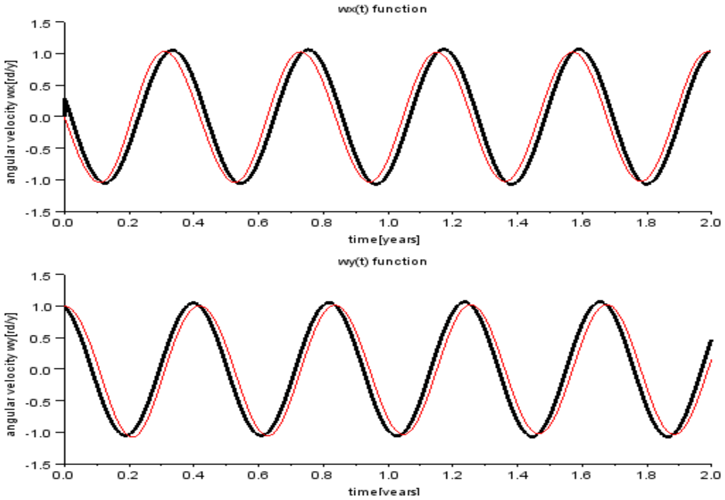

Figure 5. Numerical and analytical (bold line) solution E-L equation. Parameters: a=0.5; θ=0.5; φ=0.5; ω_{x0}=0, ω_{y0}=1;ω_{z0}=30;ω_S=1; t_k=2years, n=1000.

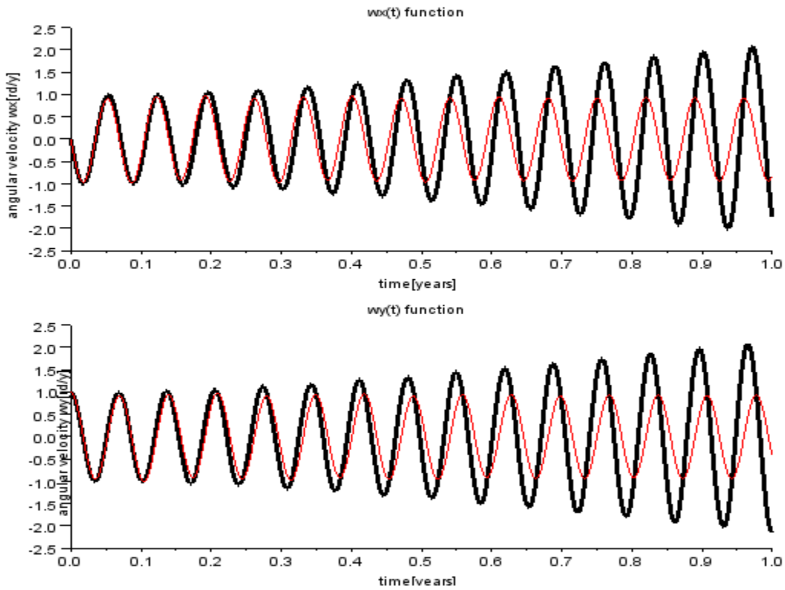

Figure 6. Numerical and analytical (bold line) solution E-L equation. Parameters: a=3; θ=0.5; φ=0.5; ω_{x0}=0, ω_{y0}=1;ω_{z0}=30;ω_S=1; t_k=2years, n=1000.

Most authors provide an averaging solar gravitational force moment over fast variable **φ** due to the rotation around the principal body axis [1, 6]. This simplifies the problem, but seems sometimes not reasonable. Long period simulation exhibits properties not evident in short period resolutions, as we, can see in fig. 7, 8, 9.

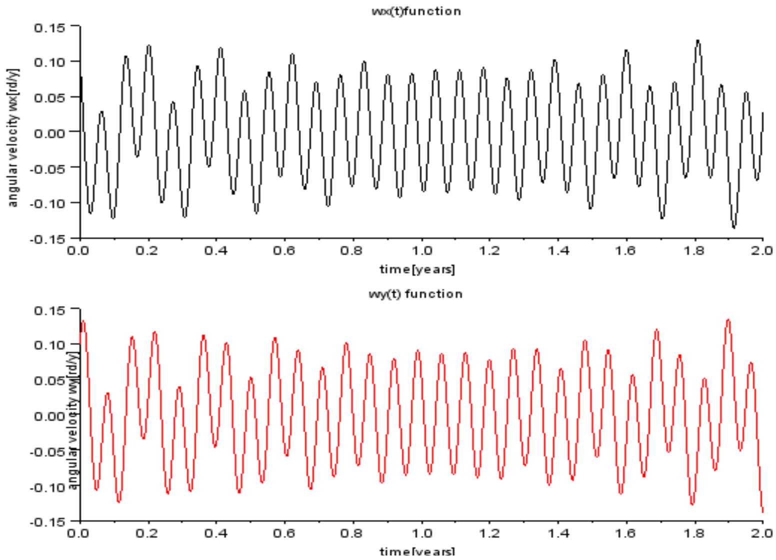

Figure 7. Numerical solution E-L equation. . Parameters: a=3; θ=0.9; φ=0.5; ω_{x0}=0.1, ω_{y0}=0.1;ω_{z0}=30;ω_S=1; t_k=2years, n=1000.

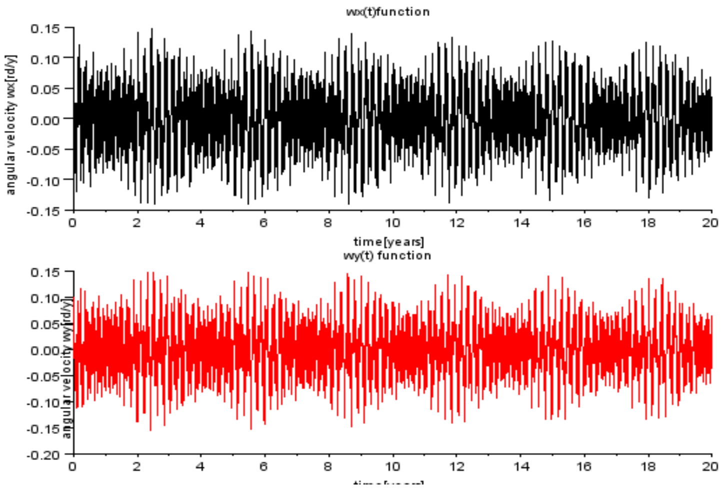

Figure 6. Numerical solution E-L equation. . Parameters: a=3; θ=0.9; φ=0.5; ω_{x0}=0.1, ω_{y0}=0.1;ω_{z0}=30;ω_S=1; t_k=20years, n=1000.

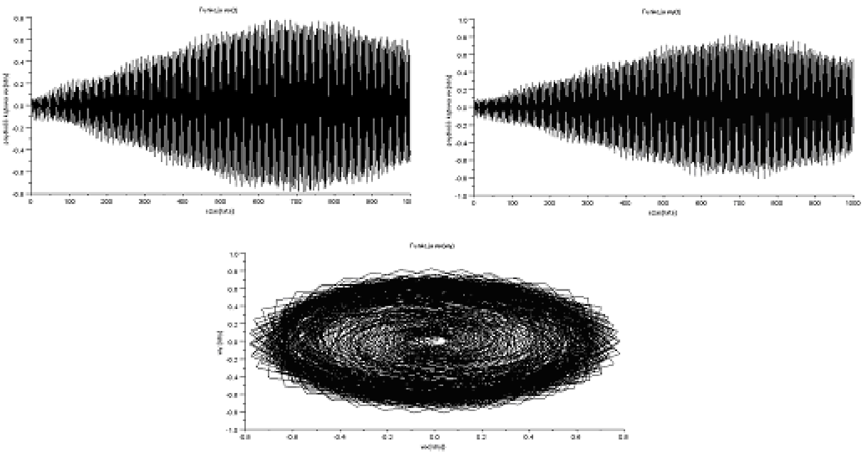

Figure 9. Numerical solution E-L equation. . Parameters: a=3; θ=0.9; φ=0.5; ω_{x0}=0.1, ω_{y0}=0.1;ω_{z0}=30;ω_S=1; t_k=2000years, n=1000. Below wyvs.wx plot.

Perpendicular components of angular velocity show periodical short time dependence and stable amplitudes. But very long period simulation shows a very slow periodical amplitude change. Adams algorithm was used and the solution was stable. Because we have not got observational data from so long time period, there is no confirmation this computational phenomenon yet. Additional simulations were done at case of the impact interaction. Usual duration time of that interaction in cosmic scale is negligibly small. For numerical impact simulations short-time force moment $\vec{M}(t_1 < t < t_2) = [M_X, M_Y, M_Z]$ and zero at the rest simulation period, t_2-t_1≈3•10-7y was added to $\vec{M}(t)$ at any time t1 . "Ode" procedures ignore this additional torque even if a smaller step was chosen and simulations were made by R-K algorithm with an appropriate small step (Fig.10).

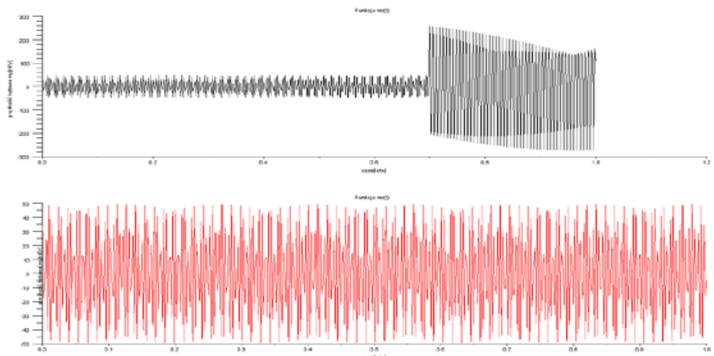

Figure 10. Simulation of impact interaction. Above R-K procedure, below “ode”.

4 Conclusions

The standard algorithms of numerical integration of systems ordinary differential equations are sufficient for most parameters E-L equation. Even if very fast variables were used (fast rotating asteroids) standard "ode" procedure worked correctly and solutions were stable. Some difficulties were observed, when θ angle neared quantity 0 or π, then function 1/sinθ or cotθ in (15) becomes of undefined value. In this case, we were warned that the result may be inaccurate. R-K algorithms were used also in some cases, they worked well, even in impact simulation, although appreciably slower. E-L equation is non-linear in all variables and it does not exist an analytical solution of it. A certain approximate solution was pointed out and it was compared with a numerical one. Although the solution for a short time is good approximation, and shows periodical properties of resolution, especially correct periods time, after a few years grows linearly with time. This property is weaker for a small θ angle and little elongated bodies and cannot be observed in nature. There were no numerical simulations for very long time periods, because all have done the assumption of stability resolution. This is almost true, but there exists a resolution changing slowly in time with periods of hundreds and thousands of years. A short observation doesn't allow to preclude such a behavior of rigid rotating bodies in the gravitational field. Numerical algorithms were also tested to in simulation of free bodies impacts in space. In the tested time scale, this events duration time is so small that ode algorithms simply ignored this interaction no matter of step quantity. The R-K algorithm may be used with a properly small step of time. It significantly elongates time of numerical process, but results are satisfactory. This method may be useful in simulation of effects of random multiple small impacts on the orbit and spin of the rigid body moving in the gravitational field.

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