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Piotr PIERZ

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Department of Electrical Power Engineering
Wrocław University of Science and Technology
Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland
phone: +48 71 320 35 41
www: http://www.weny.pwr.edu.pl/instytuty,52.dhtml; http://www.psc.pwr.edu.pl
e-mail: wydz.elektryczny@pwr.edu.pl

OFICYNA WYDAWNICZA POLITECHNIKI WROCŁAWSKIEJ
Wybrzeże Wyspiańskiego 27, 50-370 Wrocław
http://www.oficyna.pwr.edu.pl
e-mail: oficwyd@pwr.edu.pl
zamawianie.ksiazek@pwr.edu.pl

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Valeri ZHUIKOV*
Andrii MATIIKO*

CHAOTIC PROCESSES IN PWM CONVERTER

The possibility of chaotic oscillations in them is determined. The structure of the chaotic oscillations in the DC converter is considered. The system stability at various gain values of the feedback signal is investigated. The task is building a precision DC buck converter. The goal is achieved by means of providing high feedback signal gain converter operation mode. The work of the converter is commended in a case of: a) changing the input voltage; b) changing the value of the load resistance. It is shown that while ensuring the appropriate converter operation mode it is possible to improve stabilising properties and reduce the system operation frequency.

1. INTRODUCTION

Chaos is largely unpredictable long-term evolution occurring in deterministic, nonlinear dynamical system because of sensitivity to initial conditions. Power electronics circuits are rich in nonlinear dynamics. Their operation is characterised by the cyclic switching of circuit topologies which gives rise to a variety of chaotic behaviour [1–3].

The first publications in scientific power electronics journals, which investigated issues related to the chaotic work regimes appeared in the 80s of the 20th century in the wake of interest in chaos and synergy. The most common bifurcations in power electronics are Hopf bifurcation (creation of a limit cycle), and the doubling of the period, which can be observed in other nonlinear systems. Besides that a special phenomenon can be observed in power electronics systems – border collision, related to the fact that due to the large velocity of the output parameter change it is possible to miss the commutation points and transition to lower subharmonics [4].

* National Technical University of Ukraine "Kiev Polytechnic Institute", Faculty of Electronics, Department of Industrial Electronics, 16, Polytechnichna str., off. 313, Kyiv, 03056, Ukraine, e-mail: amdrew.matiyko@gmail.com

There would seem to be two reasons for studying nonlinear dynamics in the context of power electronics:

– to understand better the nonlinear phenomena that occur in power converters, and thereby avoid undesirable effects,
– to allow converters to be engineered that deliberately make use of effects such as chaos.

Though the first objective has been achieved to some extent, there are yet few practical power electronics applications in which subharmonics or chaos bring a distinct advantage. Nevertheless, with increasing awareness among power electronics practitioners of nonlinear dynamics, perhaps engineering uses will soon be found for nonlinear effects. It may be helpful to list the characteristics of chaos, and indicate some possible application areas [5].

2. PWM CONVERTER MODEL

Processes occurring in the power part of the buck PWM DC (Fig. 1) converter are described by a system of equations:

$$\begin{cases} (di(t)/dt) = -(sr/L)i(t) - u(t)/L + sE/L, \\ du(t)/dt = i(t)/C - u(t)/RC, \end{cases} \tag{1}$$

and the processes in the feedback loop are described with the following system of equations:

$$\begin{cases} U_p = k(U_r - k_d U), \\ U_k = U_g - U_p, \\ s = s(U_k), \end{cases} \tag{2}$$

where E – the power supply voltage; L, C, R, r – linear inductance, capacitance, load resistance and the internal power supply resistance; $u(t)$, $i(t)$ – the capacitor voltage and the inductor current respectively; s – switching function, k_d – divider transfer coefficient, U_r – voltage reference element.

In accordance with the control system operation algorithm voltage U_p (the output voltage of the error signal amplifier) is compared in a comparator with voltage U_g ($U_g(t) = U_{ag}(t - nT)/T$, $n = 1, 2, 3, ...$; $0 \le t \le T$; U_{ag} – the amplitude voltage value of the scanning voltage ramp generator). On the comparator's output rectangular pulses that correspond to the intervals of the closed (switching function $s = 1$ for $U_k > 0$) and open ($s = 0$ for $0 \le U_k$) states of converter's power key are formed [3, 4].

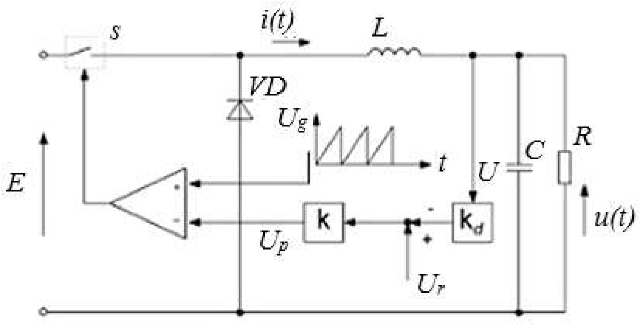

Fig. 1. PWM DC converter

3. CHAOTIC OSCILATIONS STRUCTURE

The structure of the chaotic oscillations varies depending on the gain value. With gradual increase of it system moves sequentially from steady state to chaotic processes and vice versa, forming zones of chaotic oscillations corresponding to a certain range of the coefficient k. For circuit parameters of buck converter: R = 100 Ohms; r = 0.1 Ohm, L = 0.1 H, $C = 10^{-6}$ F, E = 1000 V, U_r = 10 V, U_M = 10 V, T = 0.001 s, k_d = 0.01, the characteristic polynomial roots p_1, p_2 are valid (if $s = 0$ – $p_1 = -8873.0$, $p_2 = -1127.0$, while at $s = 1$ – $p_1 = -8872.8$, $p_2 = -1128.2$). Bifurcation diagram for time between the switching moments for the range of gain change k = 1 – 200 with increment Δk = 0,01 was built (Fig. 2). On this diagram, chaotic oscillations, which alternate with local zones of stability (ZS), are displayed [6].

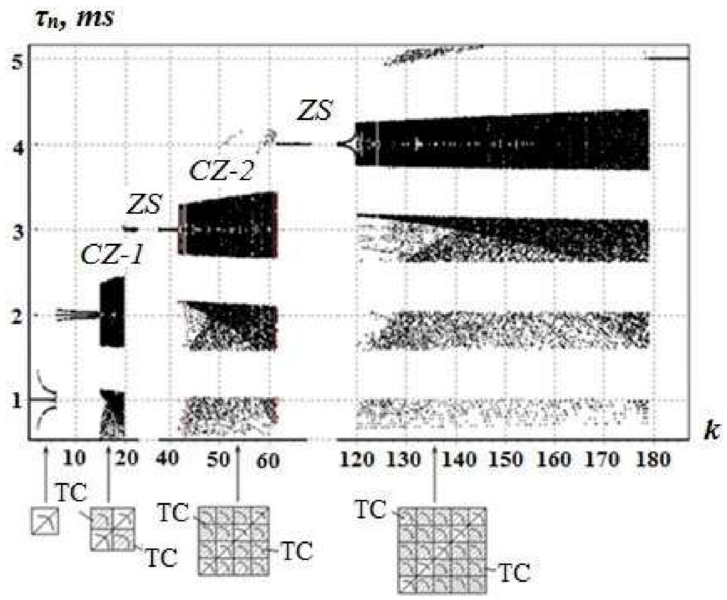

Fig. 2. Bifurcation diagram for time between the switching moments.
CZ – chaotic zone, ZS – zone of stability, TC – transfer curves for different chaotic zones

Figure 2 shows the chaotic zone CZ-2, CZ-3, which alternate with zones of stability (ZS) with a repetition period of 3T, 4T, 5T. Clusters of chaotic oscillations constitute the separate area of chaotic processes with relevant transfer curves (TC) [9]. Relation law between the number of homogeneous chaotic processes and the value of the gain is not linear. Thus, during the transition from the second chaotic zone (CZ-2) in the third (3-CZ) the number of homogeneous chaotic processes increased 2 times at 4, while the other parameters in [9], for a cluster following the CZ-2, the number of homogeneous random processes is equal to 3. The number of transfer curves of the corresponding map is also changed. With further increase of the gain (more than 400), a decrease in the number of homogeneous random processes from 8 to 4 with a decrease in the number of transfer curves is observed. This fact also indicates the existence of a non-linear relationship between the numbers of homogeneous random processes contained in the cluster to the gain of the feedback loop.

The mapping for the first zone and the resulting process is not different from the logistic map, and the corresponding process. A different situation is observed for the chaotic processes corresponding to zones CZ-2, CZ-3. Map, constructed in the usual manner, leads to an ambiguous map, as shown in Fig. 3, a. Otherwise, the map is obtained if after each bifurcation point, corresponding to the intersection of the reference voltage with the feedback signal, transfer origin for the start of the sweep voltage following the bifurcation point. In this case, the map takes the form shown in Fig. 3b, where on the central line there are maps for all homogeneous chaotic process for k = 50; and so-called TC, which provide a transition from one homogeneous process to another (over and under the central line) [8].

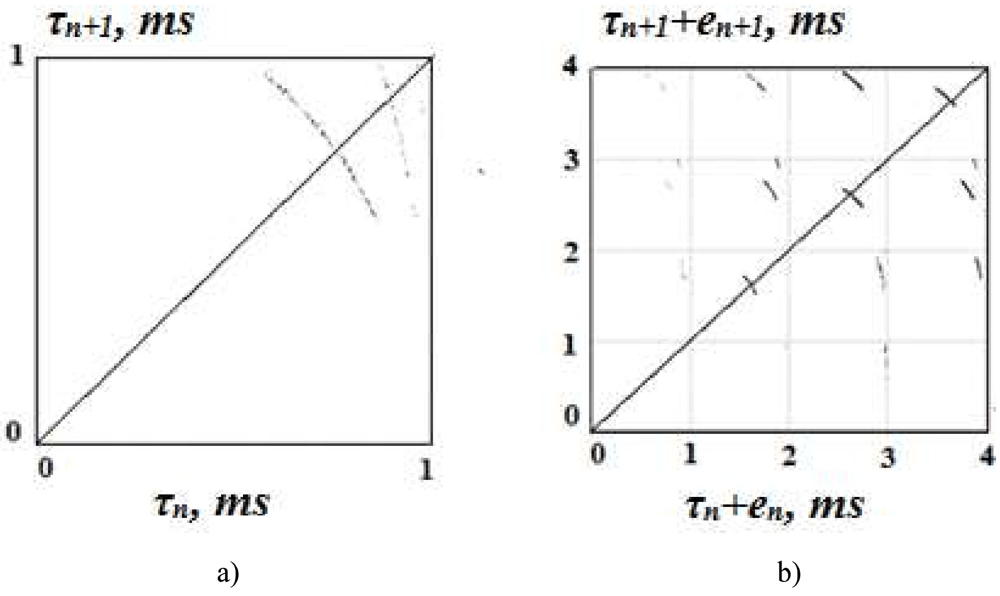

Fig. 3. Poincare map for k = 50

The nonlinear dependence of the sequence of numbers formed from the integral periods of the sweep voltage between moments of switching is preserved. For example, for the third chaotic zone CZ-3, with a gain $k = 50$, the formed sequence of integers, corresponding to the number of sweep voltage periods between moments of switching is given in Table 1 in a second row, where the first row indicates the number of switching moment.

Table 1. Sequence of switching moments

Switch moment number.	1	2	3	7	...	15	16	17
Sweep voltage periods number	3	2	2	2	...	2	2	2
Number of repetitions	1	16						
Switch moment number.	18	19	20	21	22	23	24	25
Sweep voltage periods number	1	1	2	2	1	1	1	1
Number of repetitions	2		2		4			

The third row of the table shows the number of repetitions of the amount of switching interval, that forms the second sequence that probably should also have a chaotic character [7]. The corresponding timing diagram is shown in Fig. 4.

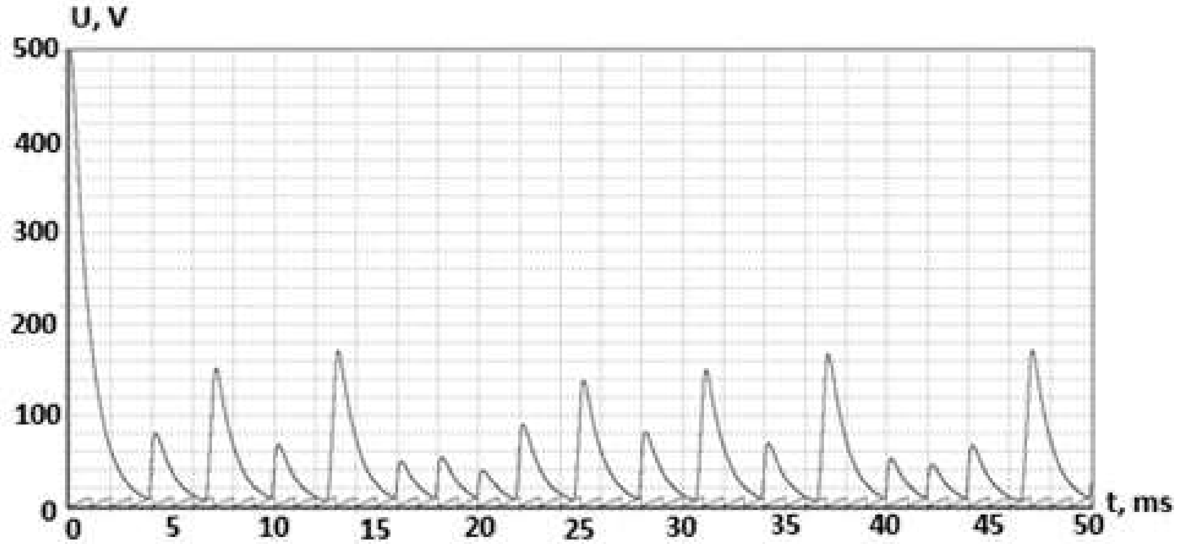

Fig. 4. Time diagram for $U_{zw}(t)$

4. SYSTEM STABILITY

Table 2 shows the characteristic equation roots values for different gain values.

Table 2. The characteristic equation roots values

Gain values	Characteristic equation roots values	
	$Z1$	$Z2$
6	0.5	0.9
15	–264	–14
20	0.26	–126.66
45	–0.65	–765.875
70	0.153	0.945
80	0.0175	–1163.07

Figure 5 schematically shows the characteristic equation roots location; as can be seen, in all cases, when the system is unstable, except for k = 15, one of the characteristic equation roots is in a unit circle, and root is shifted to the right with the gain value increase; second root with the gain value increase is shifted to the left. For both gain values at which the system is stable, both roots are in the right part of the unit circle.

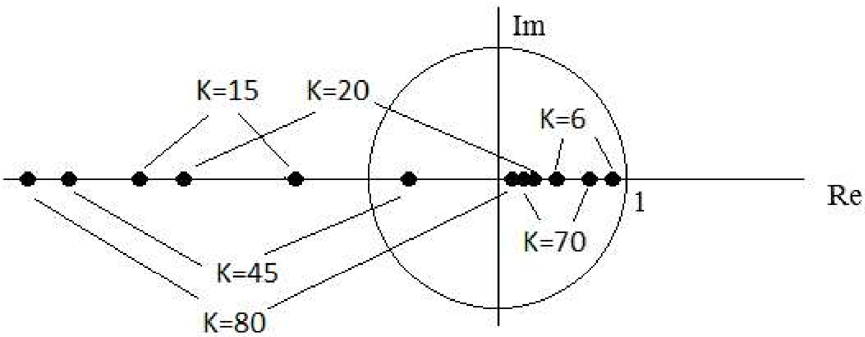

Fig. 5. The characteristic equation roots location

5. HIGH FEEDBACK SIGNAL GAIN PWM CONVERTER OPERATION MODE

Figure 6 shows a schematic representation of the bifurcation diagram, which was built gain increments $\Delta k = 0.01$ in the range of $k = 1\div90$; modelling is completed for 500 periods of scanning voltage for each value of k.

From Fig. 6 it can be seen that the 10% input voltage reduction causes the chaotic oscillations zone moves in the direction of the gain value reduction. When using high gain it is necessary to consider the position of the local zones of stability. With the big changes of the input voltage chaotic zone shifted so much that it cover the local zone of stability. Thus, it is possible to switch the converter to operate with the high gain, providing around 10% input voltage change.

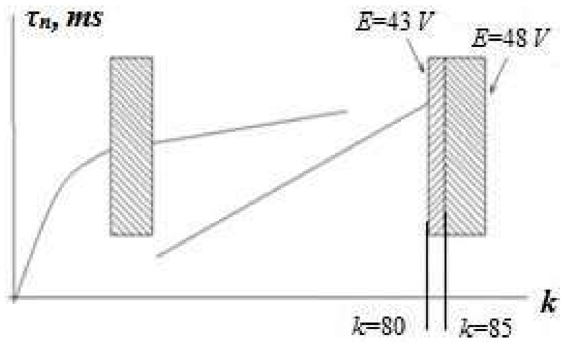

Fig. 6. Schematic representation of the bifurcation diagram

Figure 7 shows model's bifurcation diagram studied with R_1 = 4.8 Ohm, with relevant lines chaotic zones borders changes with 10% load resistance increasing (R_2 = 5.6 Ohm) and decreasing (R_3=4.3 Ohm) are shown. As it is shown in Fig. 7, with increasing of the load resistance value local zones of stability becomes wider (k = 23 ... 92), and with decreasing – becomes narrower (k = 26 ... 57).

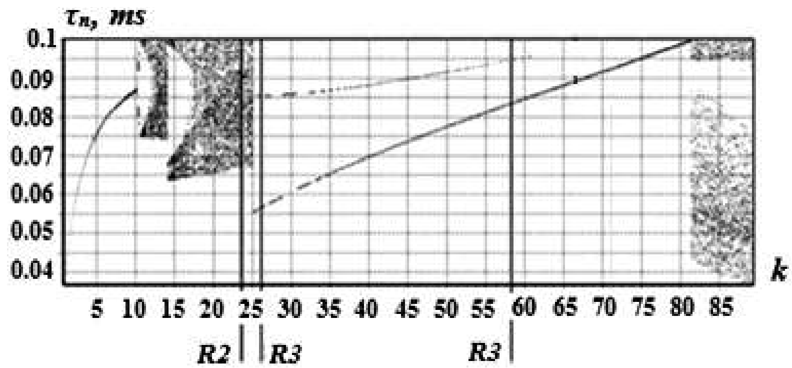

Fig. 7. Bifurcation diagram for different load values

Now the possible range of feedback signal gain can be determined. To ensure the stable operation mode of converting feedback signal gain should be in two bands.

The first range – from k = 1 to the start of the first chaotic zone. However, this option cannot provide necessary performance quality of converter.

The second range is located between the first and second chaotic zones and the feedback signal gain value must be in the range where switching occurs at a constant frequency. The critical value of gain is the beginning of the second chaotic zone for the converter with the load resistance R_3 = 4.3 Ohm.

Thus, to ensure the stable operation mode the feedback signal gain must be in the range of k = 67 ... 75. As the calculations for k = 70 showed, the load voltage ripple factor for the converter with the high feedback signal gain almost hasn't changed – $K_R = u_{max}(t)/u_{mid}(t)$ = 1.036 (with k = 6 K_R = 1.013), but the stabilisation rate has increased – $K_{ST} = (\Delta E/E)/(\Delta u(t)/u(t))$ = 22.25, while for k = 6 K_{ST} = 5.11.

The simulation results indicate that it is possible to build a precision system with the high feedback signal gain.

Figure 8 shows the voltage and current diagrams for k = 70. As it can be seen, switching frequency decreases 4 times from: 1) f = 10 kHz to 2) f = 2500 Hz.

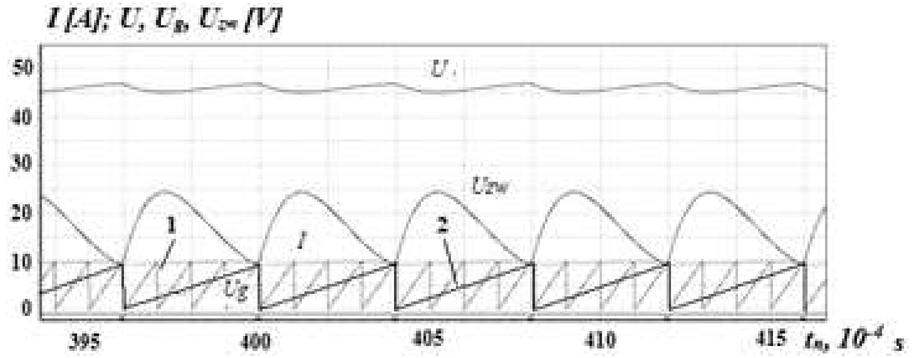

Fig. 8. Voltage and current diagrams for k = 70

As shown in Fig. 8, the reduced switching frequency did not change the output voltage, the frequency change also has not affected the voltage ripple factor and the stabilisation rate; roots-mentioned characteristics equation and the boundary of the converter vary slightly. Therefore, by increasing the feedback signal gain it is possible to reduce the switching frequency of the power key for reducing the switching losses in the system.

6. COCLUSIONS

The chaotic oscillations processes should not be described by differential equations of the less than second order. The equation order to boost converter circuit does not increase and chaotic oscillations in the steady state are not observed.

The cluster of chaotic oscillations does exist, but it is hard to specify how much homogeneous random processes it consists of and how much transfer curves mapping it contains because the cluster is described in more difficult conditions of existence. Relation law between the number of homogeneous chaotic processes and the value of the gain is not linear

Calculations show that it is advisable to ensure converter operation at a high feedback signal gain value to increase the stability and reliability of the system. In addition, the increasing of the feedback signal gain value may reduce the switching frequency of the system.

An ability of high feedback signal gain operation mode is shown. the load voltage ripple factor almost hasn't changed. However, as expected, the stabilisation rate has increased.

Also, by increasing the gain of the feedback signal may reduce the switching frequency and the frequency of the system, which reduces switching losses.

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