

## Modified calculation model for loads of bearing systems for tricone roller bits

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### INTRODUCTION

The values of the loads acting on cone bearing systems are significant parameters that support the work of a constructor of tricone roller bits. Knowledge of the value and distribution of these loads allows the adoption or correction of the basic structural dimensions of the bearing system, and thus the determination of the structure of this system at the stage of the initial technical design.

It should be emphasised that the values and distribution of loads on the cone bearing system during the actual drilling process depend on a number of factors. Apart from the given technological parameters (torque and pressure), the size of the loads is influenced by geological conditions, properties of the drilled rock, drilling technology used, etc. Determining the loads acting on separate types of bearings of the system is difficult, because a 3- or 4-row bearing system is a statically indeterminate case. The degree of difficulty of this task is increased due to the fact that during one rotation of the cone, the load changes many times, depending on which rim at the moment is in contact with the bottom of the hole. It is virtually impossible to accurately determine the influence of these factors on bearing load in real drilling conditions.

Therefore, the developed mathematical model of loads of cone bearings systems includes a series of simplifications verified in terms of mathematical correctness and reliability of the description of the drilling process with a bit.

### ASSUMPTIONS FOR LOAD MODEL OF THE BEARING SYSTEM

The mathematical model of the operation of a cone bit includes the following assumptions (Czaja and Dominiczak 1979, Kozielski 2003):

- the force applied to each particular tooth or cone column is distributed to its surroundings in the form of a continuous load  $q(x)$ ,
- the nature of the unit loads acting on the cone is known, so it is possible to distribute the load into three mutually perpendicular components:  $q_w$ ,  $q_n$ , and  $q_p$ , (Fig.1),

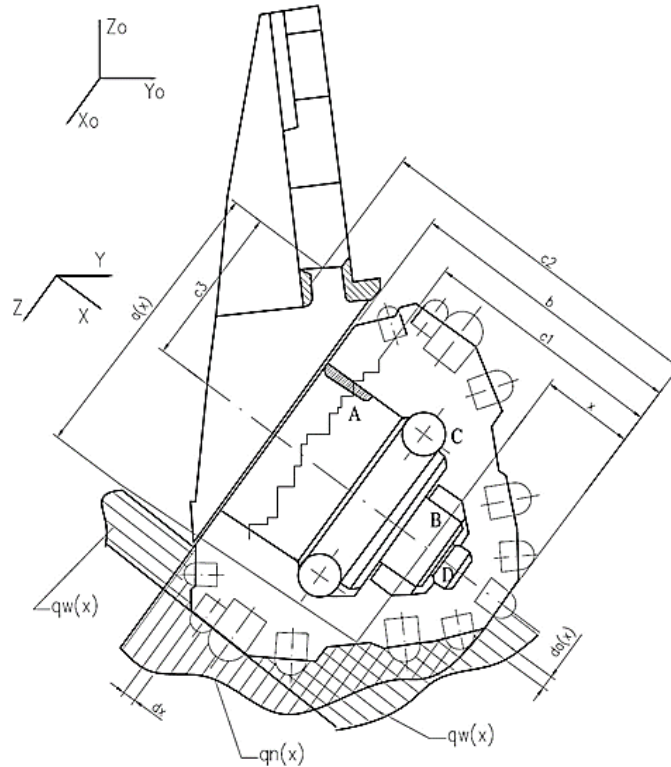


Fig. 1 The principle of determining the loads per cone bit

- the components of  $q_w$  and  $q_n$  loads lie in the vertical plane  $x$ - $z$  passing through the axis of the cone,
- the component  $q_p$  lies in the  $x$ - $y$  plane and it is perpendicular to the axis of the cone, this means that the  $q_p$  component has a direction tangent to the circles being cross-sections of the cones in the  $x_0$ - $z_0$  plane led by the points lying on the plane forming the cone, touching the bottom of the drilled hole,
- there are functional relationships between the individual components of the load:

$$q_w = f_1(q_n) \tag{1}$$

$$q_p = f_2(q_n) \tag{2}$$

- to determine the distribution of the total load of the bit on individual cones, tensometric measurements of the variability of the loads impacting on the body of bit and measurements of bending moments on pin connections and legs (segments) were carried out on real drills (Czaja and Dominiczak 1979),
- obtained results allow to determine  $q_w(x)$  loads for each forming value  $x$  within the  $0 \leq x \leq b$  with the following relationships:

$$\begin{cases} dM_1 = q_n(x) \cdot (c_1 - x)dx - q_w(x) \cdot [a(x) - c_3]da(x) \\ dM_2 = q_n(x) \cdot (c_2 - x)dx - q_w(x) \cdot a(x)da(x) \end{cases} \tag{3}$$

$$\begin{cases} M_1 = \int_0^b q_n(x) \cdot (c_1 - x)dx - \int_0^b q_w(x) \cdot [a(x) - c_3] \cdot a(x)dx \\ M_2 = \int_0^b q_n(x) \cdot (c_2 - x)dx - \int_0^b q_w(x) \cdot a(x) \cdot a(x)dx \end{cases} \tag{4}$$



In any given section of the cone in the direction of its rolling (section E-E, Fig. 3), the reaction changes according to the following relation:

$$q(\eta) = K \cdot \delta + \mu \cdot \frac{d\delta}{dt} \tag{7}$$

where:

- $K$  – stiffness of the drilled rock,
- $\mu$  – coefficient of internal friction,
- $\delta$  – deformation of ground.

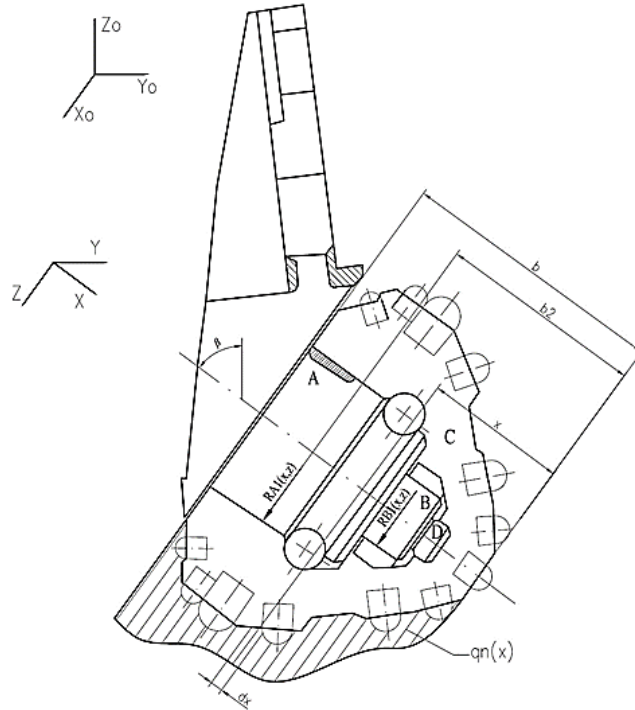


Fig. 3 Formula for determining the reaction of bite bearings from the component action  $q_n(x)$

Dependence describing the deformation of the ground as a function of the distance of a given section of a cone from its top:

$$\delta(r) = \overline{BD} = \delta_0 - [R(r) - \overline{BO}] = \delta_0 - [R(r) - \sqrt{R^2(r) - \eta^2}] \tag{8}$$

After extracting  $R(r)$  before the element and decomposition of the element into a power series, the following is obtained:

$$\delta(r) \cong \delta_0 - \frac{\eta^2}{2R(r)} \tag{9}$$

where:

$\delta_0$  – maximum ground deformation resulting from dependence:

$$R(r) = R'_0 + r'_0$$

$$R'_0 = r \cdot \sin\alpha ; \quad r'_0 = r_0 \cdot \sin\alpha$$

After transformation, the following is obtained:

$$\delta(r) \cong \delta_0 - \frac{\eta^2}{2(r_0 + r) \cdot \sin\alpha} \tag{10}$$

During the rolling of the cone around the axis of the bit with a given speed  $\omega$ , the cut zone of the contact between the cone and the ground will move away at the speed of:

$$\delta(r) \cong \delta_0 - \frac{\eta^2}{2(r_0 + r) \cdot \sin\alpha}$$

that is, with  $r = \text{constant}$

$$\frac{d\delta(r)}{dt} = \frac{-\eta}{(r_0 + r)\sin\alpha} \cdot \frac{d\eta}{dt}$$

hence:

$$\frac{d\delta(r)}{dt} = \frac{\omega}{\sin\alpha} \cdot \eta$$

$$q(\eta) = K \left[ \delta_0 - \frac{\eta_2^2(r)}{2(r_0 + r)\sin\alpha} \right] + \frac{\mu \cdot \omega}{\sin\alpha} \cdot \eta_1(r)$$

The dependence describing the change of the component at  $r = \text{const}$ , takes the following form:

Assuming that at the beginning of contact between the cone and the drilled rock, the deformation is equal to zero, and at the end of the contact the unit reaction assumes a zero value, the integration limits  $\eta_1$ ,  $\eta_2$  take the following form:

$$\begin{cases} \delta(\eta_2(r)) = 0 \\ q(\eta_1(r)) = 0 \end{cases} \quad (15)$$

hence:

$$\delta_0 - \frac{\eta_2^2(r)}{2(r_0 + r)\sin\alpha} = 0 \quad (16)$$

$$K \left[ \delta_0 - \frac{\eta_2^2(r)}{2(r_0 + r)\sin\alpha} \right] + \frac{\mu\omega\eta_1(r)}{\sin\alpha} = 0 \quad (17)$$

The above relationships allow to determine the values of  $\eta_1$  and  $\eta_2$ , that is the same zone of contact between the cone and the drilled rock, taking into account the parameters of the cone and the rotational speed.

## DIVISION OF REACTION VALUE IN INDIVIDUAL BEARINGS IN THE BEARING SYSTEMS

The values of the load components  $q_w$ ,  $q_p$  and  $q_n$  determined on the basis of the relations given in section 2 of the paper, allow to calculate the reactions in individual bearings. The dependencies for calculating the values of these reactions are presented below.

For slide bearings, the reactions occurring due to the action of the load component  $q_n(x)$  in the x-y plane (Fig. 3) are:

$$R_{A1(x,z)} + R_{B1(x,z)} = \int_0^b q_n(x) dx$$

$$R_{A1(x,z)} \cdot b_2 + R_{B1(x,z)} \cdot b_1 = \int_0^b q_n(x) \cdot x dx \quad (18)$$

Similarly, reactions in transverse bearings A and B and in thrust bearings C and D (the x-z plane) occurring due to the action of the load component  $q_w(x)$  can be calculated using the following relation:

$$\frac{R_C}{R_D} = \frac{F_C}{A_D} \quad (19)$$

$$R_C + R_D = \int_0^b q_w(x) \cdot a(x) dx$$

$$R_{A2(x,z)} + R_{B2(x,z)}$$

$$R_{A2(x,z)} \cdot (b_2 - b_1) = \int_0^b q_w(x) \cdot [a(x) - c_3] \cdot a(x) dx \quad (20)$$

where:

$F_C$  – additional stop surface field,

$F_D$  – surface of the stopper.

The dependence for calculating the reaction of the radial bearings A and B from the action of the load component of the cone  $q_p(x)$  in the x-y plane can be determined from the relation:

$$R_{A(x,y)} + R_{B(x,y)} = \int_0^b q_p(x) dx \quad (21)$$

$$R_{B(x,y)} \cdot b_1 + R_{A(x,y)} \cdot b_2 = \int_0^b q_p(x) \cdot x dx$$

Thus, the reaction values can be calculated from the following relation:

$$R_A = \sqrt{(R_{A1(x,z)} + R_{A2(x,z)})^2 + R_{A(x,y)}^2} \quad (22)$$

$$R_B = \sqrt{(R_{B1(x,z)} + R_{B2(x,z)})^2 + R_{B(x,y)}^2} \quad (23)$$

$$R_B = \sqrt{(R_{B1(x,z)} + R_{B2(x,z)})^2 + R_{B(x,y)}^2} \quad (24)$$

$$R_C = R_{C(x,z)}$$

$$R_D = R_{C(x,z)}$$

The knowledge of the components  $q_n(x)$ ,  $q_w(x)$  and  $q_p(x)$  allows also to calculate two basic parameters of the work of a cone bit, namely:

– the value of the total longitudinal force acting on the bit from dependence:

$$P_{ct} = \sum_{i=1}^3 P_{st}(i) \quad (25)$$

$$P_{ct}(i) = \int_0^b q_n(x) \cdot \cos(0,5\pi - \beta) dx + \int_0^b q_w(x) \cdot \cos\beta dx \quad (26)$$

– the value of the total longitudinal force acting on the bit from dependence  
where:

$$M_{ct} = \sum_{i=1}^3 M_{st}(i) \quad (27)$$

$$M_{ct}(i) = R_{A(x,y)}(i) \cdot r_A + R_{B(x,y)}(i) \cdot r_B \quad (28)$$

where:

$P_{ct}, M_{ct}$  – longitudinal force and torque from the  $n^{\text{th}}$  cone,

$r_A, r_B$  – radii of action of bearing transverse in the x-y plane.

Determining the values of  $\eta_1(r)$  and  $\eta_2(r)$ , that is, the zone of contact between the cone and the drilled rock can be determined using the formulas (16) and (17).

After transforming the equation (16), the formula takes the following form:

$$\eta_2 = \sqrt{2\delta_0(r_0 + r)\sin\alpha} \quad (29)$$

After transforming the equation (17), the formula takes the following form:

$$\eta_1 = -\frac{K\sin\alpha}{\mu\omega} \left[ \delta_0 - \frac{\eta_2^2}{2(r_0 + r)\sin\alpha} \right] \quad (30)$$

The calculation of the dependence describing the deformation of the ground as a function of the distance of a given section of the cone from its top and the contact zone of the cone with the drilled rock  $\alpha_{nr}$  are determined by the following relation:

$$\delta(\eta, r) \cong \delta_0 - \frac{\eta^2}{2(r_0 + r)\sin\alpha} \quad (31)$$

Loads  $q(\eta, r)$  can be determined using the formula (7) from the following relation:

$$q(\eta, r) = K \cdot \delta(\eta, r) + \mu \frac{d\delta(\eta, r)}{dt} \quad (32)$$

The total strength acting on the cone is determined by the formula:

$$G = \int_{\eta_1}^{\eta_2} \int_{r_0}^{R_0-r_0} K \cdot \delta(\eta, r) + \mu \frac{d\delta(\eta, r)}{dt} \cdot d\eta \cdot dr \quad (33)$$

The values of the load component  $q_n(x)$  is determined from the following dependence:

$$q_n(x) = \left( K \cdot \delta(\eta, r) + \mu \frac{d\delta(\eta, r)}{dt} \right) \cdot \cos\alpha \quad (34)$$

The values of the load component  $q_w(x)$  can be determined from the following formula:

$$q_w(x) = \left( K \cdot \delta(\eta, r) + \mu \frac{d\delta(\eta, r)}{dt} \right) \cdot \sin\alpha \quad (35)$$

The value of the load component  $q_p(x)$  is determined by the following dependence:

$$q_p(x) = \frac{2}{3} (q_n(x) + q_w(x)) \quad (36)$$

Determination of cone loads allows to calculate the reaction of slide bearings with dependence (18-21) with high accuracy. The calculation formulas take the following form:

$$R_{B1(x,z)} = \frac{\int_0^b q_w(x) \cdot x dx - b_2 \int_0^b q_n(x) dx}{b_1 - b_2} \quad (37)$$

$$R_{A1(x,z)} = \int_0^b q_n(x) dx - R_{B1(x,z)} \quad (38)$$

$$R_{A2(x,z)} = \frac{\int_0^b q_w(x) \cdot [a(x) - c_3] \cdot a(x) dx}{b_2 - b_1} \quad (39)$$

$$R_{B2(x,z)} = R_{A2(x,z)} \quad (40)$$

$$R_{B(x,y)} = \frac{\int_0^b q_p(x) \cdot x dx - b_2 \int_0^b q_p(x) dx}{b_1 - b_2} \quad (41)$$

$$R_{A(x,y)} = \int_0^b q_p(x) dx - R_{B(x,y)} \quad (42)$$

$$R_A = \sqrt{(R_{A1(x,z)} + R_{A2(x,z)})^2 + R_{A(x,y)}^2} \quad (43)$$

$$R_B = \sqrt{(R_{B1(x,z)} + R_{B2(x,z)})^2 + R_{B(x,y)}^2} \quad (44)$$

The value of the total longitudinal force acting on the bit takes the following final form:

$$P_{ct}(i) = \int_0^b q_n(x) \cdot \cos(0,5\pi - \beta) dx + \int_0^b q_w(x) \cdot \cos\beta dx \quad (45)$$

The value of the total torque transmitted to the bit during drilling is determined by the following formula:

$$M_{ct}(i) = R_{A(x,y)}(i) \cdot r_A + R_{B(x,y)}(i) \cdot r_B \quad (46)$$

## CONCLUSION

The presented calculation model of loads of cone bearing systems allows to determine the reaction value in bearings and to calculate the longitudinal force and torsional moment acting on the bit during drilling. It includes the variability of loads of cone bearings resulting from the geometry of the teeth of individual cones. Another step that would contribute to the research would be to supplement the presented model with calculation scheme of real contact zones for specific systems of reinforcement of cones composed of both teeth and columns made of sintered carbides. This requires access to documentation of the cone reinforcement geometry.

Preliminary calculations of the reaction values in the cone bearing systems proved that the model of reaction value calculation presented in the article fulfilled the assumed function.

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**Abstract.**

The article presents a modified calculation model of loads of cone bearing systems in tricone roller bits. The presented model includes the actual nature of the loads of a tricone roller bit and the effect of these loads on the construction of individual elements of the cone bearing systems. The presented calculation model was created in order to develop a computer program that allows the calculation and verification of structural parameters of cone bearing systems even at the stage of their initial design. It will allow an optimal selection of these parameters to geological properties of drilled rocks.

**Keywords:** drilling, drilling tools, tricone roller bit, cone bearings reaction calculation