

Stefan BROCK\*  
Maciej GNIADEK\*

## **ROBUST INPUT SHAPING FOR TWO-MASS SYSTEM WITH VARIABLE PARAMETERS**

Various mechanical systems are characterized with flexible joints. Exemplary this class of systems are the conveyers motors, the industrial robots and the cranes. The mechanical properties of those objects can be approximated by two-mass system. The paper presents the time-optimal input shaping methods for two-mass system. The control signal has to reach the set value of displacement in the shortest time, without of rising the mechanical oscillations. It was established, that the summary moment of inertia is constant, but division of mass is changing. For this case robustness for wide range of parameters deviations was analyzed. The model included PMSM motor with limited dynamics of current regulation. The research were conducted in Matlab/Simulink enviroment.

KEYWORDS: two mass system, input shaping, robustness

### **1. INTRODUCTION**

Many mechanical systems in can be characterized with resonant characteristics. This kind of systems is represented by the cranes [1], multi-mass systems [2][3], the conveyers and other systems.

Usually one of the requirements for the control system is to drive the machine without of vibrations. One of the control method used to complete this requirement is input shaping. Very often the jerk is limited, what effects with oscillations limitation, but leads to significant enlargement of reaction time. The paper shows another method, based on input shaping leading to prevent the oscillations. Simultaneously the dynamics parameters of driving system are maintained. The subject of the paper is the analysis of the robustness of this types of control methods for wide range of parameters deviations.

### **2. SIMULATION MODEL**

The research was based on two-mass system with flexible joint. The system is shown on Figure 1.

---

\* Poznan University of Technology.

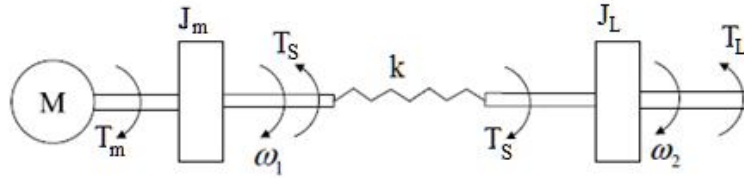


Fig. 1. Two-mass system

To build the simulation model of system it is necessary to have the physical equations describing it. The torque motor rotor  $J_m$  is equal to:

$$T_m = T_e - T_s, \quad (2.1)$$

where  $T_e$  is the electromagnetic torque and  $T_s$  is the torque transmitted through the shaft. For the load side mass, analogically:

$$T_l = T_s - T_L, \quad (2.2)$$

where  $T_l$  is the torque of second mass,  $T_L$  is the total torque of load including friction. The torque  $T_s$  can be calculated from equation

$$T_s = k_s \cdot (\theta_L - \theta_m) + d_s \cdot \left( \frac{d\theta_L}{dt} - \frac{d\theta_m}{dt} \right), \quad (2.3)$$

where  $\theta_L$  and  $\theta_m$  are the positions of both masses, and  $k_s$  and  $d_s$  are the coefficients of springiness and damping of the shaft respectively. The motor side and load side position are determined from:

$$\frac{d^2\theta_m}{dt^2} = \frac{T_m}{J_m} \quad (2.4)$$

and

$$\frac{d^2\theta_L}{dt^2} = \frac{T_l}{J_L}. \quad (2.5)$$

### 3. INPUT SHAPING

#### 3. 1. Command generation

The research presented in following chapters are based on impulse commands for vibration reduction. The base of command generation is to replace a single burst of force with multiple lower bursts, that summary supplies same amount of energy, but in smaller portions, applied in proper time. Each input signal can be calculated and shaped as a convolution of determined impulses. The amplitudes and times are possible

to calculate, if the basic characteristic values (like  $T_d$  – period of damped vibration and  $\omega_n$  – natural vibration pulsation) of the system are known. If the ratio  $R$  of load inertia to motor inertia is defined  $R=J_L/J_m$  then the natural vibration pulsation is determined by

$$\omega_n = \omega_a \sqrt{I+R} = \sqrt{\frac{k_s}{J_L}} \sqrt{I+R} \quad (3.1)$$

where  $\omega_a$  is the anti-resonance pulsation. The damping of the system is related to internal damping of the shaft  $d_s$ .

$$\zeta = \frac{d_s(I+R)}{2\omega_n J_L} \quad (3.2)$$

According to equations presented in [1], the times and amplitudes of impulses can be presented as:

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \frac{I}{I+K} & \frac{K}{I+K} \\ 0 & 0,5T_d \end{bmatrix} \quad (3.3)$$

where  $K$  is

$$K = e^{\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \quad (3.4)$$

If the model of two-mass system is known, the  $\omega_d$  and  $\omega_n$  are possible to calculate [2]. Equations shown above will provide the time-optimal control for a specific  $T_d$ , but if any of parameters will be changed, the control will not be optimal any more. This problem can be solved by modifications of the control algorithm. The modifications are shown in chapters 3.2 and 3.3. Robustness of the solution is shown in part 4.

### 3.2. Robust input shaping

The solution shown in part 3.1 is working really good if every part of process does not contain any uncertainties or inaccuracies. If the resonant frequency would change, one of the impulses will be set in not exact time or with not exact amplitude the whole process of control without of vibrations will fail. Additional problems, like uncertain parameters, will cause vibrations of whole system. To omit this problem it's necessary to use the robust version of the control algorithm. The first algorithm of robust control with input shaping was developed by Singer, Sighose and Seering [4]. The shaper was designed by requiring the partial derivative of the residual vibration to be equal to zero at the modeling frequency.

Enforcing this constraint has the effect of keeping the vibration near zero when the frequency starts to differ from the modeling frequency. The robust shaper is the simplest way to solve problem of uncertain parameters of the system. If all of the

parameters are nearly constant during the system exploitation and research this solution is usually sufficient to control the system with insignificant vibrations.

The robust shaper can be described with equations:

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \frac{I}{(I+K)^2} & \frac{2K}{(I+K)^2} & \frac{K}{(I+K)^2} \\ 0 & 0,5T_d & T_d \end{bmatrix} \quad (3.5)$$

The shaper described with equation (3.5) will cause vibrations at level not bigger than 5% for modeling frequency [1], but will provide robustness of the control.

### 3.3. Super-robust input shaping

The solution shown in part 2.2 works well if the parameters of the systems are not changing significantly during the exploitation. To omit this limitation the super-robust shaper has to be used.

The situation, when parameters of the system are changing in a wide range can be found in various real systems. For example the moment of inertia of an empty and fully loaded crane can be even hundreds time smaller. When the moment of inertia is changing also the resonant frequency is varying. In situations like this the solutions shown in parts 3.1 and 3.2 will be insufficient. The method of projecting the super-robust shapers is called “multi-hump EI shapers”. The input is divided into many small impulses convolution what effects with broaden the range of proper work of the control system. The main problem of the method is to select the frequency, that will be treated as the basis (modeling) frequency. The frequency of resonations will be changing with the parameters. The modeling frequency can be selected for example as the middle of the expected range. If the damping coefficient is very low, the super-robust shaper can be described with equation:

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} 0.125 & 0.375 & 0.375 & 0.125 \\ 0 & 0.5T_d & T_d & 1.5T_d \end{bmatrix} \quad (3.6)$$

where  $T_d$  is the period of modeling frequency.

## 4. SIMULATION TEST

The simulation tests were executed at the model described in section 2. To check the robustness of shapers the resonant frequency of the system has to be modified. To change the frequency the moments of inertia has to be modified. The sum of moments of inertia is constant and equal to the moment of inertia of single mass system. The moments of inertia are equal:

$$J_M = J_T \frac{I}{R+I} \quad (4.1)$$

$$J_L = J_T \frac{R}{R+I} \quad (4.2)$$

where  $J_L$  is the moment of inertia of load,  $J_M$  is the moment of inertia of the machine and  $J_T$  is sum of moments of inertia of both masses.  $R$  is the coefficient of mass division.

The test was based on supplying the torque to the system. The torque was selected to obtain the time-optimal movement for the reference system with non-flexible joint between the masses. The system was modeled as a single, rotating mass, characterized by the moment of inertia  $J_T$ . The control task was to rotate the mass by 40 rad, by speed limitation to 100 rad/s. The movement can be divided into three parts: acceleration, full-speed driving and deceleration. The reference and dual-mass characteristics were compared.

Figure 2 shows the input torque characteristics for selected input shapers. The time of movement was extended from 0.2 s. (for simple input shaper) up to 0.8 s (for the robust shapers).

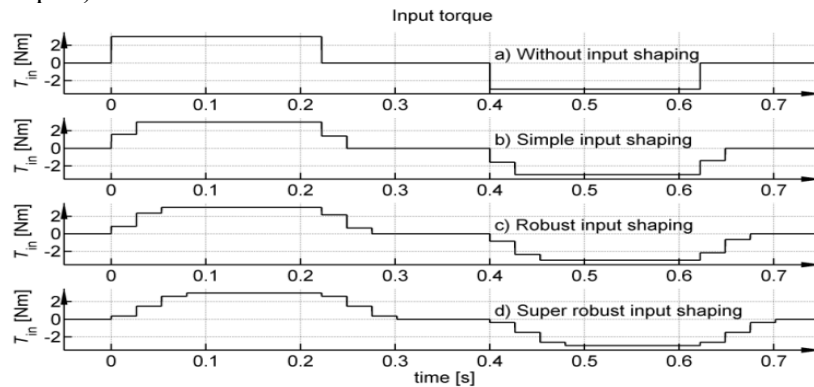


Fig. 2. Input torque for shapers

The shapers were calculated for the nominal situation, where the moments of inertia of drive and load are equal ( $R = I$ ). Afterwards, the solutions robustness for mass division changes was checked in range between  $R = 0.2$  up to  $R = 5$ .

Figure 3 shows the characteristics for  $R = I$ . The input shaper works properly, any significant oscillations do not appear. At right column the final phase of movement was shown. For the simple input shaper some small oscillations appear, for robust and super robust shaper oscillations are equal to zero. To compare the results, figure 4 presents the  $R=5$  case. For every shaper oscillations appear, but for the robust and super-robust shapers are less and less significant. For  $R = 0.2$  same conclusions can be set.

As the criteria of robustness the maximal difference between position of the machine and the load and the single mass was selected. The characteristics are shown at figure 5.

The characteristics shown on figure 5 confirms the theoretical deliberations described in part 3. The differences between all of strategies for the model frequency are not significant. If the frequency is deviating, the robustness is becoming more important. For both criteria the super-robust strategy is giving the best results. The simplest strategy will be sufficient for clearly specified model without of uncertainties. Applying the robust and super robust input shapers insignificantly enlarges the movement time, but the oscillations amplitude reduction is pointing.

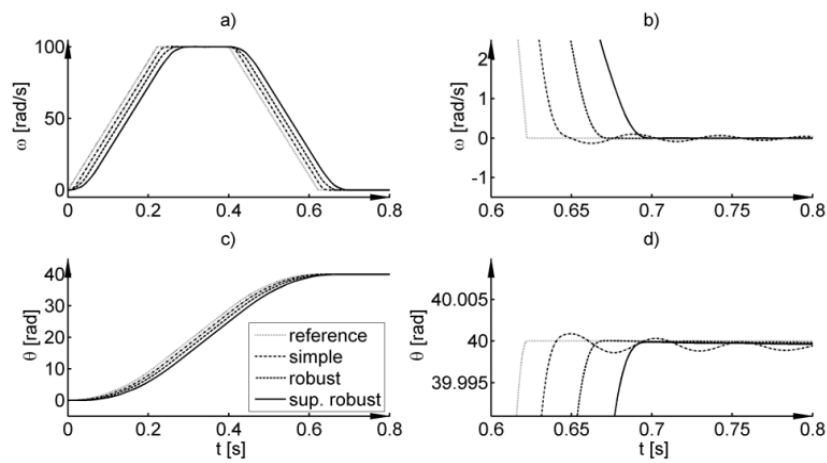


Fig. 3. Speed and position characteristics for various input shapers in case  $R = 1$ ;  
a) whole movement, b) final phase

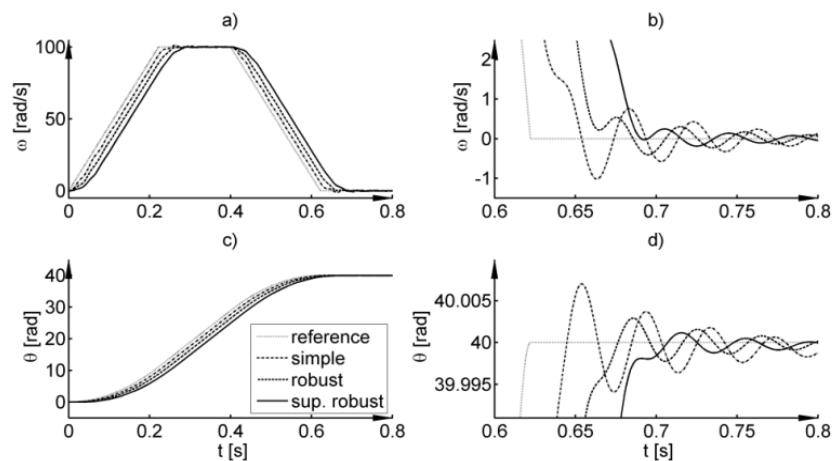


Fig. 4. Speed and position characteristics for various input shapers in case  $R = 5$ ;  
a) whole movement, b) final phase

## 5. CONCLUSION

The research has confirmed high robustness of presented methods of the input shaping. Those methods can be characterized by a small computational complexity and simple on-line realization. Approximated parameters of objects are sufficient to project the shaper. Shaping of closed-loop regulation signals for selected objects will be the subject of future research.

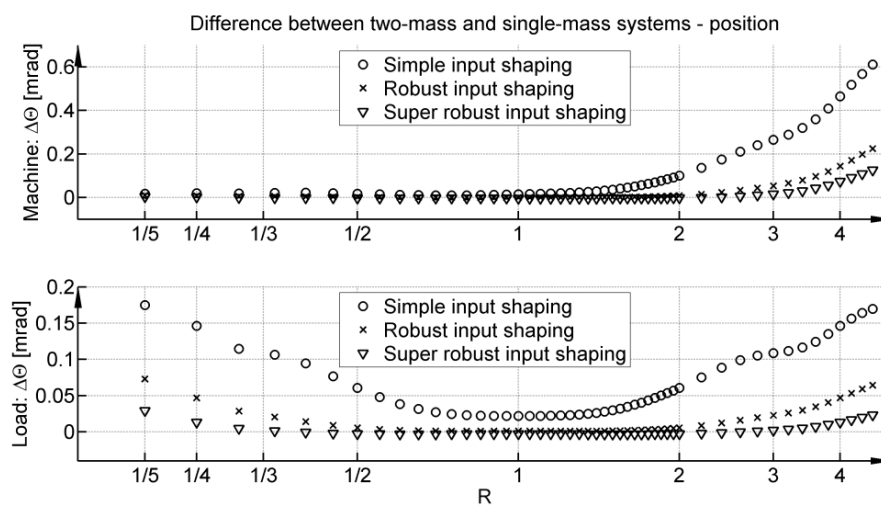


Fig. 5. Difference of the angular position between machine, load and single mass

## REFERENCES

- [1] W. Signhose and S. Warren, Command Generation for Dynamic Systems. 2011.
- [2] K. Szabat, "Direct and indirect adaptive control of a two-mass drive system—a comparison," in *Industrial Electronics, 2008. ISIE 2008. IEEE International Symposium on*, 2008, pp. 564–569.
- [3] N. Avdiu and S. Buza, "Analysis of the mutli-mass drive system dynamics with induction motor," Prague, Sep. 2011.
- [4] W. Signhose, N. Singer, and W. Seering, "Preshaping command inputs to reduce system vibration," vol. 1212, pp. 76–82, 1990.