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Memory response on generalized thermoelastic medium in context of dual phase lag thermoelasticity with non-local effect

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Theory of non-local continuum is contemporary appraised and is found to be supplementary coherent to capture the impacts of each and every point of the material at its single point. The conviction of memory dependent derivative is also newly appraised and is observed to be more intuitionistic for predicting the realistic character of the real-world obstacles. Attractiveness of the belief of a memory dependent derivative lies in its unique properties such as its significant constituents – a kernel function and time-delay are freely selected according to the requirement of a problem. The present study comprises a new meticulous thermoelastic heat conduction model for the homogeneous, isotropic, thermoelastic half space medium concerning memory effects and non-local effects. Governing equations are constructed on the basis of the newly appraised non-local generalized theory of thermoelasticity with two phase lags in the frame of a memory dependent derivative. Exact analytical solutions of the physical fields such as dimensionless temperature, displacement as well as thermal stress are evaluated by using a suitable technique of the Laplace transform. Quantitative results are determined in a time-domain for different values of time by taking the numerical inversion of the Laplace transform. Noteworthy role of the constituents of the memory dependent derivative such as kernel function as well as time-delay factor has been scrutinized on the crucial field variables of the medium through computational outcomes. Moreover, the impact of non-local parameter is examined on the variations of field quantities through the quantitative results.

Key words: memory dependent derivative, time-delay, kernel function, non-local continuum theory, dual phase lags.

Notation

e_{ij}	strain tensor,
λ, μ	Lame constants,
$\alpha = (3\lambda + 2\mu)\alpha_T$	α_T denotes volume expansion coefficient,

η entropy density,
T_0 reference temperature,
T temperature change over T_0,
ρ mass density,
K thermal conductivity,
C_e specific heat at constant strain,
τ_q phase lag of heat flux,
τ_T phase lag of temperature gradient,
σ_{ij} stress tensor,
ω time-delay,
λ_q non-local length parameter,
δ_{ij} Kronecker delta,
$\vec{u}$ displacement vector.

1. Introduction

The idea of the generalized theory of thermoelasticity was appraised forty years ago. This theory attracted the researchers due to its capability of predicting promising positive outcomes of real world obstacles as it shows the finite nature of thermal waves. In contrast to the generalised thermoelastic theory, the classical theory of thermoelasticity indicated the infinite speed of thermal waves and turned out to be physically absurd. To eliminate this absurdity, several new generalized theories of thermoelasticity were established. These new theories were derived by adding the relaxation parameters in a classical theory and found to be hyperbolic in nature which ensures the finite speed of the heat transport. Biot [1] established an extensive formulation of the classical theory of thermoelasticity (CTE) which was based on Fourier's law. Lord and Shulman [2] initiated the concept of the generalized thermoelasticity theory by evolving one relaxation parameter for heat flux in Fourier's law. Since, improvement is always required in the existing law; therefore, several refinements have been performed to the heat conduction law such as the idea of phase-lags, modifying consecutive equations, inserting new consecutive variables as well as involving a non-local phenomenon, etc. (Green and Lindsay [3], Cattaneo [4], Vernotte [5], Green and Naghdi [6–8]).

After that it was noticed that solutions for the micro and nano engineering problems are on higher demand. Then, Tzou [9] and Chandrasekharaiah [10] flourished the theory of two phase lags in the generalized thermoelasticity. The phase lag related to the heat flux was already established, however one new phase lag related to the temperature gradient was announced by Tzou [9]. The same studied theory achieved popularity as the dual phase lag (DPL) theory of thermoelasticity. Numerous investigations are enacted by using the idea of generalized thermoelasticity with phase lags such as Tiwari and Misra [11], Kumar, Tiwari, and Kumar [12], Tiwari and Mukhopadhyay [13], Tiwari, Kumar, and Kumar [14].

Generally, real world problems need data of the recent past for predicting accurate as well as realistic results. Hence, a new theory reflecting the memory effect was developed by Wang and Li [15].

For a function f, the memory dependent derivative (MDD) of the first order is simply defined as an integral form of a common derivative with a kernel function $K(t-\xi)$ on a slipping interval provided below

$$D_\omega f(t) = \frac{1}{\omega}\int\limits_{t-\omega}^{t} K(t-\xi)f'(\xi)\,d\xi; \tag{1.1}$$

ω denotes time delay which is assumed to be very small and positive for a realistic approach. Usually, the memory response requires weight $0 < K(t-\xi) \leq 1$ for $\xi \in [t-\omega t]$ which is a very small quantity so that the magnitude of the memory dependent derivative (MDD) is found to be smaller than that of the common derivative $f'(t)$. The right-hand side of Eq. (1.1) is denoted as a weighted mean of the common derivative $f'(t)$. We notice from Eq. (1.1) that $\xi \in [t-\omega t]$ the function $f(\xi)$ assumes the values from the interval $[t-\omega, t]$ considering the present time t and past time $t-\omega$. Since ω reflects a very small quantity, therefore, $t-\omega$ is considered recent past time for the present time t. Most beautiful character of MDD is found in the behaviour of the kernel function as it can be selected freely in the definition of the memory dependent derivative (MDD).

When $K(t-\xi) = 1$ and$\omega \to 0$, we have

$$D_\omega f(t) = \frac{1}{\omega}\int\limits_{t-\omega}^{t} f'(\xi)\,d\xi = \frac{f(t)-f(t-\omega)}{\omega} \to f'(t). \tag{1.2}$$

In this case, the first order memory dependent derivative becomes an ordinary derivative.

In the similar pattern, for the r-times differentiable function$f(t)$,

$$D_\omega^r f(t) = \frac{1}{\omega}\int\limits_{t-\omega}^{t} K(t-\xi)f^r(\xi)\,d\xi \tag{1.3}$$

is called the r-order ($r \in N$) ‘memory-dependent derivative’ of f at t for the time-delay parameter ω, where the kernel, $K(t-\xi)$ is the r-times differentiable function of t and ξ.

Several studies were carried out by using the definition of memory dependent derivative [16–24].

If the space is nanoscale of point type, the local theory is found to be suitable as a local theory is point dependent. But in the case of macro scale space, the non-local theory gives better description as a nonlocal theory is not point specific. The nonlocal elasticity theory exhibits the elasticity globally and is independent of the concept of the single point while the local theory is restricted to a single point of the solid. Consequently, the nonlocal theory is observed to be more capable for expressing more information on long-range forces of atoms or molecules. An internal length scale parameter is added in the arrangement of the nonlocal elasticity model. In the limiting case, when the effects of strain at different points other than a single point are negligible, the classical (local) elasticity model can be retrieved from the nonlocal model. Yu *et al.* [25] propounded the size-dependent generalized thermoelasticity model by adopting Eringen's nonlocal model. Further, Yu *et al.* [26] established the theory of thermoelasticity with a nonlocal phenomenon based on nonlocal elasticity and nonlocal heat conduction.

Challamel *et al.* [27] modified Fourier's law with a nonlocal theory idea. Tzou and Guo [28] investigated the heat conduction model involving a phase-lag response as well as a nonlocal response. Recently, Bachher and Sarkar [29] have examined the nonlocal theory for thermoelastic materials with voids. Gupta and Mukhopadhyay [30] presented a report on generalized thermoelasticity theory based on a nonlocal heat conduction model in context of dual-phase-lag thermoelasticity. Abouelregal *et al.* [31] presented the solution of the Moore–Gibson–Thompson equation for the unbounded medium with a cylindrical hole. Application of the Chebyshev–Ritz method for static stability and vibration analysis of nonlocal microstructure-dependent nanostructures was demonstrated by Ebrahimi *et al.* [32]. Dynamic analysis of nano rods was described by Numanoğlu *et al.* [33].

The objective of the current work is to analyse the influence of a new heat conduction model with two phase lags concerning the memory effect and the nonlocal effect on the transmission of thermoelastic vibrations in homogeneous, isotropic, thermoelastic half space medium. In order to reveal the clear picture of memory response, quantitative outcomes are derived for various kernel functions as well as time-delay for different times. Achieved outcomes for a memory dependent derivative are compared to the results obtained for without memory dependent derivative. An influence of a nonlocal parameter has also been examined on the field variables through computational results. Noteworthy results of the memory dependent derivative as well as the nonlocal parameter are attributable to the studied fields.

2. Basic equations

Heat conduction equation: Considering the concept of Tzou and Guo, the heat conduction equation with a nonlocal effect is given by [28, 30, 34],

$$\left(1+(\lambda_q)_k\frac{\partial}{\partial x_k}+\tau_q\frac{\partial}{\partial t}\right)q_i=-K\left(1+\tau_T\frac{\partial}{\partial t}\right)T_{,i}. \tag{2.1}$$

Energy equation [30]:

$$-q_{i,i} = \rho T_0 \frac{\partial S}{\partial t}. \tag{2.2}$$

Entropy equation [30]:

$$T_0 \rho S = \rho C_e T + \alpha T_0 e_{kk}. \tag{2.3}$$

Equation of motion [30]:

$$\sigma_{ij,j} = \rho \frac{\partial^2 u_i}{\partial t^2}. \tag{2.4}$$

Stress-strain-temperature relation [30]:

$$\sigma_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} - \alpha T \delta_{ij}. \tag{2.5}$$

Strain-displacement relation [30]:

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{2.6}$$

The subscripts i, j, and k assume the values 1, 2, and 3 and the subscripted comma notation denotes the partial derivative with respect to the space variable.

Combining Eqs. (2.1)–(2.3) and (2.6) and introducing the notion of a memory dependent derivative, we achieve the following heat conduction equation for the nonlocal dual-phase-lag (DPL) model in context of the memory-dependent derivative [15, 30],

$$K(1 + \tau_T D_\omega) T_{,ii} = \left(1 + (\lambda_q)_k \frac{\partial}{\partial x_k} + \tau_q D_\omega\right)\left(\rho C_e \frac{\partial T}{\partial t} + \alpha T_0 \frac{\partial u_{k,k}}{\partial t}\right). \tag{2.7}$$

With the help of Eqs. (2.4)–(2.6), the equation of motion takes the following form:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = (\lambda + \mu) u_{j,ji} + \mu u_{i,jj} - \alpha T_{,i}. \tag{2.8}$$

The stress–displacement–temperature relation is determined as follows:

$$\sigma_{ij} = \lambda u_{k,k} \delta_{ij} + \mu(u_{i,j} + u_{j,i}) - \alpha T \delta_{ij}. \tag{2.9}$$

Equations (2.7)–(2.9) represent the basic governing equations of the generalized nonlocal dual phase lag thermoelasticity theory in the frame of a memory-dependent derivative.

3. Formulation of the problem

We assume a problem of isotropic, homogeneous thermoelastic one-dimensional half-space ($x \geq 0$) medium. The boundary of the half-space is considered to be traction free but subjected to a time dependent thermal shock. All the physical field variables are assumed to be bounded and diminish as $x \to \infty$.

Since the problem is taken for the one-dimensional half-space, therefore the displacement vector is taken as $\vec{u} = (u(x,t), 0, 0)$.

Governing equations describing the present problem are written as

$$K(1+\tau_T D_\omega)\frac{\partial^2 T}{\partial x^2} = \left(1+\lambda_q \frac{\partial}{\partial x} + \tau_q D_\omega\right)\left(\rho C_e \frac{\partial T}{\partial t} + \alpha T_0 \frac{\partial^2 u}{\partial x \partial t}\right), \tag{3.1}$$

$$\rho \frac{\partial^2 u}{\partial t^2} = (\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} - \alpha \frac{\partial T}{\partial x}, \tag{3.2}$$

$$\sigma_{xx} = (\lambda + 2\mu)\frac{\partial u}{\partial x} - \alpha T. \tag{3.3}$$

For the present study, we have chosen the following kernel functions [12, 16]:

$$K(t-\xi) = 1 - \frac{2b}{\omega}(t-\xi) + \frac{a^2}{\omega^2}(t-\xi)^2 = \begin{cases} 1, & a = b = 0, \\ \left(\frac{\xi - t}{\omega} + 1\right), & a = 0,\, b = \frac{1}{2}, \\ \left(\frac{\xi - t}{\omega} + 1\right)^2, & a = b = 1. \end{cases} \tag{3.4}$$

The case $K(t-\xi) = 1$ presents the case of the constant kernel function whereas dimensionless second and third kernel functions are found to be linear and non-linear in nature, respectively.

Non-dimensionalization

To simplify the governing equations, the following non-dimensional quantities are introduced into them

$$\begin{gathered} T' = \frac{T}{T_0}, \quad (t', \tau_T', \tau_q', \omega') = c_1^2 \zeta (t, \tau_T, \tau_q, \omega), \quad \lambda_q' = c_1 \zeta (x, \lambda_q), \\ u' = \frac{c_1 \zeta (\lambda + 2\mu)}{\alpha T_0} u, \quad \sigma_{xx}' = \frac{1}{\alpha T_0}\sigma_{xx}, \quad c_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad \zeta = \frac{\rho C_e}{K}. \end{gathered} \tag{3.5}$$

Introducing non-dimensional quantities Eq. (3.5) into Eqs. (3.1)–(3.3), we obtain the following equations in the terms of dimensionless form (after suppressing primes)

$$(1+\tau_T D_\omega)\frac{\partial^2 T}{\partial x^2} = \left(1+\lambda_q \frac{\partial}{\partial x} + \tau_q D_\omega\right)\left(\frac{\partial T}{\partial t} + \beta \frac{\partial^2 u}{\partial x \partial t}\right), \tag{3.5}$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} - \frac{\partial T}{\partial x}, \tag{3.6}$$

$$\sigma_{xx} = \frac{\partial u}{\partial x} - T, \tag{3.7}$$

where $\beta = \frac{\alpha^2 T_0}{\rho^2 C_e c_1^2}$ represents the thermoelastic coupling constant.

4. Initial and boundary conditions

Since, a boundary of the medium is taken as traction free but it is suffering from a sudden heat input, therefore boundary conditions are assumed as [30]:

$$\begin{aligned} &\sigma_{xx}(0,t) = 0, \quad u(\infty, t) = 0 \quad \text{for } t > 0, \\ &T(0,t) = \theta_0 H(t), \quad T(\infty, t) = 0 \quad \text{for } t > 0. \end{aligned} \tag{4.1}$$

Initial conditions are assumed to be homogeneous.

5. Analytical solutions

For deducing the exact solutions of the present problem, the Laplace transform method has been employed.

The Laplace transform is defined as

$$L[f(x,t)] = \int_0^\infty e^{-st} f(x,t)dt = \bar{f}(x,s). \tag{5.1}$$

On taking the Laplace transform of Eqs. (3.6)–(3.8), we obtain

$$\left(q\frac{d^2}{dx^2} - p - s\lambda_q \frac{d}{dx}\right)\bar{T} = \beta\left(p\frac{d}{dx} + s\lambda_q \frac{d^2}{dx^2}\right)\bar{u}, \tag{5.2}$$

$$\left(\frac{d^2}{dx^2} - s^2\right)\bar{u} = \frac{d\bar{T}}{dx}, \tag{5.3}$$

$$\bar{\sigma}_{xx} = \frac{d\bar{u}}{dx} - \bar{T} \tag{5.4}$$

with

$$p = s\left(1 + \frac{\tau_q}{\omega} G(\omega)\right), \quad q = \left(1 + \frac{\tau_T}{\omega} G(\omega)\right),$$

where

$$G(\omega) = 1 - \frac{2b}{\omega} + \frac{2a^2}{\omega^2} - e^{-s\omega}\left[(1 - 2b + a^2) + \frac{2(a^2 - b)}{\omega s} + \frac{2a^2}{\omega^2 s^2}\right].$$

Taking Laplace transform of the boundary conditions (Eq. (4.1)) we obtain

$$\bar{\sigma}_{xx}(0,t)=0,\quad \bar{u}(\infty,t)=0,\qquad \bar{T}(0,t)=\frac{\theta_0}{s},\quad \bar{T}(\infty,t)=0. \tag{5.5}$$

On solving Eqs. (5.2) and (5.3), the following decoupled equation involving $\overline{T}$ and $\overline{u}$ are obtained as:

$$\left[q\frac{d^4}{dx^4}-(1+\beta)s\lambda_q\frac{d^3}{dx^3}-(qs^2+p+p\beta)\frac{d^2}{dx^2}+s^3\lambda_q\frac{d}{dx}+ps^2\right](\bar{u},\bar{T})=0. \tag{5.6}$$

The corresponding auxiliary equation is

$$ql^4-(1+\beta)s\lambda_q l^3-(qs^2+p+p\beta)l^2+s^3\lambda_q l+ps^2=0. \tag{5.7}$$

All the variables are vanishing for $x\to\infty$, therefore for obtaining the feasible solutions, we consider the roots with negative real parts of Eq. (5.7).

Special case:

It was found that the forth degree differential equation (Eq 5.6) of a research paper where the absence of a memory dependent derivative appears, is similar to the differential equation in the published article [30] which provides the authentication and validation of the results of the manuscript and is also cited in the present work where symbols are $q=n$, $p=m$ and $\beta=\varepsilon$ according to the reference paper.

The solution of the differential equation (4.7) can be determined as

$$\overline{T}(x,s)=C_1e_1^{-l_1x}+C_2e_1^{-l_2x}, \tag{5.8}$$

where l_1 and l_2 are the roots of Eq. (5.6) such that $\operatorname{Re}(l_i)>0$, $i=1,2$. Similarly, with the help of Eq. (5.3), the exact analytical expression of displacement is given by

$$\overline{u}(x,s)=C_1\left(\frac{l_1}{s^2-l_1^2}\right)e_1^{-k_1x}+C_2\left(\frac{l_2}{s^2-l_2^2}\right)e_1^{-k_2x}. \tag{5.9}$$

Using Eqs. (5.4), (5.8) and (5.9), we obtain the following analytical expression of the thermal stress:

$$\bar{\sigma}_{xx}(x,s)=C_1\left(\frac{l_1^2}{l_1^2-s^2}-1\right)e_1^{-k_1x}+C_2\left(\frac{l_2^2}{l_2^2-s^2}-1\right)e_1^{-k_2x}. \tag{5.10}$$

With the help of boundary conditions (Eq. (5.5)), we obtain the following expressions of C_1, C_2:

$$C_1=\frac{\theta_0(s^2-l_1^2)}{s(l_2^2-l_1^2)}, \tag{5.11}$$

$$C_2=\frac{\theta_0}{s}-\frac{\theta_0(s^2-l_1^2)}{s(l_2^2-l_1^2)}. \tag{5.12}$$

6. Quantitative results

The present section derives the computational results of the physical fields – dimensionless temperature, dimensionless displacement and dimensionless thermal stress in the time-domain. For this, we have chosen a suitable technique of the numerical Laplace transform proposed by Bellman. Software 'MATLAB' is used to determine the numerical results. Graphical results are sketched for two different times – 0.69 and 1.21. The main motive of this section is to evaluate the impact of a newly constructed heat conduction model (nonlocal dual phase lag model with a memory dependent derivative) on the studied fields and also to compare it to the previously existing generalized thermoelastic models.

Computational results have been analysed in three different subsections. The first subsection exhibits the effect of the various kernel functions on the behaviour of all physical fields. The second subsection reveals the effect of the time-delay on the physical fields of the medium while the third subsection reveals the influence of a nonlocal parameter on the field variables.

The copper material has been chosen for studying numerical results. Physical parameters are considered in the following way [13, 30]:

$$\lambda = 7.76 \times 10^{10}\,\mathrm{N\cdot m^{-2}}, \quad \mu = 3.86 \times 10^{10}\,\mathrm{N\cdot m^{-2}}, \quad \rho = 8954\,\mathrm{kg\cdot m^{-3}},$$
$$\theta_0 = 1, \quad K = 386\,\mathrm{W\cdot m^{-1}K^{-1}}, \quad \tau_q = 0.2,$$
$$C_e = 383.1\,\mathrm{J\cdot kg^{-1}\cdot K}, \quad \tau_T = 0.15, \quad T_0 = 293\,\mathrm{K}.$$

6.1. Effect of kernel function

In the current subsection, we attempted to analyse the influence of different kernel functions for time-delay $\omega = 0.1$ and the nonlocal length parameter $\lambda_q = 0.05$. Moreover, results under different kernel functions are compared to those corresponding to the constant kernel function which represents the case where the memory dependent derivative is found to be absent in the heat conduction equation. Results are carried out for two values of non-dimensional time – 0.69 and 1.21; in this way, we realize the influence of the time factor on the variations of physical fields.

Figure 1 states the variations of the temperature field against the distance at time $t = 0.69$. This figure describes the temperature behaviour for different values of kernel functions by taking $K(t-\xi) = 1$, $\left(\frac{\xi-t}{\omega}+1\right)$, $\left(\frac{\xi-t}{\omega}+1\right)^2$. The case $a = b = 0$ represents the case of a constant kernel function which expresses the absence of memory dependent derivative i.e. traditional nonlocal generalized thermoelasticity with two phase lags $a = 0$, $b = 1/2$ shows the linear kernel and the third kernel exhibits the nonlinear kernel function.

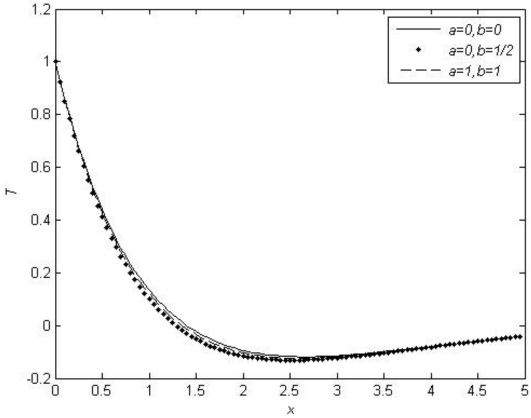

FIG. 1. Variation of temperature T with distance x for time $t = 0.69$.

Figure 1 describes that the temperature curves start with a constant value nearer to 1 at the boundary of the half-space satisfying the thermal boundary condition. The maximum value of the temperature is observed to be 1. However, it gradually decreases in the direction of increasing the distance until it approaches zero. During its journey, profiles of the temperature achieve negative values before reaching zero. The profiles of the temperature field obtain high values for the constant kernel function compared to the non-constant kernel functions. This result shows that the traditional nonlocal dual phase lag model (without memory dependent derivative) provides higher values of temperature compared to the nonlocal dual phase lag model with memory dependent derivative. High values of the temperature field become a good cause for producing higher stress and reduce the strength of the material. Hence, it can be thought that the heat conduction model containing the memory dependent derivative is efficient to provide better results and proves itself a better model compared to the conventional model without memory dependent derivative.

Figure 2 depicts the variations of the temperature field for higher time $t = 1.21$. A trend of variations of the temperature profiles are found to be similar (Fig. 1). However, the influence of the chosen kernel function becomes more significant on the profiles of the temperature field for a high value of time. Also, we observe that the profiles of temperature vanish earlier at a higher value of time.

Figure 3 demonstrates the nature of the displacement for all kernel functions at time $t = 0.69$. The variance pattern of the displacement profile is such that starting from a constant but negative value, it starts to move towards the positive direction as the value of the distance increases and after reaching the peak value, it starts decreasing with increasing a distance and ultimately becomes zero for

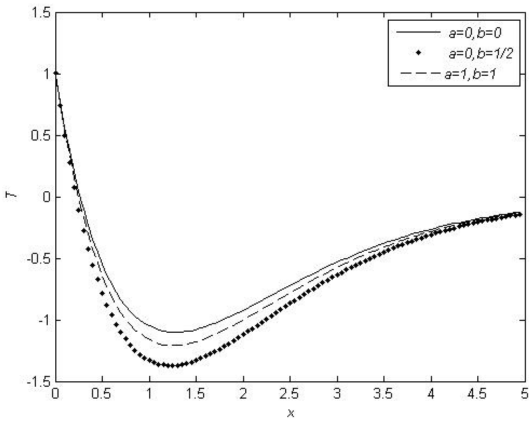

FIG. 2. Variation of temperature T with distance x for time $t = 1.21$.

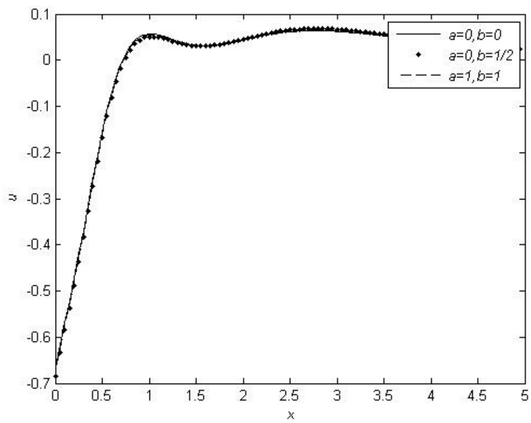

FIG. 3. Variation of displacement u with distance x for time $t = 0.69$.

the highest distance. In contrast to the temperature field, the impact of the kernel functions are found to be less effective on the variations of the profiles of the displacement field. Still, some effect is observed at the peak and it is observed that the profile of the displacement field under a constant kernel function obtains the highest value at the peak. Hence, we can conclude that the profiles of the displacement are compressive in nature in the presence of a memory dependent derivative.

Figure 4 exhibits the nature of the displacement field at a higher time $t = 1.21$. The variance pattern is similar to that from Fig. 3 but the effects of the kernel function are detected to be more prominent on the variations of the displace-

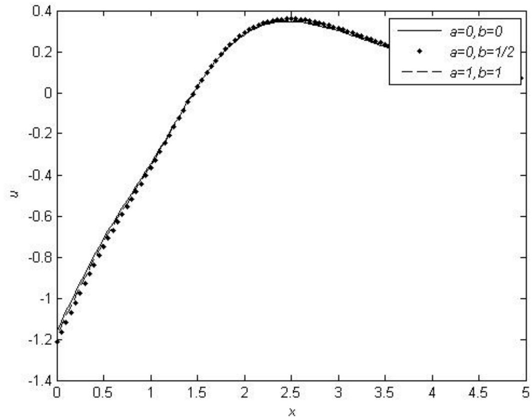

FIG. 4. Variation of displacement u with distance x for time $t = 1.21$.

ment profiles. The profile of a displacement field suffers less number of jumps at a higher time.

Figure 5 presents the behaviour of stress profiles against the distance for different kernel functions at time $t = 0.69$. The patterns of variations of stress profiles are such that satisfying the boundary condition, they start from zero and provide peaks in positive as well as negative directions on increasing the distance and finally vanish for a higher distance. An influence of the kernel function is observed to be less significant in this case.

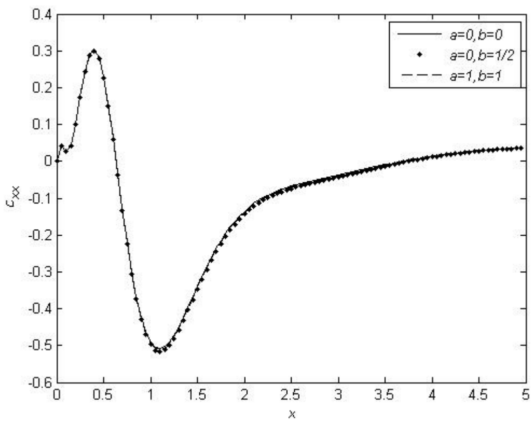

FIG. 5. Variation of stress σ_{xx} with distance x for time $t = 0.69$.

Figure 6 describes the behaviour of stress field versus distance for different kernel functions at time $t = 1.21$. The mode of variations of stress profiles are found to be similar as Fig. 5 shows. The beauty of the kernel functions is determined in this case. We observe that the stress profile under a constant kernel

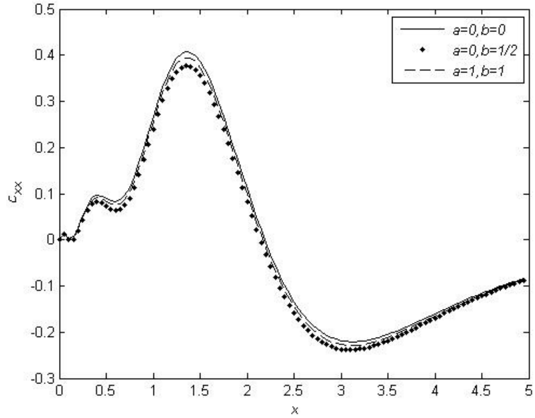

Fig. 6. Variation of stress σ_{xx} with distance x for time $t = 1.21$.

function achieves the highest thermal stress and differences among the kernel functions become more significant at the peaks. In this way, authors concluded that the heat conduction model with memory dependent derivative produces lower stress compared to the case of without memory dependent derivative. This fact builds the memory dependent derivative model as an efficient and powerful model compared to the traditional heat conduction model without memory dependent derivative.

6.2. Effect of time-delay

In this subsection, we analyse the influence of the time-delay ω on the variations of the studied fields–temperature, displacement as well as stress for the nonlinear kernel function $(t-\xi) = \left(\frac{\xi-t}{\omega}+1\right)^2$. For this, the values of time-delay ω are considered as 0.1, 0.3 and 0.5. The value of the nonlocal length parameter $\lambda_q = 0.05$ is taken into account. Graphical results are evaluated for the non-dimensional time $t = 1.21$.

Figure 7 characterizes the nature of the temperature field against the distance for different values of time-delay $\omega = 0.1, 0.3, 0.5$ at time $t = 1.21$. The effect of time-delay is found to be significant on the profiles of temperature. High values of time delay enhance the values of the temperature. This result can be understood in such a sense that the definition of a memory dependent derivative lies in an interval and a smaller interval provides realistic results according to the definition of a memory dependent derivative since a smaller interval represents the data of recent past and authors observe that if the value of time-delay is smaller, then the length of the interval will also be smaller simultaneously. From the present figure, we find the similar result and we conclude that the lower value of time-delay predicts good results compared to the higher values of time-delay.

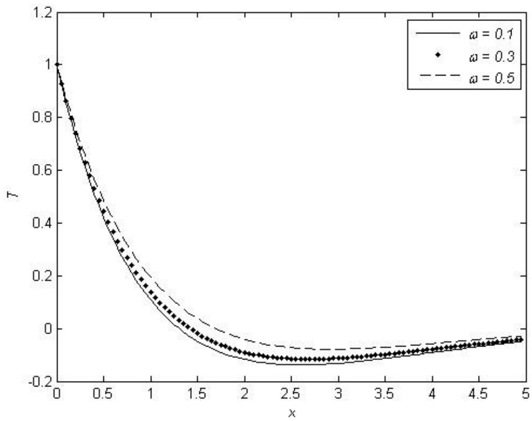

FIG. 7. Variation of temperature T with distance x for time $t = 1.21$.

Figure 8 depicts the nature of the displacement profiles for different values of the time delay parameter. The higher value of the time-delay enhances the domain the influence of the displacement field and this result is found to be similar to the temperature field.

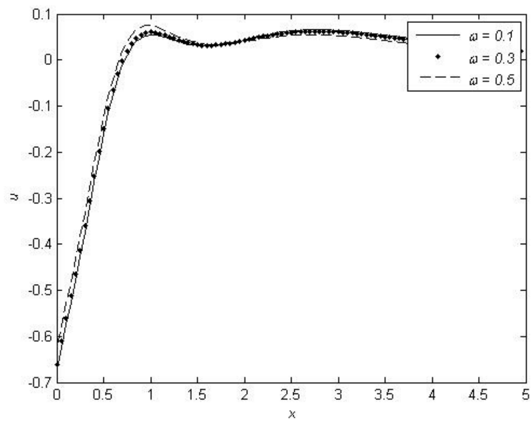

FIG. 8. Variation of displacement u with distance x for time $t = 1.21$.

Figure 9 shows the stress behaviour versus distance for various values of the time-delay parameter at time $t = 0.69$. From this figure, we observe that values of the time-delay affect most significantly the variations of the stress field. The highest value of time delay causes the highest stress which may become to enhance the risk of the failure of the strength of a material. We know that an event in immediate time is dependent on the successive past time. However, a big difference between the past and the present time fails to provide a realistic prediction for any event; then it will be very hectic to calculate accurate and true

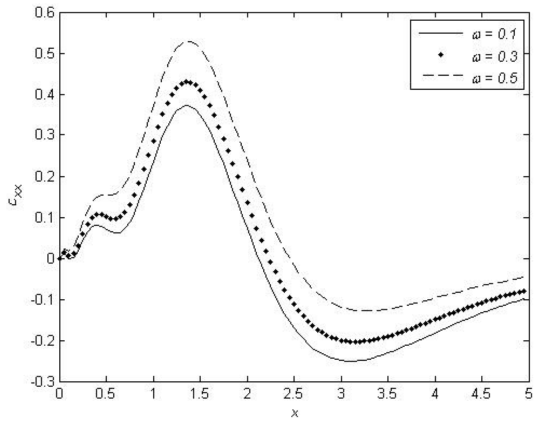

Fig. 9. Variation of stress σ_{xx} with distance x for time $t = 1.21$.

predictions of the event. Therefore, the measures of the time delay value should be very small and this conclusion has been disclosed in the present results.

6.3. Effect of non-local parameter

The present subsection derives the role of the nonlocal length parameter λ_q on the physical fields for time $t = 1.21$. We have sketched the graphical results for the non-linear kernel function $K(t-\xi) = \left(\frac{\xi-t}{\omega}+1\right)^2$ and the time-delay parameter $\omega = 0.1$. Values of the nonlocal length parameter λ_q are assumed as 0, 0.05. 0.1. The case $\lambda_q = 0$ represents the case of generalized thermoelasticity without nonlocal effect.

Figure 10 demonstrates the variations of temperature field T versus distance x at time $t = 1.21$. The present figure explains the nature of the temperature field

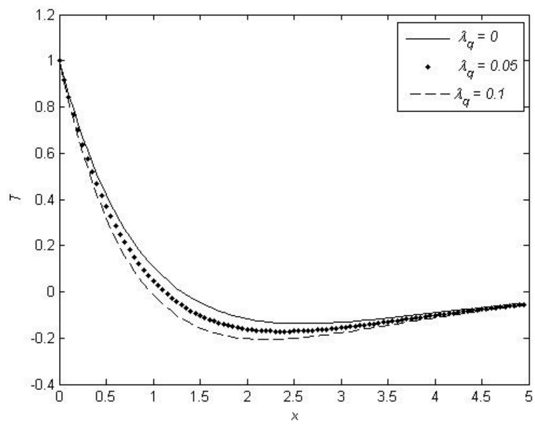

Fig. 10. Variation of temperature T with distance x for time $t = 1.21$.

for different values of the nonlocal length parameter λ_q. The influence of the nonlocal parameter λ_q has been observed by taking its different values such as 0, 0.05 and 0.1. The case $\lambda_q = 0$ presents the case of the absence of the nonlocal effect.

The variance pattern of the temperature profiles is found to be similar as shown in Fig. 1. The temperature values for the nonlocal dual phase lag model ($\lambda_q > 0$) are observed to be smaller than those for the local thermoelastic dual phase lag model ($\lambda_q = 0$). It means that the effects of nonlocality reduce the temperature values. As authors know that high temperature generates the higher stress which reduces the strength of a material. Therefore the nonlocal dual phase lag model ($\lambda_q > 0$) proves itself a better model compared to the local thermoelastic model ($\lambda_q = 0$).

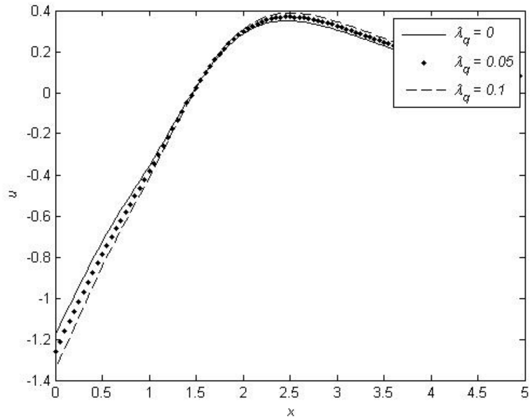

FIG. 11. Variation of displacement u with distance x for time $t = 1.21$.

The behaviour of the displacement distribution u against the distance x is presented in Fig. 11, which provides the values of the displacement for the nonlocal and local dual phase lag models. It is inspected from the figure that the magnitude of the displacement has a maximum value on the boundary of the half-plane. Starting from a negative constant value on the boundary, it moves towards positive direction and after providing peak, it tends towards zero value. From the figure, we achieve that the curve which represents the local theory varies more prominently in the beginning from the curves of the nonlocal theory.

Figure 12 exhibits the nature of the thermal stress σ_{xx} against the distance for various values of the non-dimensional length parameter λ_q. It is noticed from the figure that the parameter λ_q has a major effect on the stress distribution. We observe from the figure that the thermal stress meets the mechanical conditions on the surface, as in all cases it begins at zero. The stress values for the local dual phase lag model are higher than for the nonlocal dual phase lag model for

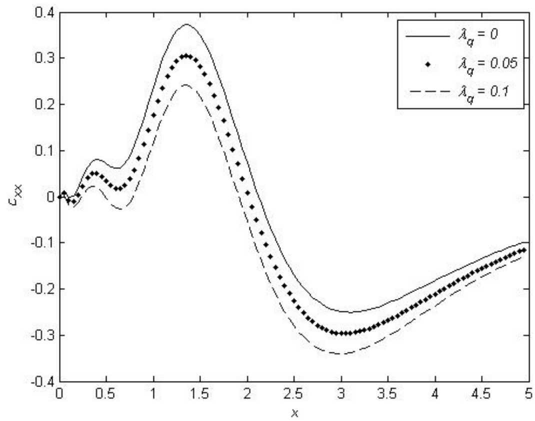

FIG. 12. Variation of stress σ_{xx} with distance x for time $t = 1.21$.

thermal conductivity. Hence it is concluded that the nonlocal phenomenon in the thermal conduction model predicts accurate and better results compared to the model without nonlocal effect.

7. Conclusions

In the current research paper, a new mathematical model is derived on the basis of the nonlocal dual phase lag heat conduction model in the frame of a memory dependent derivative. This new proposed model is constructed on the foundation of the ideas investigated by TZOU and GUO [28], the theory of non-localized thermal conductivity with two-relaxation parameters in addition to the derivative based on memory influence. To illustrate the proposed model, an infinite thermoelastic half-space with a traction free boundary and subject to thermal shock is investigated. Exact analytical solutions are evaluated in the Laplace transform domain and by applying a suitable numerical technique, quantitative results of all field quantities are determined in a time-domain. The impact of the significant parameters, such as kernel, time-delay and nonlocal length parameter are analysed on the distributions of studied fields. It is believed that this study must be beneficial in tackling different problems of several areas: of physics, geophysics etc. and constructing better mathematical modelling of those systems. Salient features of the present investigation are given below.

1. Distributions of all physical quantities such as dimensionless temperature, displacement, and thermal stress occur in a finite and limited region near the surface of the half-space and these influences gradually disappear outside this region after a certain period of time. This physical phenomenon demonstrates the fact that thermoelastic vibrations transmit at a finite

speed inside the medium in contrast to the traditional theories that predict infinite speeds.

2. When the idea of memory dependent derivative is included in the heat conduction model, then all the field quantities such as temperature, displacement and thermal stress have obtained less values. Lower values of field quantities generate a low dissipation of energy and it reduces the chances for the structural failure of the material. In this way, the inclusion of memory influence becomes a better concept for obtaining the promising results.
3. Kernel functions are more influential on the profiles of temperature and stress.
4. A lower value of time-delay reveals more substantial realistic results compared to a higher value of the time delay.
5. The values of the physical fields enhance as the value of time increases monotonically.
6. Achieved physical fields become more compressive in nature, in presence of a nonlocal length parameter and consequently, authors obtained low values of field variables in the presence of nonlocal influence. Therefore, insertion of a nonlocal effect causes less energy loss and provides more satisfactory results.
7. It is concluded that the idea of the memory dependent derivative and nonlocal effect depicts the noteworthy influence that make the nonlocal thermoelasticity model in context of a memory dependent derivative more realistic compared to the existing models.

Hence, authors concluded that the present manuscript propounds a new meticulous mathematical model in context of the new definition of a memory dependent derivative where it is observed from the results, that the values of all the physical fields such as temperature, displacement and thermal stress are suppressed and, in this way, authors found that the system dissipates less energy compared to the case of absence of a memory dependent derivative. Moreover, this model becomes better compared to the existing ones.

Conflict of statement

There is no conflict of interest among the authors.

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