

Self-adaptive whale optimization for the design and modelling of boiler plant

by

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Abstract: Recently, boiler plants are have been the subject of intensive investigations in the context of energy-saving technologies and management for power saving and reduction of emissions. Modern boiler design offers several benefits with this respect. In the past, improper design of boilers has been the cause of explosions which led to the loss of life and property. Modern designs attempt to avoid such mishaps. This paper presents a novel Self-Adaptive Whale Optimization Algorithm (SAWOA) for improving the learning characteristic of the neural network, the major intention being to model the characteristics of the boiler plant and so to effectively predict the boiler behaviour. The performance analysis of the introduced model has been carried out using the three test cases with consideration of several parameters. In the experimental analysis, the introduced technique is compared with the existing ones, based on such approaches as Neural Model (NM), Firefly (FF-NM), Adaptive Firefly NM (AFF-NM), and Whale Optimization Algorithm-NM (WOA-NM). In this comparison, the error, i.e. the difference between the actual and the predicted value, was used, and the results revealed that the error is lower for the introduced technique under different experimental scenarios. The experimental results demonstrate that the performance level of SAWOA is by 18% better than those of NM, FF-NM, and AFF-NM, and by 3.74% better than that of WOA-NM. This confirms the quality of performance of the proposed approach regarding boiler plants.

Keywords: boiler, whale optimization, neural model, temperature outlet, feed water flow

1. Introduction

In the search for the energy saving technologies, the boiler plants are subject to advanced analyses with the aim of finding appropriate designs and applications. In this context, the Circulated Fluidized Bed boiler (CFB) appeared

as an advanced boiler combustion technology. It has the ability of enhancing the desulfurisation and features low nitrogen oxide discharge. In addition, the evolution of CFB boiler technology associates the commitment for maximizing the energy-saving with large capacity parameters. Optimization and adequate parameter management are obligatory for meeting the requirements of the effective future use of CFB boilers. By optimizing the working parameters, it is possible to push forward the CFB boiler technology, leading to minimization of emissions and improvement of energy saving in boiler plants.

The first principle based modelling (see Flynn and Malley, 1999, or Wei, Wang and Wu, 2007) and the experimentally based modelling (Astrom and Bell, 2000; Kocaarslan and Cam, 2007) are the two leading boiler plant modelling schemes. The experimental modelling approaches are meant to manage the way the reality is reflected through the model and to adequately treat the essential nonlinear dynamics. The relationship between physics and the engineering principles as well as the true plant parameters are the main features of the first principle based modelling. Besides, the first principle based modelling has the ability of securing the algorithm evaluation. In any case, novel approaches have to be developed, with the use of the leading optimization methods for enhancement of boiler effectiveness.

The traditional approach and the so-called intelligent approach are the two broad approaches to optimizing the boiler operation parameters. The traditional approaches exploit the known characteristics of the system that include physical, mechanical and chemical properties, as well as the (differential) equations to model the boiler and carry out the design process. The experimental values, actual data, design values and the optimum values are present in the traditional method. These are employed to optimize the measures of boiler operation. Additionally, the traditional methods offer some definite benefits, such as, e.g., real-time updating with good probability properties. Nevertheless, the high investment of resource and manpower, equipment aging questions, difficulty in handling multiple parameters, survey and installation errors are the drawbacks of the traditional approach. The so-called intelligent approach is based on computational intelligence technologies and data mining methods. The deviation inspection, correlation analysis, prediction, and clustering are included in the scope of techniques applied. Fuzzy logic (Kocaarslan, Ertugrul and Tiryaki, 2006), genetic algorithms, pattern recognition and neural networks (Bahman and Ali, 2011; Chandok, Kar and Suneet, 2008; Rusinowski and Stanek, 2007; Bhatnagar, Kavita and Subhash, 2017) are often included in the intelligent approaches. Here, the real-time updating, strong maneuverability and solving complex modelling issues are the advantages. However, the longer running time for obtaining the adequate model fitting is the disadvantages. Hence, the novel advanced boiler power plant modelling schemes are welcome, leading to enhanced modelling and optimization.

The major contribution of this paper is the advance in the representation of the characteristics of boiler plant, meant for attaining an effective performance prediction of boiler behaviour. The performance analysis of the implemented technique has been carried out using three test cases with several parameters. In the experimental examination, the introduced model is compared with selected existing techniques, such as Neural Model (NM), Firefly (FF-NM) algorithm, Adaptive Firefly NM (AFF-NM) algorithm, and Whale Optimization Algorithm-NM (WOA-NM). Here, the error magnitude has been examined, and the results demonstrate that the error is minimal for the introduced SAWOA methodology under different experimental scenarios. The experimental results show, therefore, that the performance of SAWOA is better than of NM, FF-NM, WOA-NM and AFF-NM techniques.

2. Literature survey

In 2015, Beyhan and Kavaklioglu (2015) presented a method for modelling of U-tube Steam Generators (UTSG). In their work, the Artificial Neural Network (ANN), online and offline fuzzy system, based on extreme learning machine (ELM) were also presented for the use in the U-tube steam generators. The detection of water level of the UTSG system was exploited to secure the sufficient cooling capacity for the nuclear reactor and, at the same time, to prevent the damage of turbine blades. The root-mean-squared error and the minimum description length were analysed to measure the performance. It was observed that the extreme learning machine, ELM, possesses the advantages such as learning ability, higher degree of modelling precision and effective learning.

Secco et al. (2015) used a computational method to minimize the NO_x discharge in a 600MW tangentially-fired pulverized coal boiler. In this case, the genetic algorithm was exploited to produce the boiler settings automatically. In addition, the genetic algorithm was connected with the CFD (Computational Fluid Dynamics) simulations of the boiler in order to enhance the attained values of the target function. The proposed method minimizes the NO_x discharges while lowering the operational cost and corrosion. It was demonstrated that the genetic algorithm has the capability to handle operational parameters.. However, it has problems with the real-time performance.

Then, Sayed, Gharghory and Kama (2015) presented a novel hybrid jump PSO algorithm that was based on the Cauchy mutation and Gaussian distribution. It was exploited to tune the setting of PI controllers for the boiler-turbine unit. This new method was functioning on the basis of the observation of the global and local optimal particles of the PSO algorithm. The simulation results, presented in the paper, showed the enhanced optimization regarding the control measures. In addition, the main benefit of PSO is its high convergence rate. Since this algorithm has the ability of avoiding local optima and is dependent upon the algorithmic measures, it performs well in the given application ori-

ented problem.

A bit earlier, Liu et al. (2013) presented the modelling of the boiler unit having 1000 MW of capacity, as well as the ultra-supercritical property. The advanced optimizing boiler model was developed on the basis of an effective genetic algorithm and neural networks. Even though the developed algorithm works well within a broad scope of system parameter values, it suffers from several drawbacks, like, e.g. poor performance under the inaccurate formulation of the objective function.

In 2012, Kljajic, Gvozdenac and Vukmirovic (2012) presented an approach meant to improve the effectiveness of boilers. The approach relies on the operating performance measurement. The scheme was exploited for evaluating the effectiveness and the performance of arbitrarily chosen 65 boilers, located in Northern Serbia in 50 sites. The applied neural network shows an enhanced learning capability. Nevertheless, in this technique, the problems, related to randomness occurs because of the random initial settings.

Hengyan, Lingmei and Huahua (2011) considered the Circulating Fluidized Bed Boiler (CFB), where the structure of the network was optimized by using the Absolute Mean Impact Value (AMV). This was highly helpful for predicting the efficiency of the boiler, which, in turn, enhanced the predictive ability of the system. On the top of this, the best value was selected by the Genetic Algorithm (GA), performing successfully also under different loads.

3. Modelling of boiler plant

3.1. The preliminaries

Figure 1 demonstrates the procedure of the intelligent system model for the optimal design of boiler plant. Here, u_1 , u_2 and u_3 are the inputs, with u_1 denoting the fuel flow, u_2 signifying the input of the governor valve, and u_3 is the feed water flow. Then, y_1 , y_2 and y_3 are the three outputs from the system, where y_1 indicates the electric power, y_2 indicates the steam pressure, and y_3 indicates the outlet steam temperature. It should be emphasised that the output and input variables that are selected have a close association with the stability and the quality of performance of the power plants. A machine learning algorithm of Neural Network is used for predicting the output for the available input parameters.

3.2. Theoretical model

The thermal analysis of the boiler BP-1150 is carried out on the basis of the DIN 1942 norm. In Eq. (1), the balance, associated with the energy for the

boiler is presented:

$$\dot{E}_{IE_1} + \dot{E}_{IE_2} = \dot{Q}_{he_1} + \dot{E}_{el_1} + \dot{E}_{el_2} + \dot{Q}_{hl}. \quad (1)$$

Here, \dot{E}_{IE_1} and \dot{E}_{IE_2} represent the input energy flux, \dot{Q}_{he_1} and \dot{Q}_{hl} refer to the fuel flux, and \dot{E}_{el_1} and \dot{E}_{el_2} refer to the energy loss flux.

The energy flux, which appears in Eq. (1), can also be represented as follows.

At first, the input energy flux of the fuel is always proportional to the fuel flux, this being shown in Eq. (2). In Eq. (2), W_d^* indicates the fuel, which is characterised by a low heating value, and it is defined through Eq. (3). Subsequently, the input energy flux is independent of the fuel flux, as this is represented in Eq. (4). In addition, the flux of the consumed fuel is proportional to the energy failure flux, and it is represented in Eq. (5). Eq. (6) defines the loss due to the fuel gas used and unburned combustibles. Then, Eq. (7) represents the loss due to the unburned combustibles and enthalpy in the slag. The loss, due to the unburned combustibles in fuel dust and enthalpy is represented in Eq. (8).

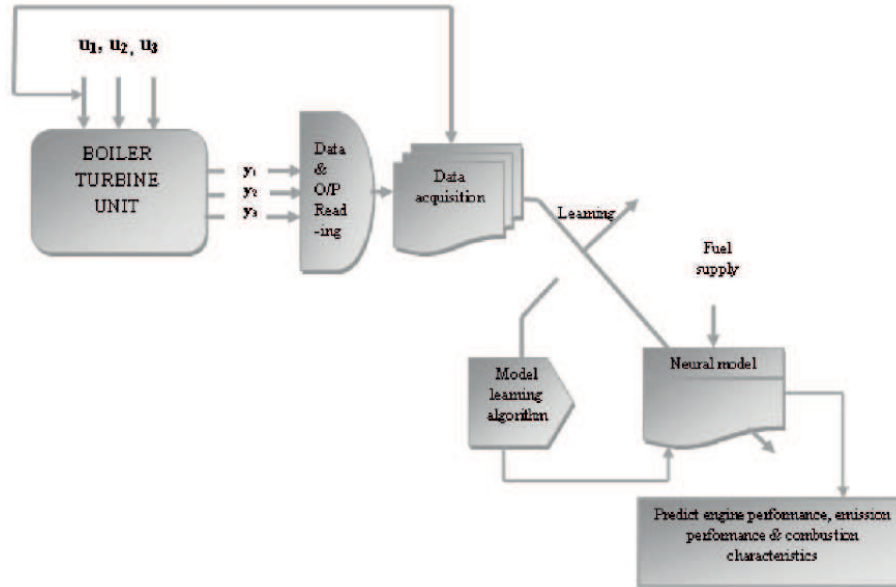


Figure 1. Block diagram of intelligent method for boiler design

$$\dot{E}_{IE_1} = \dot{C}W_d^* \quad (2)$$

$$W_d^* = W_d + i_{e_1} + j_{e_2} \quad (3)$$

$$\dot{E}_{IE_2} = \dot{Q}_{he_2} + N_{pfm} \quad (4)$$

$$\dot{E}_{el1} = \dot{E}_{gl} + \dot{E}_{enl1} + \dot{E}_{enl2} \quad (5)$$

$$\dot{E}_{gl} = \dot{C} (S_{pe} + S_{ce}) = \dot{C}S \quad (6)$$

$$\dot{E}_{enl1} = \dot{C}g_{A1} (i_{A1Pe} + i_{A1Ce}) = \dot{C}g_{A1} i_{A1} \quad (7)$$

$$\dot{E}_{enl2} = \dot{C}g_{A2} (i_{A2Pe} + i_{A2Ce}) = \dot{C}g_{A2} i_{A2} \quad (8)$$

From Eq. (9) it results that the energy loss flux is independent of the fuel flux. On the basis of the DIN 1942 norm, referred to, Eq. (10) is formulated. As we substitute the Eqs. (2) through (9) in Eq. (1), we obtain Eq. (11). We formulate the effectiveness of the boiler energy as the ratio of the advantageous heat flux per input energy of the boiler, as given in Eq. (12), which can also be formulated as Eq. (13).

$$\dot{E}_{el2} = \dot{Q}_{hCS} \quad (9)$$

$$\dot{Q}_{hl} = 0.0315\dot{Q}_{mup}^{0.7} \quad (10)$$

$$\dot{C}W_d^* + \dot{Q}_{hé2} + N_{pfm} = \dot{Q}_{hé1} + \dot{C} (S + g_{A1}i_{A1} + g_{A2}i_{A2}) + \dot{Q}_{hcs} + \dot{Q}_{hl} \quad (11)$$

$$\eta_{EK} = \dot{Q}_{hé1} \frac{\dot{Q}_{hé1}}{\dot{C}W_d^* + \dot{Q}_{hé2} + N_{pfm}} = \frac{\dot{Q}_{hé1}}{\dot{E}_{IE3}} \quad (12)$$

$$\eta_{EK} = 1 - \frac{\dot{C}S_{pe}}{\dot{E}_{IE3}} - \frac{\dot{C}S_{ce}}{\dot{E}_{IE3}} - \frac{\dot{C}(g_{A1}i_{A1} + g_{A2}i_{A2})}{\dot{E}_{IE3}} \dot{C} - \frac{\dot{Q}_{hcs} + \dot{Q}_{hl}}{\dot{E}_{IE3}}, \quad (13)$$

where

$$L_{rel} = \frac{\dot{C}S_{pe}}{\dot{E}_{IE3}} \quad (14)$$

$$L_{rel2} = \frac{\dot{C}(g_{A1}i_{A1pe} + g_{A2}i_{A2pe})}{\dot{E}_{IE3}} \quad (15)$$

$$L_{rel3} = \frac{\dot{C}(g_{A1}i_{A1ce} + g_{A2}i_{A2ce})}{\dot{E}_{IE3}} L_{rel4} = \frac{\dot{Q}_{hcs} + \dot{Q}_{hl}}{\dot{E}_{IE3}}. \quad (16)$$

Further, Eq. (16) can also be written as Eq. (17), and the obtained model is estimated on the basis of the indirect method. Among the factors, which influence boiler effectiveness, L_{rel} indicates the loss in fuel gas due to the unburned combustibles in slag and L_{rel3} denotes flue dust. This paper focuses on putting together the empirical methodology on the basis of the result presented in Liu et al. (2013) and Flynn and Malley (1999).

$$\eta_{EK} = 1 - L_{rel} - L_{rel1} - L_{rel2} - L_{rel3} - L_{rel4} \quad (17)$$

3.3. The NLARX model

Fig 2 presents the architecture of the model, referred to as the Nonlinear Autoregressive Exogenous Input (NLARX), see Deng, Stobart and Maass (2011). In Eq. (18), the NLARX model is schematically represented, with $Y(k)$ being a given output and $X(k)$ a given input. In addition, the number of past outputs is N and the number of past inputs is M , respectively.

$$Y(k) = F(Y(k-1), \dots, Y(k-N), X(k), \dots, X(k-M+1)) \quad (18)$$

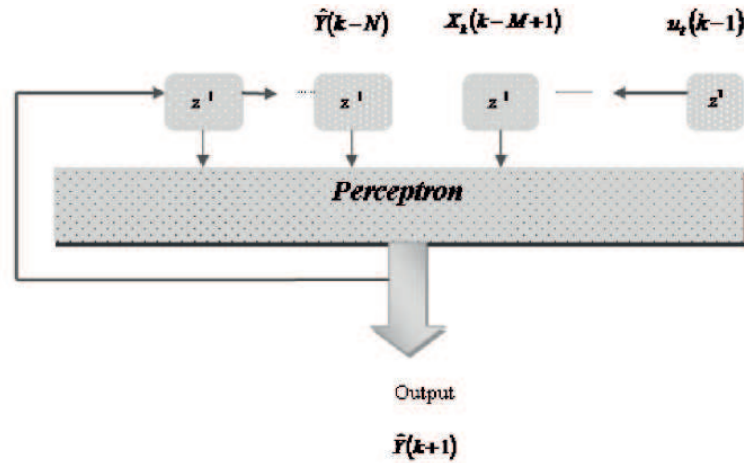


Figure 2. Diagrammatic representation of the NLARX model for modelling of the boiler plant

Here, the current output is predicted by employing the past input terms. So, the output from LARX is the effect of the changes, taking place in the inputs and the preceding output values. Hence, the combined regression function with two types of blocks – linear and nonlinear – is applied. In some cases also the conventional regressors are formed with the delayed input and output variables. Therefore, the problem of nonlinear unconstrained optimization has to be solved within the NLARX model. Eq. (19) represents again the essence of the model. In this equation, p represents the number of weighing parameters, Z_T represents the training library, $Y_t(k)$ represents the desired output, $Y_t^\wedge(k|\omega)$ represents the output from NLARX, and ω represents the weights. Further, $\|\cdot\|^2$ represents the L2-norm. Eq. (20) represents the training library and Eq. (21) shows the vector of weights.

$$\min_{\omega} e(\omega, z_T) = \frac{1}{2T} \sum_{t=1}^T \|Y_t(k) - Y_t^\wedge(k|\omega)\|^2 \quad (19)$$

$$Z_T = [Y_t(k), X(k)] \quad k = 1, \dots, T \quad (20)$$

$$\omega = [\omega_1, \dots, \omega_i, \dots, \omega_p]. \quad (21)$$

An error measure is treated as the performance index of the network, and it is represented in Eq. (19). The performance index represents the error of the network estimation for the definite training patterns. The values of ω , corresponding to the network parameters, have to be changed in order to possibly reduce the value of the $e(\omega, z_T)$ index over the whole trajectory.

The network must identify the characteristics of the boiler plant for effective modelling of the boiler plant. This identification consists, actually, appropriate setting of the associations among the inputs and the outputs of the boiler plant. An accurate knowledge base is needed to determine the relationship. A theoretical model or an empirical investigation is utilized to construct the knowledge base.

In empirical methodology, selected input and output variables of the boiler plant are considered. The set of these variables includes such magnitudes as water flow, spray flow, steam pressure in the throttle, steam temperature, water level in drum, electrical power, outlet temperature, and outlet steam pressure. It is known that the dynamic behaviour of the concrete boiler plant cannot be assessed for practical purposes through the theoretical computations. Hence, the effects of electric power and enthalpy on each of the inputs is determined. These are measured for other magnitudes possibly kept constant.

4. Learning of the boiler plant characteristics

4.1. The preliminaries

The mathematical description of single neurons was developed by the McCulloch and Pitts in 1942 (see McCulloch and Pitts, 1943; Hopfield, 1982). This description served as the basis for the construction of artificial neural networks. The input from one neuron and the output from the other neuron produce the signals, which move to the dendrite of the neuron. In addition, the estimation of the dendrite weights is employed to evaluate the intensity of the signal, arriving at the dendrite. Subsequently, taking the sum of those signals, or taking their sum multiplied by their corresponding weights provides the neurons activation functions argument. However, the representation model of the single neuron is considered as the issue due to the computational complexity. Here, the single neuron is interlinked in a net, which is the organized form of a lot of single neurons. Moreover, the signal moves from one layer to another layer in the organized form of the neural network, and that signal represents the input signal to a given layer. Generally, for the whole of the network, there is one input signal, one output signal and one or more hidden layers of a neural network. In the present work, the feed forward neural network model is implemented that has the capacity of performing multidimensional nonlinear estimation.

For the artificial neural network, a single training is required, and this capability is different and more effective than in the traditional methods. In each method, multiple iterative training is required to find the weights of neurons. These weights determine the error, i.e. the difference between the expected value z and the actual value, y . The applied objective function of the mean squared error (MSE) is presented in Eq. (22). It is not simple to realize the error delta factor $\delta = z - y$, appearing in Eq. (22), because the expected values z are not identifiable in hidden layers. Therefore, the individual error values are identified by employing the back propagation delta rule (see McCulloch and Pitts, 1943; Hopfield, 1982). It depends upon the weights of links between the consecutive hidden layers and the delta factor values δ of the next hidden layer. The back propagation method assesses the delta factor values of the output layers, which is returned back due to error propagation. In minimizing the objective function the gradient approach can be made use of, as represented in Eq. (23) – see below for the explanation of the working of this formula.

$$E = \frac{1}{2} \sum_{l=1}^N (z_j - y_j)^2 \quad (22)$$

$$w(s+1) = w(s) - \eta \nabla E(w(s)) + \alpha \Delta w(s-1) \quad (23)$$

The weight of the neurons is established by employing the Eq. (23). The error back propagation method in neural networks has been affirmed as providing good results in neural network training. Nevertheless, for a system showing strongly nonlinear relationships between the output and input data, the standard back propagation method may not be sufficiently effective. Consequently, the SAWOA technique is proposed in this study in order to replace the standard back propagation procedure, so that, as a result, the nonlinearities can be accurately identified and rendered.

4.2. The proposed learning method

In 2016, Mirjalili and Lewis (2016) (see also Ling, Zhou and Luo, 2017) introduced the meta-heuristic whale optimization algorithm (WOA), which is derived from the humpback whale hunting behaviour, referred to as bubble net hunting. The WOA is a population-based technique. This optimization algorithm follows three basic behavioural concepts, namely bubble-net foraging behaviour, encircling prey, and search for prey.

In this paper, the SAWOA is exploited to replace the back propagation model for the purpose of exact identification of the nonlinearities. Here, Z refers to the optimal solution for neural network output, and z represents the actual neural network output.

In the conventional WOA, the humpback whales identify the position of the prey and encircle them. The WOA algorithm assumes that the current best candidate solution is the target prey or is close to the optimum. Once the best solution is selected, the other agents (solutions) update their positions towards

the best search agent, this being represented in Eqs. (24) and (25), where I stands for the current iteration number, B and V represent the coefficient vectors, Z^* represents the position vector of the optimal solution, and $||$ denotes the absolute value.

$$D = |V.Z^*(i) - Z(i)| \quad (24)$$

$$Z(i+1) = Z^*(i) - B.D. \quad (25)$$

The values of vectors B and V are calculated on the basis of Eqs. (26) and (27),

$$B = 2ar - a \quad (26)$$

$$V = 2r. \quad (27)$$

In Eq. (26), r is a random vector and the components a and a_2 are first generated for $i = 1$ (fixed value), as this is represented in Eqs. (28) and (29).

$$a = 2 - i \left(\frac{2}{\max^i} \right) \quad (28)$$

$$a_2 = -1 + i \left(\frac{-1}{\max^i} \right). \quad (29)$$

Now, regarding the values of a and a_2 , Eqs. (30) and (31) are applied when $i = 2$. In these equations, f_t represents the fitness function.

$$a = 2 - (\max(f_t) - f_t(i-1)) * \frac{2}{\max(f_t)} \quad (30)$$

$$a_2 = \left(-1 + \max(f_t) - f_t(i-1) * \frac{-1}{\max(f_t)} \right) \quad (31)$$

Eqs. (32) and (33) are utilized to include weights in the calculation of the components a and a_2 . Here, w indicates the weight.

$$a = \left(2 - (\max(f_t) - f_t(i-1)) * \frac{2}{\max(f_t)} \right) * w \quad (32)$$

$$a_2 = \left(-1 + \max(f_t) - f_t(i-1) * \frac{-1}{\max(f_t)} \right) * w \quad (33)$$

In the spiral position updating, the humpback whales attack the prey. Here, the distance between the prey location and the whale location is estimated. Subsequently, the helix shaped movement of a humpback is produced using Eq. (34):

$$Z(i+1) = D' . e^{bm} . \cos(2\pi m) + Z^*(i) \quad (34)$$

where $D' = |Z^*(i) - Z(i)|$ represents the distance between the optimal solution (prey) and the n^{th} whale, b is a constant, m denotes a number contained in the interval $[-1, 1]$, calculated according to Eq. (35):

$$m = (a_2 - 1) * r + 1. \quad (35)$$

The spiral position updating is given in Eq. (36), where q represents the random number, which is uniformly distributed between 0 and 1.

$$Z(i+1) = \begin{cases} Z^*(i) - B \cdot D & \text{if } q < 0.5 \\ D' \cdot e^{bm} \cdot \cos(2\pi m) + Z^*(i) & \text{if } q \geq 0.5 \end{cases} \quad (36)$$

In the search for prey, the humpback whales are the search agents and the search for prey is the search for the optimal solution (which can, in general, also change its position). The agents change their positions on the basis of the positions of other whales. To force the search agent to go away from the reference whale (avoiding of local solutions), B is set as greater than 1, otherwise it is lower than 1 (no enforcing of bigger distance from the reference whale). The search for prey is represented in Eqs. (37) and (38):

$$D = |V \cdot Z_{rand}(i) - Z(i)| \quad (37)$$

$$Z(i+1) = Z_{rand} - B \cdot D \quad (38)$$

where X_{rand} refers to the random position vector that is selected from the current population.

The verbal description of the pseudocode of the algorithm, given further on, is as follows:

1. The population of the whales (of the search agents) is initialized as Z_i , where $i = 1, 2, \dots, n$.
2. Subsequently, the fitness of each search agent is computed and Z^* is assigned to the best search agent.
3. The values of B , V , a , m , q are updated using Eq. (26), (27), (28) and (35).
4. If $q < 0.5$, the position of current search agent is updated by Eq. (24) for $|B| < 1$, while it is updated using Eq. (38) for $|B| \geq 1$.
5. Current positions of the search agents are updated using Eq. (34) for $q \geq 0.5$.
6. The fitness of each search agent is computed and Z^* is updated.
7. The process is repeated until the entire iteration is completed.

5. Results and discussion

5.1. The procedure

The proposed SAWOA was tested and compared with selected existing methods such as NM, FF-NM, AFF-NM, WOA-NM, and SAWOA-NM. The experimental

Pseudocode 1: Proposed SAWOA algorithm for learning the boiler plant characteristics	
Initialize the population Z_i ($i = 1, 2, \dots, n$) and M_{gen} (Maximum number of generations)	
Let $i = 1$	
Compute the fitness of Z_i ($i = 1, 2, \dots, n$)	
	For every Z_i ($i = 1, 2, \dots, n$), update a, V, B, m, q
	if ($q < 0.5$)
	if ($ B < 1$)
	Update the current search agent position using Eq. (24)
	Else if ($ B \geq 1$),
	Choose a random search solution Z_{rand}
	Update the current search agent position using Eq. (38)
	End if
	Else if ($q \geq 0.5$)
	Update current search agent position using Eq. 34
	End if
	End for
	Verify if any search agent has moved away from the search area and alter it.
	Compute the fitness of Z_i ($i = 1, 2, \dots, n$)
	if there is an enhanced search solution, update Z^* .
	Let $i = i + 1$
	When i reaches M_{gen} , the algorithm is completed
	Return the optimal solution Z^* and the optimal value of the fitness function

study has been carried out in the MATLAB platform. In this experiment, ten parameters are examined in order to effectively analyse the developed model. The parameters considered include steam pressure, outlet temperature, steam pressure at the throttle, electrical power, spray water flow, water level in the drum, steam flow feed, steam pressure in the drum, water flow, and steam temperature. In addition, the experimentation was performed for three test cases, and the values of the parameters considered were obtained from Liu et al. (2013) and Flynn and Malley (1999). In the first test case, such parameters as electrical power, outlet temperature and steam pressure have been considered. The remaining parameters have been considered in the test cases 2 and 3. Furthermore, the test case 2 has been split into the “a” and “b” subcases.

5.2. Performance analysis

We shall now present some illustrations, related to the comparative analysis of performance of the techniques considered for the particular test cases, as introduced above, against the background of the plant model. It must be emphasised that with respect to numerous characteristics the techniques behave very similarly, and so we are showing here only those diagrams, which actually present some visible differences between these techniques in terms of performance. Thus, Fig. 3 refers to case 1, Fig. 4 to case 2 (a), Fig. 5 to case 2 (b), and finally Fig. 6 to case 3. As said, in the first case three variables were analysed: electrical power, steam pressure, and outlet temperature. For this case, only the performance with respect to electrical power differed in any visible manner, and it is shown here in Fig. 3.

In Fig. 4, selected results for the test case 2 (a), with seven parameters, are presented. The quantities considered in this test case are: 1. feed water flow, 2. spray water flow, 3. steam flow, 4. steam pressure at the throttle, 5. steam pressure in the drum, 6. water level in the drum, and 7. steam temperature. Here, the sole really telling differences occur for feed water flow (Fig. 4 (a)), equal 4.22% for the proposed SAWOA-NM and for the FF-NM. For the above listed parameters no significant differences in performance are observed with regard to parameters 2 through 5, and hence the respective curves are not shown. However, for water level in the drum and for steam temperature, the differences are observed of 18% and 3%, respectively, for the proposed and the compared techniques, as this is illustrated in Figs. 4 (b) and 4 (c).

In Fig. 5, corresponding to the test case 2 (b), the selected courses from among those of seven parameters, i.e. 1. feed water flow, 2. spray water flow, 3. steam temperature, 4. steam flow, 5. steam pressure at the throttle, 6. steam pressure in the drum, and 7. water level in drum are shown for the proposed and the compared methodologies. Here, the deviation of 3.74% occurs for the proposed SAWOA-NM in terms of feed water flow (Fig. 5 (a)). Two other illustrated variables, for which distinct differences are observed, and are

here illustrated, are steam flow (Fig. 5 (b)), and water level in drum (Fig. 5 (c)).

In Fig. 6, the comparison between the proposed and the other techniques for test case 3 is illustrated. Namely, it can be seen that the proposed technique differs from the AFF-NM technique with respect to feed water flow. For other variables, some differences occur for individual samples or their short segments, but they are not illustrated here, since these differences are, actually, not visible.

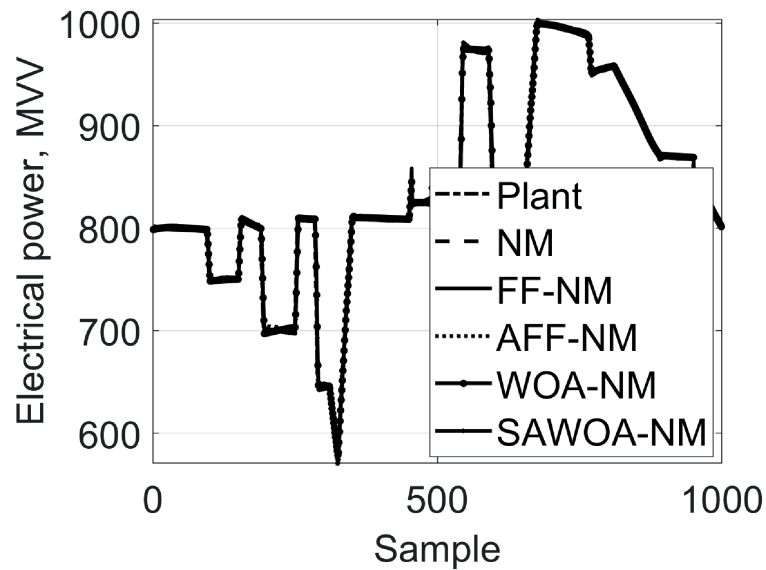


Figure 3. Graphical representation of comparison among the outputs of the proposed and conventional approaches for test case 1 regarding the variable of electrical power, performance with regard to other variables being very similar

Table 1 and the following ones contain the values of the error, appearing between the plant and the proposed SAWOA-NM as well as the conventional NM, FF-NM, AFF-NM, and WOA-NM. The three considered cases, Case 1, Case 2 and Case 3 are shown, and the error for different parameters is provided. In addition, the proportions of 25% to 75% of the experimental data have been also considered. From those tables, it can be concluded that the proposed model is better than the conventional models in terms of error reduction.

NM: neural model; WOA: whale optimization

5.3. Shrinking effect

The shrinking effect has been studied here by varying the value of q , so as to control the shrinking or spiral updating behaviour of the whales. If q is greater than or equal to 0.5, then the Shrinking effect will occur. If q is below 0.5, no

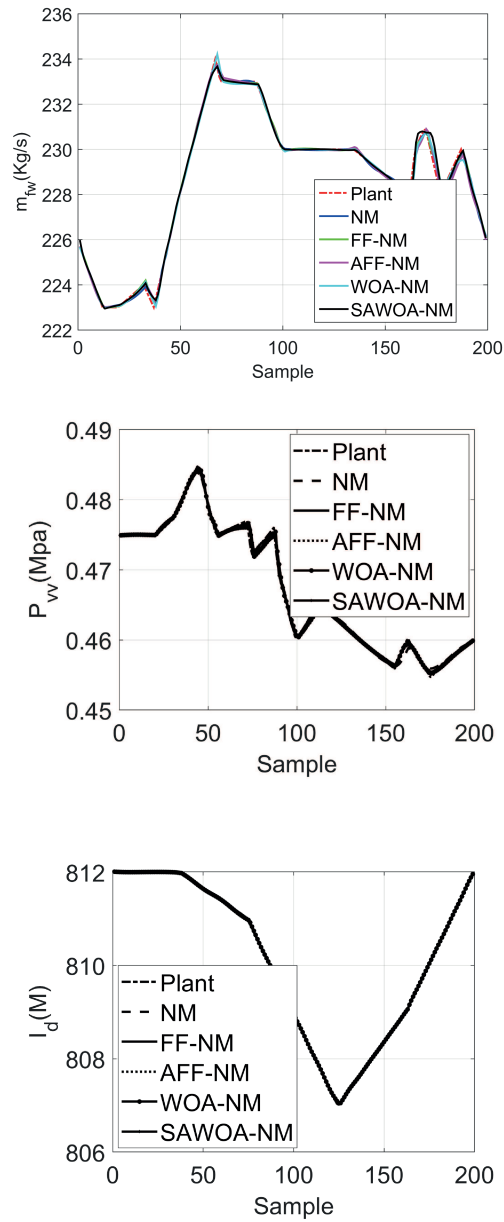


Figure 4. Graphical representation of comparisons between the output of the proposed and other compared approaches for the test case 2(a): (a) feed water flow, (b) water level in the drum, and (c) steam temperature

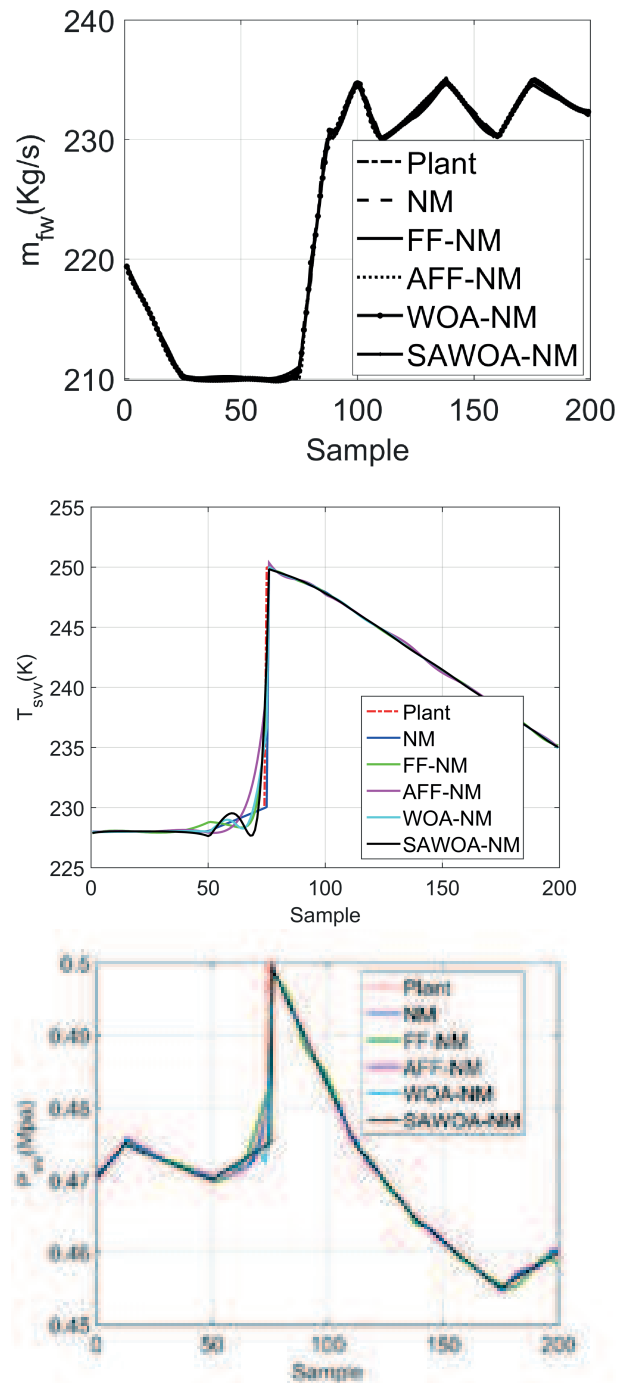


Figure 5. Graphical representations of comparisons among the outputs of proposed and conventional approaches for the test case 2(b): (a) feed water flow, (b) steam flow, and (c) water level in drum

Table 1. Error values for the proposed and conventional models for the test case 1

Percentage of experimental data	25					50					75					
	Approaches	NM	FF-NM	AFF-NM	WOA	WOA-NM	NM	FF-NM	AFF-NM	WOA	WOA-NM	NM	FF-NM	AFF-NM	WOA	WOA-NM
Case 1																
Electrical power	13.87	5.37	4.24	9.66	5.66	4.34	8.53	6.62	4.38	8.62	6.98	4.55	7.56	4.42	3.95	
Steam pressure	.15	.1	.098	.14	.09	.099	.13	.09	.1	.15	.09	.1	.12	.08	.095	
Outlet temperature	.79	.54	.61	1.1	.5	.62	1.03	.54	.62	1.19	.49	.61	.99	.48	.61	

Table 2. Error values for the proposed and conventional models for the test case 2a

Percentage of experimental data	25					50					75				
	NM	FF-NM	AFF-NM	WOA	WOA-NM	NM	FF-NM	AFF-NM	WOA	WOA-NM	NM	FF-NM	AFF-NM	WOA	WOA-NM
Case 2 (a)															
Feed water flow	.93	3.74	1.31	1.63	7.96	1.21	1.21	8.37	1.63	1.02	1.52	1.44	1.49	1.4	.87
Steam pressure at throttle	.03	.03	.04	.03	.03	.05	.02	.03	.03	.03	.03	.02	.02	.03	.02
Steam pressure in drum	.02	.03	.02	.02	.03	.02	.02	.03	.02	.02	.03	.01	.02	.03	.02
Spray water flow	.27	.03	.02	.25	.03	.02	.42	.03	.02	.33	.06	.01	.23	.03	.01
Water level in the drum	.0031	.0037	.0041	.002	.005	.0034	.0017	.0062	.008	.0026	.0059	.0029	.0014	.0038	.0022
Steam flow	.75	3.87	.1	1.33	3.19	.96	.78	2.72	.87	2.64	3.89	2.62	.68	4.38	.81
Steam temperature	.24	.29	.42	.22	.28	.78	.21	.33	.48	.22	.28	.57	.2	.28	.42

Table 3. Error values for the proposed and conventional models for the test case 2b

Percentage of experimental data	25					50					75					
	Approaches	NM	FF-NM	AFF-NM	WOA	WOA-NM	NM	FF-NM	AFF-NM	WOA	WOA-NM	NM	FF-NM	AFF-NM	WOA	WOA-NM
Case 2 (b)																
Feed water flow	.8	.59	1.02	.33	.64	.59	.34	.61	.42	.32	.66	.51	.3	.59	.28	
Steam pressure at throttle	.01	.01	.02	.0095	.01	.01	.0097	.01	.01	.01	.01	.0095	.0094	.01	.0089	
Steam pressure in drum	.01	.01	.0083	.0078	.01	.0079	.0078	.01	.01	.0079	.01	.0084	.0077	.01	.0058	
Spray water flow	.09	.01	.009	.09	.01	.0074	.09	.01	.0078	.09	.01	.0071	.09	.01	.007	
Water level in drum	.0007	.0017	.001	.0011	.0017	.001	.00078	.002	.0015	.00099	.0021	.0009	.0007	.0017	.0012	
Steam flow	.34	1.64	1.17	.31	1.16	.37	.36	1.33	.64	1.28	1.67	.32	.66	1.15	.39	
Steam temperature	.08	.11	.19	.08	.11	.26	.08	.11	.18	.08	.12	.18	.08	.11	.17	

Table 4. Error values for the proposed and conventional models for the test case 3

Percentage of experimental data	25					50					75				
	NM	FF-NM	AFF-NM	WOA	WOA-NM	NM	FF-NM	AFF-NM	WOA	WOA-NM	NM	FF-NM	AFF-NM	WOA	WOA-NM
Case 3															
Feed water flow	.15	.25	.12	0.15	0.25	0.11	0.17	0.16	0.86	0.17	0.18	0.32	0.18	0.1	0.09
Steam pressure at throttle	.00094	.00045	.00022	0.00079	0.00034	0.0029	0.001	0.0006	0.0009	0.001	0.0005	0.00096	0.0006	0.00007	0.00023
Steam pressure in drum	.00022	.00011	.0013	0.0002	0.00011	0.0013	0.0002	0.00037	0.0001	0.00016	0.00074	0.0013	0.0013	0.00059	0.0012
Spray water flow	.0059	.00068	.0015	.0059	.00021	.0014	.0067	.00053	.0011	.0071	.00025	.0013	.0008	.0002	.00099
Water level in drum	.0005	.0013	.00019	.00035	.0016	.00028	.00036	.14	.00027	.0004	.00097	.00019	.00035	.0018	.00018
Steam flow	.09	1.41	.05	.07	.92	.05	.05	1.19	.05	.06	1.09	.05	.09	0.86	.04
Steam temperature	.0032	.01	.02	.0048	.0061	.02	.0044	.0096	.02	.0097	.0089	.02	.0026	.0079	.016

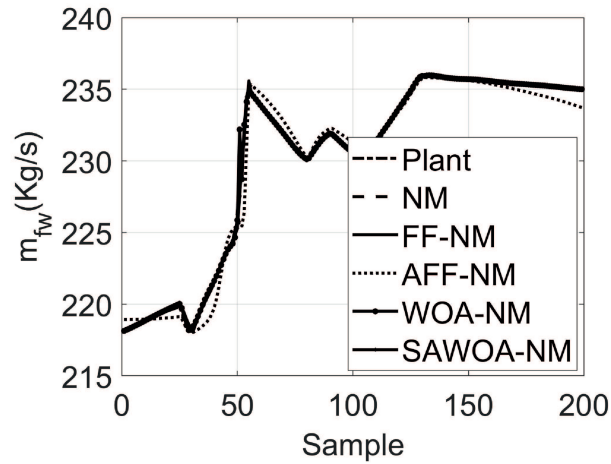


Figure 6. Graphical representation of comparison among the outputs of the proposed and other approaches for the test case-3, for feed water flow

shrinking effect occurs. In the experiment, q is varied from 0.1 to 1 and the results obtained are shown in Tables 2 through 5.

Table 5 shows the error values for the proposed model for test case 1, involving three parameters, depending upon the shrinking threshold values (0.1 to 1). Likewise, Tables 6, 7 and 8 show the error values for the proposed model for, respectively, the test cases 2 (a), 2 (b), and 3, with respect to the changing value of the shrinking threshold.

5.4. Analysis of weight influence

On the basis of Eqs. (32) and (33), by varying the weights between 0.1 and 1, the analysis of weights has been performed. In Table 9, error values for the proposed model are shown, obtained for varying weight values (0.1 to 1) for the three parameters of the test case 1. Tables 10, 11 and 12 show analogous results for the test cases 2 (a), 2 (b) and 3, respectively.

6. Conclusions

In this paper, a novel self-adaptive WOA-NM methodology was proposed for purposes of modeling the characteristics of a boiler plant. Using this novel technique, effective prediction of boiler behaviour can be obtained. Moreover, the here proposed self-adaptive WOA was utilized to enhance the learning characteristic of the neural network. The performance of the introduced methodology was assessed for four different test cases. The test cases involved different subsets of parameters, including those related to spray water flow, feed water flow, steam temperature, steam pressure at the throttle, steam pressure in the drum, steam

Table 5. Error values for the proposed model for test case 1 corresponding to varying shrinking threshold

Shrinking threshold	Electrical power	Steam pressure	Temperature
0.1	4.63	0.07	0.58
0.2	4.42	0.09	0.67
0.3	4.44	0.1	0.62
0.4	4.09	0.06	0.6
0.5	4.64	0.07	0.61
0.6	4.6	0.08	0.5
0.7	4.59	0.09	0.52
0.8	4.47	0.07	0.41
0.9	3.89	0.06	0.48
1	4.35	0.09	0.56

Table 6. Error values for of the proposed model for test case 2 (a) corresponding to varying shrinking threshold value

Shrinking threshold	feed water flow	steam pressure in drum	spray water flow	steam flow	steam pressure at throttle	water level in drum	steam temperature
0.1	0.17	0.0003	0.0061	0.33	0.0013	0.0005	0.0039
0.2	0.13	0.004	0.0079	0.05	0.00049	0.004	0.0039
0.3	0.12	0.31	0.0048	0.08	0.0013	0.00034	0.0054
0.4	0.17	0.0002	0.0066	0.09	0.0013	0.00038	0.0043
0.5	0.17	0.00032	0.0078	0.05	0.0011	0.00035	0.0053
0.6	0.16	0.00035	0.0057	0.78	0.00095	0.00036	0.01
0.7	0.16	0.00041	0.0066	0.13	0.00075	0.00041	0.0061
0.8	0.15	0.00028	0.0079	0.05	0.0013	0.00039	0.0061
0.9	0.16	0.0001	0.0052	0.06	0.0009	0.00046	0.004
1	0.14	0.00053	0.0076	0.07	0.0013	0.00047	0.0051

Table 7. Error values for the proposed model for test case 2 (b) corresponding to varying shrinking threshold value

Shrinking threshold	feed water flow	steam pressure in drum	spray water flow	steam flow	steam pressure at throttle	water level in drum	steam temperature
0.1	0.1	0.00034	0.00038	1.39	0.00026	0.0018	0.0065
0.2	0.19	0.00048	0.00089	1.13	0.00082	0.0017	0.0068
0.3	0.19	0.00059	0.00047	1.18	0.0006	0.0097	0.01
0.4	0.15	0.0004	0.00043	1.07	0.00042	0.0013	0.0062
0.5	0.14	0.00037	0.00037	1.08	0.0004	0.0018	0.012
0.6	0.13	0.00074	0.00045	1.08	0.00031	0.0018	0.0074
0.7	0.15	0.00016	0.0004	0.8	0.00024	0.0018	0.008
0.8	0.15	0.00046	0.00068	1.41	0.00031	0.002	0.0068
0.9	0.46	0.00054	0.00056	1.41	0.00038	0.0011	0.0067
1	0.19	0.00083	0.00032	1.09	0.00021	0.0018	0.0097

Table 8. Error values for the proposed model for the test case 3 corresponding to the varying shrinking threshold values

Shrinking threshold	feed water flow	steam pressure in drum	spray water flow	steam flow	steam pressure at throttle	water level in drum	steam temperature
0.1	0.35	0.0011	0.0051	0.07	0.00098	0.00022	0.02
0.2	0.81	0.0014	0.0013	0.08	0.0015	0.00024	0.01
0.3	0.19	0.0019	0.0011	0.1	0.00087	0.00048	0.02
0.4	0.15	0.0024	0.0011	0.05	0.0041	0.00038	0.01
0.5	0.23	0.0012	0.0014	0.18	0.0013	0.00031	0.01
0.6	0.37	0.001	0.0012	0.05	0.0014	0.00064	0.02
0.7	0.13	0.0015	0.0017	0.06	0.0012	0.00041	0.03
0.8	0.18	0.0023	0.0014	0.05	0.0061	0.00032	0.02
0.9	0.15	0.001	0.0012	0.05	0.0036	0.00043	0.02
1	0.12	0.0014	0.0014	0.09	0.0013	0.00021	0.02

Table 9. Error values for the proposed model for test case 1 with varying weights

Weights	Electrical power	Steam pressure	Temperature
0.1	4.6	0.097	0.57
0.2	4.56	0.089	0.5
0.3	4.27	0.046	0.55
0.4	4.15	0.09	0.61
0.5	3.99	0.1	0.37
0.6	4.32	0.1	0.59
0.7	4.39	0.09	0.61
0.8	4.19	0.09	0.59
0.9	4.49	0.1	0.6
1	4.53	0.1	0.61

Table 10. Error values for the proposed model for test case 2 (a), corresponding to varying weights

Weights	feed water flow	steam pressure in drum	spray water flow	steam flow	steam pressure at throttle	water level in drum	steam temperature
0.1	0.16	0.00025	0.0079	0.16	0.00085	0.00031	0.0038
0.2	0.16	0.00046	0.0088	0.27	0.00093	0.00039	0.0072
0.3	0.15	0.0003	0.012	0.08	0.00078	0.0005	0.0034
0.4	0.17	0.00026	0.0058	0.05	0.00093	0.00042	0.0056
0.5	0.11	0.00024	0.0062	0.26	0.00099	0.00041	0.01
0.6	0.12	0.0027	0.01	0.05	0.00095	0.00054	0.0048
0.7	0.15	0.00042	0.0061	0.08	0.00012	0.0004	0.003
0.8	0.1	0.00033	0.007	0.07	0.0005	0.00036	0.0047
0.9	0.16	0.0003	0.0083	0.09	0.00011	0.00038	0.0077
1	0.21	0.00024	0.0075	0.08	0.00011	0.00048	0.0054

Table 11. Error values for the proposed model for the test case 2 (b), corresponding to varying weight values

Weights	feed water flow	steam pressure in drum	spray water flow	steam flow	steam pressure at throttle	water level in drum	steam temperature
0.1	0.13	0.00031	0.00022	1.21	0.00039	0.0014	0.0089
0.2	0.12	0.00042	0.00052	1.02	0.000418	0.0012	0.0072
0.3	0.084	0.00073	0.0003	1.4	0.00011	0.0015	0.01
0.4	0.16	0.00057	0.00028	1.42	0.00019	0.0018	0.0097
0.5	0.1	0.00046	0.00019	1.41	0.00036	0.0018	0.0086
0.6	0.18	0.00033	0.00037	1.1	0.00034	0.0016	0.0087
0.7	0.18	0.00029	0.0013	1.15	0.0010	0.0018	0.01
0.8	0.12	0.00076	0.00011	1.19	0.00051	0.0018	0.01
0.9	0.21	0.00025	0.00035	1.13	0.00047	0.0018	0.0081
1	0.18	0.0006	0.00016	0.85	0.00045	0.0018	0.0077

Table 12. Error deviation of the proposed model for test case 3, corresponding to varying weight values

Weights	feed water flow	steam pressure in drum	spray water flow	steam flow	steam pressure at throttle	water level in drum	steam temperature
0.1	0.29	0.0011	0.0018	0.03	0.0019	0.00033	0.01
0.2	0.11	0.0011	0.0015	0.04	0.0015	0.00042	0.02
0.3	0.13	0.0014	0.0023	0.04	0.0028	0.00049	0.03
0.4	0.12	0.0015	0.0015	0.03	0.0036	0.00025	0.023
0.5	0.15	0.0012	0.0014	0.08	0.0021	0.00038	0.02
0.6	0.96	0.0018	0.0013	0.04	0.002	0.00043	0.02
0.7	1.56	0.0032	0.0019	0.05	0.0013	0.00015	0.02
0.8	0.15	0.0013	0.0019	0.07	0.0054	0.00027	0.02
0.9	0.13	0.0013	0.0013	0.13	0.0022	0.00028	0.03
1	0.11	0.0015	0.0014	0.06	0.00039	0.00019	0.03

flow, and water level in the drum. The proposed methodology was compared with the such existing methodologies as NM, FF-NM, AFF-NM, WOA-NM in order to verify the efficiency of the proposed approach. In the analysis, error values have been calculated for respective boiler parameters and their dependence upon the parameters of the procedure was also analysed. Finally, the simulation results demonstrated that the model, obtained with the new methodology, coincides with the behaviour of the actual boiler plant with minimum error.

References

- ASTROM, K.J AND BELL, R.D. (2000) Drum boiler dynamics. *Automatica*, **36**(3): 363–378, March.
- ATHANASIOS, N., NIKOLAOS, N., NIKOLAOSA, M., PANAGIOTIS, G. AND KAKARAS, E. (2015) Optimization of a log wood boiler through CFD simulation methods. *Fuel Processing Technology*, **137**, 75-92, September.
- BAHMAN, Z. N. AND ALI, A. (2011) Accurate prediction of the dew points of acidic combustion gases by using an artificial neural network model. *Energy Conversion and Management*, **52**(2): 911–916, February.
- BEYHAN, S. AND KAVAKLIOGLU, K. (2015) Comprehensive Modeling of U-Tube Steam Generators Using Extreme Learning Machines. *IEEE Transactions on Nuclear Science*, **62**(5): 2245-2254.
- BHATNAGAR, K. AND SUBHASH, G. (2017) Extending the Neural Model to Study the Impact of Effective Area of Optical Fiber on Laser Intensity. *International Journal of Intelligent Engineering and Systems*, **10**, 274-283.
- CHANDOK, J. S., KAR, I. N. AND SUNEET, T. (2008) Estimation of furnace exit gas temperature (FEGT) using optimized radial basis and back-propagation neural networks. *Energy Conversion and Management*, **49**(8): 1989–1998, August.
- DENG, J., STOBART, R. AND MAASS, B. (2011) The Applications of Artificial Neural Networks to Engines. In: K. Suzuki, ed., *Artificial Neural Networks - Industrial and Control Engineering Applications*. IntechOpen, 309-332, DOI: 10.5772/15783
- DIN 1942 STANDARD (1994) Acceptance testing of steam generators, *edition 02*.
- FLYNN, M. AND MALLEY, M. (1999) A drum boiler model for long term power system dynamic simulation. *IEEE Trans Power Syst*, **14**(1): 209–217, February.
- HE, X. AND YANG, X. S. (2013) Firefly Algorithm: Recent Advances and Applications. *Int. J. of Swarm Intelligence*, **1**, 1, 36-50.
- HENGYAN, X., LINGMEI, W. AND HUAHUA, C. (2011) The study of optimizing circulating fluidized bed boiler operational parameters based on neural network and genetic algorithm. In: D.X. Hunan, L.G. Changsha, eds, *Fourth International conference on Intelligent Computation technology and automation*, Shenzhen, Guangdong (ICICTA'2011). IEEE Computer Society, 287-290.

- HOPFIELD, J. J. (1982) Neural Networks and Physical Systems with Emergent Collective Computational Abilities. *Proc. Nat. Acad. Sci.*, **79**(8): 2554–2558, April.
- KLJAJIC, M., GVOZDENAC, D. AND VUKMIROVIC, S. (2012) Use of Neural Networks for modeling and predicting boiler's operating performance. *Energy*, **45**(1): 304-311, September.
- KOCAARSLAN, I. AND CAM, E. (2007) Experimental modelling and simulation with adaptive control of power plant. *Energy Conversion and Management*, **48**(3): 787–796, March.
- KOCAARSLAN, I., ERTUGRUL, C. AND TIRYAKI, H. (2006) A fuzzy logic controller application for thermal power plants. *Energy Conversion and Management*, **47**(4): 442–458, March.
- LING, Y., ZHOU, Y. AND LUO, Q. (2017) Levy Flight Trajectory-Based Whale Optimization Algorithm for Global Optimization. *IEEE Access*, **5**, 6168–6186.
- LIU, X. AND BANSAL, R. C. (2014) Integrating multi-objective optimization with computational fluid dynamics to optimize boiler combustion process of a coal fired power plant. *Applied energy*, **130**, 658-669, October.
- LIU, X. J., KONG, X. B., HOU, G. L. AND WANG, J. H. (2013) Modeling of a 1000 MW power plant ultra super-critical boiler system using fuzzy-neural network methods. *Energy Conversion and Management*, **65**, 518–527, January.
- MAXWELL, J. CLERK (1892) *A Treatise on Electricity and Magnetism*, 3rd ed., 2. Oxford: Clarendon, 68-73.
- MCCULLOCH, W. S. AND PITTS, W. (1943) A logical calculus of the ideas immanent in nervous activity. *Bulletin Math Biophys*, **5**, 115-133.
- MIRJALILI, S. AND LEWIS, A. (2016) The Whale Optimization Algorithm. *Advances in Engineering Software*, **95**, May 2016, 51-67.
- RUSINOWSKI, H. AND STANEK, W. (2007) Neural modelling of steam boilers. *Energy Conversion and Management*, **48**(11): 2802–2809, November.
- SAYED, M., GHARGHORY, S.M. AND KAMA, H. (2015) Gain Tuning PI controllers for Boiler Turbine Unit using a New Hybrid Jump PSO. *Journal of Electrical Systems and Information Technology*, **2**(1): 99-110, May.
- SECCO, S. D., JUAN, O., LOUISY, M. L., LUCAS, J. Y., PLION, P. AND PORCHERON, L. (2015) Using a genetic algorithm and CFD to identify low NOx configurations in an industrial boiler. *Fuel*, **158**, 672–683, October.
- SONG, J., ROMERO, C. E., YAO, Z. AND HE, B. (2016) Improved artificial bee colony-based optimization of boiler combustion considering NOX emissions, heat rate and fly ash recycling for on-line applications. *Fuel*, **172**, 20-28, May.
- SZEGA, M. AND NOWAK, G. T. (2015) An optimization of redundant measurements location for thermal capacity of power unit steam boiler calculations using data reconciliation method. *Energy*, **92**, 135-141, December.
- VANDANI, A. M. K., BIDI, M. AND AHMADI, F. (2015) Exergy analysis and evolutionary optimization of boiler blowdown heat recovery in steam

- power plants. *Energy Conversion and Management*, 106, 1-9, December.
- WEI, J. L., WANG, J. H. AND WU, Q. H. (2007) Development of a multi-segment coal mill model using an evolutionary computation technique. *IEEE Transactions on Energy Conversion*, **22**(3): 718-27, August.
- YANG, T., CUI, C., SHEN, Y. AND LV, Y. (2016) A novel denitration cost optimization system for power unit boilers. *Applied Thermal Engineering*, 96, 400-410, March.
- YANG, X. S. (2008) *Nature-Inspired Metaheuristic Algorithms*. Luniver Press, UK.
- YANG, X. S. (2009) Firefly algorithms for multimodal optimisation. *Proc. 5th International Symposium on Stochastic Algorithms, SAGA 2009: Stochastic Algorithms: Foundations and Applications*, 5792, 169-178.