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AN INVENTORY MODEL TO STUDY THE EFFECT OF THE PROBABILISTIC RATE OF CARBON EMISSION ON THE PROFIT EARNED BY A SUPPLIER

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The inventory of suppliers providing raw materials to industries producing green products faces two challenging problems. The first one is that raw materials are usually deteriorating items and the second one is that they emit carbon-based gases during deterioration. Moreover, each item has its unique carbon emission rate and composition, called the pattern of carbon emission, which is a function of the rate of carbon emission. In this present research, we attempt to develop a stochastic inventory model with price, stock, and pattern of carbon emission-dependent demand to maximise the profit of a supplier selling a single product. The rate of deterioration is a function of the rate of carbon emission and effective investment in preservation. The cost of carbon emission is a function of green investment and the pattern of carbon emission. Holding costs and purchase costs are constant. We consider three patterns of carbon emission, and each pattern is defined by a negative exponential function. The rate of carbon emission is assumed to be probabilistic and follows one of the three probabilistic distributions: uniform, triangular, and beta. Numerical validation is provided together with sensitivity analysis of the parameters for managerial insights. Analysis of the effect of carbon emission on the profit earned is made and results are interpreted. Particle swarm optimisation (PSO) and genetic algorithm (GA) are applied to solve the model, while statistical analysis and sensitivity analysis of the parameters of the algorithm are provided along with the graphical representation of convergence.

Keywords: *inventory modelling, carbon emission, green investment, preservation, PSO, GA*

1. Introduction

The awareness for green products increases with time, and simultaneously the demand for natural and renewable resources for producing takes the hike. It is the sustain-

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able green products that mimic the actual product with minimum impact on the environment. But green products like bioethanol, paper, toilet products, utensils, etc., require environmental exploitations. Thus, it is a challenge for the decision-maker that, *to what extent we can exploit our environment to develop such items whose negative impact on the environment is less?* This part is usually taken care of by inventory practitioners and model developers who develop mathematical models to address real-world problems, although the effect of exploiting natural resources as a source of raw materials is a serious issue that needs to be discussed from both economic and ecological perspectives. In the current study, we consider the economic aspect of a supplier selling the natural resources which act as a raw material for production industries. Apart from the quantity of raw material to be utilised preserving the sustainability factor, rate of deterioration, and the emission of carbon-based gases are also considered major problems from both an economic and environmental point of view [19]. An overview of the impact of greenhouse gas on the inventory of major corporations is analysed in [39]. These two issues can be controlled by reducing the selling period, though the technique is not advantageous for raw materials, like vegetable scrap and straw which need immediate treatment to reduce the rate of deterioration and minimise the level of carbon emission. Studies have been conducted [32] to reduce carbon emissions and expand a green society for sustainable development from both economic and environmental perspectives.

Modern preservation technologies can be implemented to reduce the rate of deterioration so that the natural resources could be utilised for a longer period. The phenomenon of carbon emission and the associated tax depends on the product, so the green investment is not usually carried out. When the carbon emission in the holding is relatively large in quantity and its associated carbon tax is significant, in that case, the manager decides to invest in green technologies and policies to increase the consumption of carbon-based gases that consequently reduce the carbon particles and related gases in the surroundings.

The study of carbon policies and their footprint in different products, locality, and the situation is an active area of research, especially for the suppliers' storing the carbon-emitting products on a large scale. In the era of sustainability, the effect of carbon emission is a serious global issue that can be confronted by green policies or by choosing the raw materials/sellable products for which emission is minimum.

2. Survey of literature

In the era of sustainability, the effect of carbon emission plays a crucial role in maintaining the profit structure of a company. Excessive carbon emission can increase carbon tax so that it would have a significant impact on the total profit earned by the suppliers. The effect of carbon emission is associated with inventory [20], production [15], and transportation [18]. The carbon constraint is introduced [2] as an environmental constraint to regulate carbon emission in production facilities and transportation for a lot-sizing problem. Economic quality control modelling with carbon constraint is developed by Chen et al. [6]. Carbon tax in retailers is studied by Daryanto et al. [7] with price-dependent demand under the economic quality control model. Chen et al. [6] provide analytical treatment to reduce carbon emissions by optimising the ordering quantity in economic ordering quantity modelling with carbon constraints. The impact of carbon emission on the ordering quantity and holding the effective ordering to minimise the cost in an EOQ is studied [3]. The effect of carbon emission on the demand for a product is studied by Aliabadi et al. [1] with random emission. Shen et al. [34] propose a production inventory model with a constant demand for deteriorating items with preservation technology under carbon emission policy. Carbon emission is controllable through the proper implementation of green technologies in a sustainable production inventory [24]. The study on controllable carbon emission is further extended to the two-warehouse inventory model [28] for effective reduction in the rate of emission.

The exploitation of natural resources to produce green products is a common phenomenon in the production sector. The utilisation of correct material in optimum quantity is one of the methods to reduce environmental exploitation. The production inventory model is proposed [28] to optimise the ordering quantity and purchase cost with replenishment policies. Green measures can be incorporated to reduce the impact of carbon emission in both EPQ, [12] as well as in EOQ, [31]. The three-stage Stackelberg game is considered to address the issue of profit maximisation with emission-dependent demand [14] with green investment. Mixed-integer nonlinear programming problem [36], discuss the policies regarding the awareness, taxation, government subsidy for firms considering the sustainability under green investment. The supply chain of green products with price and sales effort-dependent demand is proposed by Ghosh et al. [16]. For more research on effective green investment, one can refer to [11] and [13].

Deterioration is another cause for reducing the profit of suppliers selling natural resources, though imposing preservation technologies can reduce the negative effect of deterioration to a considerable amount. Recently, much research has been conducted to study the effect of deterioration and preservation on minimising cost [33] and maximising profit [10]. A two-warehouse inventory model with price- and stock-dependent demand is proposed [29] for deteriorating items under alternative trade credit policy. Deteriorating items are studied in a non-deterministic environment with the parametric approach to the differential equations [30]. Time-dependent deterioration is researched with discount policy and multiple prepayments [23], under price-dependent demand. Mashud et al. examine *i*th price-dependent demand [26] under a two-level trade-credit period. The authors [27] propose an inventory model to study the carbon emission and deterioration under effective green investment and preservation technology. Bhattacharjee and Sen [5] develop an inventory model with time-dependent deterioration and shortages of taw raw material in a heuristic environment. A measure to reduce deterioration by separating defective and non-defective items is proposed by Hasan et al. [17].

Deteriorating items with time-dependent demand is proposed under inventory modelling by Xu et al. [38], together with the effect of carbon emission. In Table 1, related topics in this direction have been summarized.

Table 1. Recent literature on inventory modelling

Reference	Demand	Carbon emission	Green investment	Random emission	Deterioration /defective	Preservation /screening
Aliabadi et al. [1]	credit- period-, price- and emission-dependent	yes	no	no	yes	no
Das et al. [9]	price-, stock-, and replacement period-dependent	no	no	no	no	no
Dutta [13]	price-dependent	yes	yes	yes	no	no
Singh et al. [35]	constant	yes	no	no	no	no
Daryanto et al. [8]	constant demand for good and defective items	yes	no	no	yes	yes
Mashud et al. [25]	price-dependent	no	no	no	yes	yes
Mishra et al. [22]	price and trade credit period	yes	yes	no	yes	yes
Manna et al. [21]	price- and stock-dependent	no	no	no	yes	yes
Present paper	price-, stock-, demand emission-dependent	yes	yes	yes	yes	yes

Recent researches in the area of inventory control to study the impact of carbon emission on the profit and demand of a product have been accomplished. Although the deterioration considered is either constant or a function of time, the effect of deterioration and emission of carbon-based gases on natural products like beet-root, sugarcane, etc. are not mutually exclusive and there is no research found in the literature considering the deterioration dependent on emission. Moreover, the study of different emission rate and their impact on the profit under the demand dependent on price, stock, and random emission is not covered in the literature as per the author's knowledge. So, an effort has been made to develop an inventory model considering the above factors together with an insight into green policies and effective preservation technologies. Description of proposed work and research problem is provided in Section 4.

3. Model description and research problem

Commercial goods like bioethanol, sugar, paper, etc., need raw materials from natural resources like beet-root, wood stock, sugarcane, bamboo, etc. The utilisation of raw materials from natural resources affects the environment and economic structure of a state. So, it is important to find a sustainable inventory model to regulate the utilisation of natural resources. Further, the suppliers selling these products to the production industry need to maintain an inventory of items they are dealing with (Fig. 1).

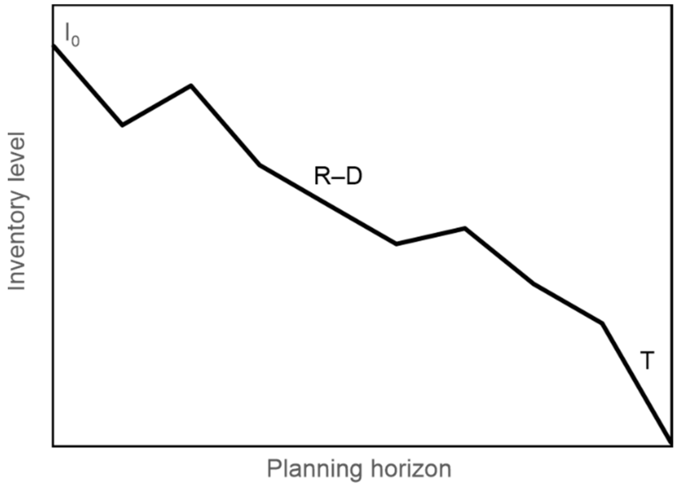

Fig. 1. Scheme of the inventory level

The major problems with these products mentioned above are their fast deterioration and the emission of carbon-based gases. It is observed that the pattern of carbon emission varies from product to product and the production house prefers those natural resources whose carbon emission is low. A pattern of carbon emission is a function of the rate of carbon emission which represents the amount of carbon-based gases released by the product from the natural resources during deterioration. So, the pattern of carbon emission plays a very important role in deciding the demand for a product. In this paper, an attempt is made to develop a single-item stochastic inventory model with price-, stock-, and emission-dependent demand. The current study is conducted on three patterns of carbon emission each of them is a negative exponential function of the rate of carbon emission. The rate of carbon emission is assumed to be probabilistic and it follows the probabilistic distribution, uniform, triangular, and beta distribution. Further, the following questions will be addressed with a numerical example.

- Optimal green investment to reduce carbon emission in inventory.
- Effective preservation technology costs to reduce the fast deterioration.
- Optimal selling price to maximise profit.
- Optimal selling period.
- Analyze the effect of carbon emission on the profit earned.

4. Mathematical model

Assumptions

- Lead time is zero.
- Demand function is

$$D(t, p, \gamma) = \left(d + I(t) - \alpha p\right) f(\gamma)$$

where $0 < \alpha < 1$ is the price sensitivity, and $f(\gamma)$ is the function of the rate of carbon emission representing the pattern of carbon emission (amount of carbon emission).

- The rate of carbon emission γ is probabilistic.
- Replenishment quantity is constant and instantaneous.
- Deterioration is a function of preservation cost and rate of carbon emission

$$\theta(\beta, \gamma) = \theta\left(1 - \mathrm{e}^{-f(\gamma)/\beta}\right)$$

- Carbon emission cost in inventory is a function of green investment

$$c_e\left(\gamma, I_g\right) = c_e \mathrm{e}^{-\left(I_g - f(\gamma)\right)}$$

- Shortages are not allowed.
- Initially the stock is full.
- Planning horizon is infinite.

Notation

We consider the following notations

I_0 – maximum inventory level

$I(t)$ – inventory level at any time t

θ – rate of deterioration, $0 < \theta < 1$

β – preservation cost, USD

I_g – green investment, USD

γ – constant rate of carbon emission, $0 < \gamma \le 1$

T – planning horizon

R – rate of replenishment

p – selling price, USD
c_e – carbon emission cost, USD
c – cost of the purchased item, USD
H – holding cost, USD
d – initial demand
$D(t, p, \gamma)$ – demand function

- The differential equation representing the flow of material from the stock to the market is given by:

$$\frac{dI}{dt} + \theta(\beta, \gamma)I = R - D(t, p, \gamma)$$

- Subject to the condition $I(0) = I_0$
- The inventory level at any time t is given by

$$I(t) = I_0 \mathrm{e}^{-t(\theta(\beta,\gamma)+f(\gamma))} + \left(\frac{R + \alpha f(\gamma)p - d}{\theta(\beta,\gamma) + f(\gamma)}\right)\left(1 - \mathrm{e}^{-t(\theta(\beta,\gamma)+f(\gamma))}\right)$$

- Holding cost

$$HC = H\left(\frac{I_0\left(1 - \mathrm{e}^{-(\theta(\beta,\gamma)+f(\gamma))T}\right)}{\theta(\beta,\gamma) + f(\gamma)} + \frac{R + \alpha f(\gamma)p - d}{\theta(\beta,\gamma) + f(\gamma)} + \left(T + \frac{\mathrm{e}^{-(\theta(\beta,\gamma)+f(\gamma))T}}{\theta(\beta,\gamma) + f(\gamma)}\right)\right)$$

- Deterioration cost

$$DC = c\theta(\beta,\gamma)\left(\frac{I_0\left(1 - \mathrm{e}^{-(\theta(\beta,\gamma)+f(\gamma))T}\right)}{\theta(\beta,\gamma) + f(\gamma)} + \frac{R + \alpha f(\gamma)p - d}{\theta(\beta,\gamma) + f(\gamma)} + \left(T + \frac{\mathrm{e}^{-(\theta(\beta,\gamma)+f(\gamma))T}}{\theta(\beta,\gamma) + f(\gamma)}\right)\right)$$

- Preservation cost

$$PC = \beta$$

- Green investment

$$GI = I_g$$

- Carbon emission cost

$$CHC = c_e\left(I_g\right)\left(\frac{I_0\left(1-\mathrm{e}^{-(\theta\,(\beta,\gamma)+f(\gamma))T}\right)}{\theta(\beta,\gamma)+f(\gamma)}+\frac{R+\alpha f(\gamma)p-d}{\theta(\beta,\gamma)+f(\gamma)}+\left(T+\frac{\mathrm{e}^{-(\theta\,(\beta,\gamma)+f(\gamma))T}}{\theta(\beta,\gamma)+f(\gamma)}\right)\right)$$

- Purchase cost

$$\Pr C = c\left(I_0 + RT\right)$$

- Average total cost is given by

$$ATC = c_0 + HC + DC + \alpha + I_g + CEC + PrC$$

$$\begin{aligned}
ATC &= c_0 + H\left(\frac{I_0\left(1-\mathrm{e}^{-(\theta\,(\beta,\gamma)+f(\gamma))T}\right)}{\theta(\beta,\gamma)+f(\gamma)}+\frac{R+\alpha f(\gamma)p-d}{\theta(\beta,\gamma)+f(\gamma)}+\left(T+\frac{\mathrm{e}^{-(\theta\,(\beta,\gamma)+f(\gamma))T}}{\theta(\beta,\gamma)+f(\gamma)}\right)\right)\\
&+ c\theta(\beta,\gamma)\left(\frac{I_0\left(1-\mathrm{e}^{-(\theta\,(\beta,\gamma)+f(\gamma))T}\right)}{\theta(\beta,\gamma)+f(\gamma)}+\frac{R+\alpha f(\gamma)p-d}{\theta(\beta,\gamma)+f(\gamma)}+\left(T+\frac{\mathrm{e}^{-(\theta\,(\beta,\gamma)+f(\gamma))T}}{\theta(\beta,\gamma)+f(\gamma)}\right)\right)+\beta+I_g\\
&+ c_e\left(I_g\right)\left(\frac{I_0\left(1-\mathrm{e}^{-(\theta\,(\beta,\gamma)+f(\gamma))T}\right)}{\theta(\beta,\gamma)+f(\gamma)}+\frac{R+\alpha f(\gamma)p-d}{\theta(\beta,\gamma)+f(\gamma)}+\left(T+\frac{\mathrm{e}^{-(\theta\,(\beta,\gamma)+f(\gamma))T}}{\theta(\beta,\gamma)+f(\gamma)}\right)\right)\\
&+ c\left(I_0+RT\right)
\end{aligned}$$

- Total revenue

$$\begin{aligned}
TR = pf(\gamma)\Bigg(\Bigg(&\frac{I_0\left(1-\mathrm{e}^{-(\theta\,(\beta,\gamma)+f(\gamma))T}\right)}{\theta(\beta,\gamma)+f(\gamma)}+\frac{R+\alpha f(\gamma)p-d}{\theta(\beta,\gamma)+f(\gamma)}\\
&+\left(T+\frac{\mathrm{e}^{-(\theta\,(\beta,\gamma)+f(\gamma))T}}{\theta(\beta,\gamma)+f(\gamma)}\right)\Bigg)+(d-\alpha p)T\Bigg)
\end{aligned}$$

• Average profit

$$
\begin{aligned}
AP = \frac{1}{T}\Big(& pf(p) \\
& \times \left(\left(\frac{I_0\left(1-\mathrm{e}^{-(\theta(\beta,\gamma)+f(\gamma))T}\right)}{\theta(\beta,\gamma)+f(\gamma)} + \frac{R+\alpha f(\gamma)p-d}{\theta(\beta,\gamma)+f(\gamma)} + \left(T + \frac{\mathrm{e}^{-(\theta(\beta,\gamma)+f(\gamma))T}}{\theta(\beta,\gamma)+f(\gamma)} \right) \right) + (d-\alpha p)T \right) \\
& - \left(c_0 + H \left(\frac{I_0\left(1-\mathrm{e}^{-(\theta(\beta,\gamma)+f(\gamma))T}\right)}{\theta(\beta,\gamma)+f(\gamma)} + \frac{R+\alpha f(\gamma)p-d}{\theta(\beta,\gamma)+f(\gamma)} + \left(T + \frac{\mathrm{e}^{-(\theta(\beta,\gamma)+f(\gamma))T}}{\theta(\beta,\gamma)+f(\gamma)} \right) \right) \right. \\
& + c\,\theta(\beta,\gamma) \left(\frac{I_0\left(1-\mathrm{e}^{-(\theta(\beta,\gamma)+f(\gamma))T}\right)}{\theta(\beta,\gamma)+f(\gamma)} + \frac{R+\alpha f(\gamma)p-d}{\theta(\beta,\gamma)+f(\gamma)} + \left(T + \frac{\mathrm{e}^{-(\theta(\beta,\gamma)+f(\gamma))T}}{\theta(\beta,\gamma)+f(\gamma)} \right) \right) + \beta + I \\
& + c_e\left(I_g\right) \frac{I_0\left(1-\mathrm{e}^{-(\theta(\beta,\gamma)+f(\gamma))T}\right)}{\theta(\beta,\gamma)+f(\gamma)} + \frac{R+\alpha f(\gamma)p-d}{\theta(\beta,\gamma)+f(\gamma)} + \left(T + \frac{\mathrm{e}^{-(\theta(\beta,\gamma)+f(\gamma))T}}{\theta(\beta,\gamma)+f(\gamma)} \right) \\
& \left. + c\left(I_0 + RT\right) \right) \Big)
\end{aligned}
$$

The decision variable we consider are

• selling price p,

• preservation cost β,

• green investment I_g,

• planning horizon T.

5. Methodology

The proposed work is designed mathematically with a linear differential equation with the initial condition. The objective function is nonlinear and maximisation type with four variables. So, it is difficult to establish optimisation with an analytical method and even if the function is optimised by introducing various restrictions on parameters, it will be a local optimum and not the global optimum. Therefore, the heuristic approach is considered in this case. The formulated objective function is solved using weighted particle swarm optimisation and genetic algorithm to make a comparison between the solutions. However, we adopt the graphical approach from [21] to analyse the convergence of both algorithms. Further, a comprehensive statistical analysis is provided, and a solution is accepted when the variance of the array of global optimum (best improvements) is less than a fixed predefined number, [4], for the present problem we consider variance = 10^{-10}. Moreover, a description of algorithms, numerical values of parameters, sensitivity analysis of the parameters of both algorithms, and the convergence curves are provided in Appendices 1–3.

The system and software information where the calculations are conducted.

3.40 GHz Intel Core i5 Processor

12 GB RAM and 64 Bit Operating System

Windows 10 environment

SciLab 6.1.0 software and MS Excel application.

6. Numerical illustration

In the lack of original data, the following values of the parameters are assumed. Consider the vendor selling beet-root scrap for ethanol production. The vendor maintains an initial stock of I_0 = 10 000 lbs and keeps the replenishment on with a constant rate R = 500 lbs. Beet-root scrap emits carbon-based gases at a rate of 0.5 in a certain pattern and to reduce that green investments have been made. The holding cost of the material is H = 10 USD and the purchase cost c of beet-root is 12 USD. The rate of deterioration θ is 0.001 but it can be reduced by incorporating preservation technologies. Further, the initial demand d for beet-root in the ethanol production industry is 450 lbs. The price sensitivity α of the beet-root is 0.2. It is important to know the selling price (USD/unit) of beet-root, the green investments (USD) and effective preservation technology cost (USD), and the time cycle (days). We consider three different patterns of carbon emission (Tables 2–13, Fig. 2).

Table 2. Optimal solution with PSO

Pattern	Carbon emission $f(\gamma)$	p^*	β^*	I_g^*	T^*	Profit
I	$e^{-(1+\gamma^2)}$	250	671.293	12.545	1.655	537 494.41
II	$e^{-(1/(1-\gamma))}$	250	774.437	12.973	2.973	283 979.21
III	$e^{-1/\gamma}$	250	1000	12.973	2.584	283 979.12

Table 3. Optimal solution with GA

Pattern	Carbon emission $f(\gamma)$	p^*	β^*	I_g^*	T^*	Profit
I	$e^{-(1+\gamma^2)}$	250	622.6292	12.527	1.655	537 494.41
II	$e^{-(1/(1-\gamma))}$	250	620.465	12.972	2.584	283 979.21
III	$e^{-1/\gamma}$	250	424.333	12.967	2.584	283 979.21

Table 4. Statistical analysis of the optimal solution

Statistical parameter	Pattern					
	Weighted PSO			Genetic algorithm		
	I	II	III	I	II	III
Mean	537 494.41	283 979.21	283 979.12	537 494.41	283 979.21	283 979.21
Median	537 494.41	283 979.21	283 979.12	537 494.41	283 979.21	283 979.21
Variance	2.089×10^{-16}	6.991×10^{-18}	2.345×10^{-19}	2.19×10^{-19}	7.605×10^{-19}	1.677×10^{-19}
Standard deviation	1.445×10^{-8}	2.64×10^{-10}	4.8×10^{-10}	4.68×10^{-10}	5.784×10^{-10}	4.095×10^{-10}
Time elapsed	2.12	2.146	2.139	89.626	86.979	86.768

Case I

γ follows uniform distribution. Let $\gamma_1 = 0.5$ and $\gamma_2 = 0.65$, then

Table 5. Optimal solution under uniform distribution for PSO

Pattern	Carbon emission $f(\gamma)$	p^*	β^*	I_g^*	T^*	Profit
I	$e^{-(1+\gamma^2)}$	250	633.8	12.545	1.743	501 156.33
II	$e^{-(1/(1-\gamma))}$	250	1000	13.193	3.022	214 773.54
III	$e^{-1/\gamma}$	250	1000	12.814	2.24	352 954.98

Table 6. Optimal solution under uniform distribution for GA

Pattern	Carbon emission $f(\gamma)$	p^*	β^*	I_g^*	T^*	Profit
I	$e^{-(1+\gamma^2)}$	250	693.386	12.545	1.743	501 156.33
II	$e^{-(1/(1-\gamma))}$	250	277.937	13.203	3.022	214 773.54
III	$e^{-1/\gamma}$	250	90.233	16.807	2.24	352 954.38

Table 7. Statistical analysis of optimal solution under uniform distribution

Statistical parameter	Pattern					
	Weighted PSO			Genetic algorithm		
	I	II	III	I	II	III
Mean	501 156.33	214 773.54	352 954.98	501 156.33	214 773.54	352 954.98
Median	501 156.33	214 773.54	352 954.98	501 156.33	214 773.54	352 954.98
Variance	5.945×10^{-18}	1.579×10^{-18}	4.706×10^{-20}	3.422×10^{-19}	2.473×10^{-19}	2.19×10^{-19}
Standard deviation	2.44×10^{-9}	1.26×10^{-9}	2.2×10^{-10}	5.85×10^{-10}	4.973×10^{-10}	4.68×10^{-10}
Time elapsed	2.156	2.169	2.178	88.48	88.746	89.082

Case II

γ follows triangular distribution. Let $\gamma_1 = 0.5$, $\gamma_2 = 0.58$, and $\gamma_3 = 0.65$ then

Table 8. Optimal solution under triangular distribution PSO

Pattern	Carbon emission $f(\gamma)$	p^*	β^*	I_g^*	T^*	Profit
I	$e^{-(1+\gamma^2)}$	250	981.902	12.586	1.745	500 322.9
II	$e^{-(1/(1-\gamma))}$	250	35.921	13.2	3.032	213 273.12
III	$e^{-1/\gamma}$	250	840.668	12.812	2.233	354 459.08

Table 9. Optimal solution under triangular distribution with GA

Pattern	Carbon emission $f(\gamma)$	p^*	β^*	I_g^*	T^*	Profit
I	$e^{-(1+\gamma^2)}$	250	868.774	12.078	1.745	500 322.85
II	$e^{-(1/(1-\gamma))}$	250	576.082	13.2	3.032	213 273.12
III	$e^{-1/\gamma}$	250	286.136	12.807	2.233	354 459.08

Table 10. Statistical analysis of optimal solution under triangular distribution

Statistical parameter	Pattern					
	Weighted PSO			Genetic algorithm		
	I	II	III	I	II	III
Mean	500 322.9	213 273.12	35 459.08	500 322.85	213 273.12	354 459.08
Median	500 322.9	213 273.12	35 459.08	500 322.85	213 273.12	354 459.08
Variance	8.822×10^{-18}	1.291×10^{-18}	7.054×10^{-18}	1.369×10^{-20}	2.139×10^{-20}	1.232×10^{-19}
Standard deviation	2.97×10^{-9}	1.14×10^{-9}	2.66×10^{-9}	1.17×10^{-10}	1.46×10^{-10}	3.51×10^{-10}
Time elapsed	2.173	2.165	2.213	105.13	88.37	97.463

Case III

γ follows beta distribution. Let $a = 0.5$, and $b = 0.65$, then

Table 11. Optimal solution under beta distribution with PSO

Pattern	Carbon emission $f(\gamma)$	p^*	β^*	I_g^*	T^*	Profit
I	$e^{-(1+\gamma^2)}$	250	1000	12.514	1.591	566 764.26
II	$e^{-(1/(1-\gamma))}$	250	144.12	12.83	2.279	344 095.10
III	$e^{-1/\gamma}$	250	1000	13.16	2.962	223 634.99

Table 12. Optimal solution under beta distribution with GA

Pattern	Carbon emission $f(\gamma)$	p^*	β^*	I_g^*	T^*	Profit
I	$e^{-(1+\gamma^2)}$	250	769.68	12.491	1.591	566 764.26
II	$e^{-(1/(1-\gamma))}$	250	499.366	12.83	2.28	344 095.10
III	$e^{-1/\gamma}$	250	182.947	13.13	2.962	223 634.99

Table 13. Statistical analysis of optimal solution under beta distribution

Statistical parameter	Pattern					
	Weighted PSO			Genetic algorithm		
	I	II	III	I	II	III
Mean	566 764.26	344 095.10	223 634.99	566 764.26	344 095.10	223 634.99
Median	566 764.26	344 095.10	223 634.99	566 764.26	344 095.10	223 634.99
Variance	7.091×10^{-18}	3.516×10^{-18}	3.518×10^{-18}	6.708×10^{-19}	4.141×10^{-19}	4.192×10^{-20}
Standard deviation	2.66×10^{-9}	1.88×10^{-9}	2.35×10^{-9}	8.19×10^{-10}	6.435×10^{-10}	2.048×10^{-10}
Time elapsed	1.896	1.903	2.18	89.759	88.992	88.804

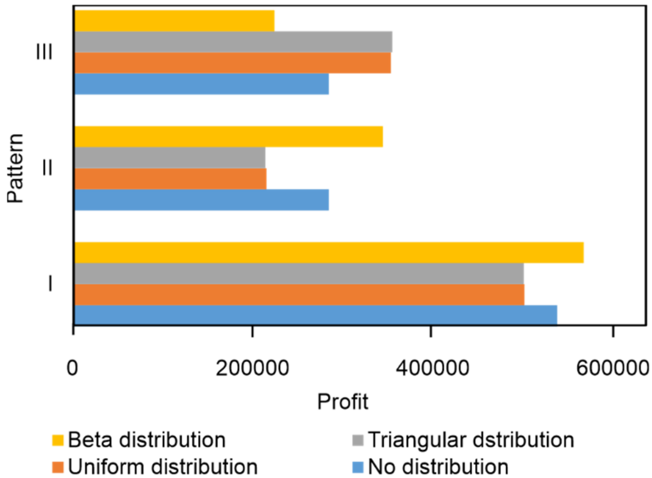

Fig. 2. Profits earned in each pattern and for each distribution

7. Sensitivity analysis

We perform the sensitivity analysis of the parameters to test the change in profit earned under different patterns.

Table 14. Sensitivity analysis of I_0

Parameter	Per cent change	Profit		
		Pattern I	Pattern II	Pattern III
I_0	20	633 211.83	331 050.99	331 050.99
	10	585 347.32	307 493.77	307 493.77
	–10	489 657.13	260 522.76	260 522.76
	–20	441 841.61	237 148.18	237 148.18

Table 15. Sensitivity analysis of θ

Parameter	Per cent change	Profit		
		Pattern I	Pattern II	Pattern III
θ	20	537 494.41	283 979.21	283 979.21
	10	537 494.41	283 979.21	283 979.21
	–10	537 494.41	283 979.21	283 979.21
	–20	537 494.41	283 979.21	283 979.21

Table16. Sensitivity analysis of R

Parameter	Per cent change	Profit		
		Pattern I	Pattern II	Pattern III
R	20	592 346.6	361 677.46	361 677.46
	10	564 238.63	319 883.00	319 883.00
	–10	511 984.82	253 213.01	253 213.01
	–20	487 583.52	226 380.05	226 380.05

Table 17. Sensitivity analysis of H

Parameter	Per cent change	Profit		
		Pattern I	Pattern II	Pattern III
H	20	592 346.6	361 677.46	361 677.46
	10	564 238.63	319 883.00	319 883.00
	–10	511 984.82	253 213.01	253 213.01
	–20	487 583.52	226 380.05	226 380.05

Table 18. Sensitivity analysis of c

Parameter	Per cent change	Profit		
		Pattern I	Pattern II	Pattern III
c	20	528 511.51	280 134.15	280 134.15
	10	532 882.59	282 005.79	282 005.79
	–10	542 381.51	286 068.76	286 068.76
	–20	547 587.20	288 292.33	288 292.33

Table 19. Sensitivity analysis of c_e

Parameter	Per cent change	Profit		
		Pattern I	Pattern II	Pattern III
c_e	20	537 494.34	283 979.18	283 979.18
	10	537 494.38	283 979.19	283 979.19
	–10	537 494.45	283 979.22	283 979.22
	–20	537 494.49	283 979.24	283 979.24

Table 20. Sensitivity analysis of d

Parameter	Per cent change	Profit		
		Pattern I	Pattern II	Pattern III
d	20	498 243.36	234 232.89	234 232.89
	10	517 398.79	257 439.56	257 439.56
	–10	558 627.89	314 762.21	314 762.21
	–20	580 899.09	350 394.70	350 394.70

Table 21. Sensitivity analysis of α

Parameter	Per cent change	Profit		
		Pattern I	Pattern II	Pattern III
α	20	538 296.88	284 546.31	284 546.31
	10	537 895.12	284 262.28	284 262.28
	–10	537 094.76	283 697.11	283 697.11
	–20	536 696.15	283 415.97	283 415.97

Table 22. Sensitivity analysis of γ

Parameter	Per cent change	Profit		
		Pattern I	Pattern II	Pattern III
γ	20	537 494.41	283 979.21	283 979.21
	10	537 494.41	283 979.21	283 979.21
	–10	537 494.41	283 979.21	283 979.21
	–20	537 494.41	283 979.21	283 979.21

Table 23. Sensitivity analysis of γ under uniform distribution

Parameter	Per cent change	Profit		
		Pattern I	Pattern II	Pattern III
γ (uniform)	20	501 156.33	214 773.54	352 954.98
	10	501 156.33	214 773.54	352 954.98
	–10	501 156.33	214 773.54	352 954.98
	–20	501 156.33	214 773.54	352 954.98

Table 24. Sensitivity analysis of γ under triangular distribution

Parameter	Per cent change	Profit		
		Pattern I	Pattern II	Pattern III
γ (triangular)	20	500 322.90	213 273.12	354 459.08
	10	500 322.90	213 273.12	354 459.08
	–10	5003 22.90	213 273.12	354 459.08
	–20	500 322.90	213 273.12	354 459.08

Table 25. Sensitivity analysis of γ under beta distribution

Parameter	Per cent change	Profit		
		Pattern I	Pattern II	Pattern III
γ (beta)	20	566 764.26	344 095.10	223 634.99
	10	566 764.26	344 095.10	223 634.99
	–10	566 764.26	344 095.10	223 634.99
	–20	566 764.26	344 095.10	223 634.99

In the light of the sensitivity analysis of the parameters, we can discuss some points to be considered during the decision-making processes of the vendor to maximise profit.

The initial I_0, replenishment quantity R, price sensitivity α, and holding cost H for a given cycle are sensitive parameters. From Tables 14, 16, 17, 21 it is clear that the profit increases with the increase in these quantities.

The cost price c and initial demand d are also sensitive parameters, although it is clear from Tables 18, 20 that the cost price and initial demand have a negative effect on profit-making. Profit reduces with the increase in these two parameters.

From Tables 15, 19 we can say that, the rate of deterioration θ and carbon emission cost c_e do not have any effect on the profit earned.

The rate of carbon emission is assumed to be probabilistic and we analyse three negative exponential patterns of emission. It has been observed that the profit is maximum when the carbon emission follows Pattern I.

For Pattern I and Pattern II, if the rate of emission follows beta distribution, the profit increases whereas the profit decreases when it follows uniform and triangular distribution. For Pattern III, the situation is exactly the opposite.

Furthermore, the carbon emission rate γ has an impact on the profit earned. From Tables 22–25 it can be observed that profit reduces with the increase in the rate of carbon emission for Patterns I and II. But for Pattern III, the situation is the opposite.

8. Analysis of the rate of carbon emission

In the sensitivity analysis, we made a mild change in the given value of γ, and it is clear that mild change in γ does not affect the profit anyway. In this section, we analyse the effect of $\gamma \in (0, 1]$ on the profit under three different patterns. We consider the values of γ as follows: $\gamma = 0.1 + 0.1k$, $0 \le k \le 9$.

Table 26. Analysis of the effect of γ on the profit

γ	Profit		
	Pattern I	Pattern II	Pattern III
0.1	662 445.88	606 560.56	no feasible solution
0.2	645 344.11	537 494.41	81 552.498
0.3	617 822.29	460 419.32	120 378.02
0.4	581 272.85	375 339.19	192 632.15
0.5	537 494.41	283 979.21	283 979.21
0.6	488 558.02	192 632.15	375 339.19
0.7	436 658.45	120 378.02	460 419.32
0.8	383 965.24	81 552.498	537 494.41
0.9	332 489.19	no feasible solution	606 560.56
1.0	283 979.21	no feasible solution	668 246.90

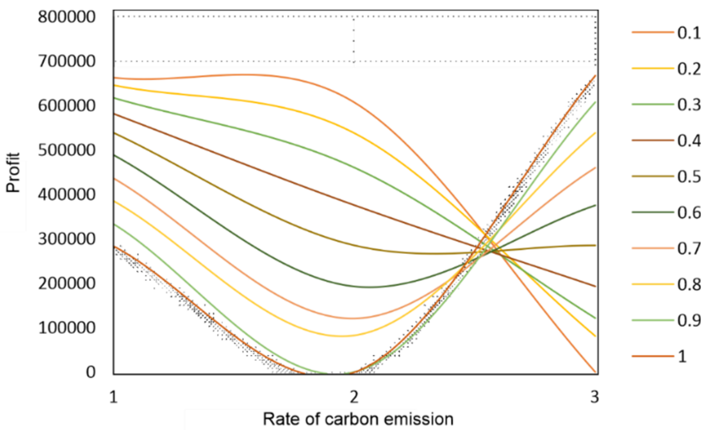

Fig. 3. Profit under three patterns with different rates of carbon emission

The change in profit is observed when we vary the rate of carbon emission (Fig. 3). A straight descent in the profit is noticed from Patterns I–III when the rate of carbon emission γ is 0.4. Moreover, the profit earned is between 200 000 and 300 000 USD for all values of γ when the amount of carbon emission follows a pattern between II and III. Maximum profit is recorded for $\gamma = 0.1$ and between patterns I and II.

9. Conclusions

This is a single-item stochastic model with price-, stock-, and emission-dependent, associated with a Pattern of carbon emission. The rate of emission is probabilistic and follows the uniform, triangular and beta distribution. Holding and purchase costs are assumed to be constant, and carbon emission cost is a function of green investment. The deterioration is dependent on the pattern of carbon emission and preservation costs. Due to a nonlinear objective function with four variables, an algorithmic approach is considered to solve the formulated objective function. Particle swarm optimisation and genetic algorithm are applied to maximise the profit and to obtain the optimum selling price, preservation cost, green investment, and planning horizon. The obtained results suggest that suppliers should consider the products for sale whose emission follows Pattern I, though the planning horizon is short as compared to the planning horizon for Patterns II and III. Therefore, the supplier must sell the product as early as possible. When the rate of emission follows the uniform distribution, suppliers have another option of products with Pattern III. Although the profit reduces, the seller gets more time to sell the products and the situation is similar when the rate of emission follows the triangular distribution. When the emission rate follows beta distribution, the profit under Pattern II is greater than that under Pattern III. However, one must consider the products whose rate of emission follows Pattern I under beta distribution, as the profit earned in beta distribution is maximum as compared to all the other cases.

The statistical analysis is provided for each solution and the observation that can be made from all the statistical data is that the array of global optimum (best improvements) converges as no significant deviation is observed between the global best and the mean and median of the array of best improvements. Further, the sequence of improvement is convergent as the variance for each solution is smaller than the predefined quantity of 10^{-10}. The convergence curve for each solution is provided in Appendix 3. However, there is a significant difference in the computational time and particle swarm optimisation extracts the optimal solution much faster than the genetic algorithm. Thus, for our mathematical model particle swarm algorithm provides results faster than the genetic algorithm, although there is no difference in the optimal values of the profit function.

The present work can be extended to include trade credit period-dependent demand with replacement policy and non-instantaneous replenishment. Further, the deterioration dependent on time, preservation, and emission can expand the study for the products whose deterioration is time-dependent. Suppliers selling multiple products gathered from natural resources can be studied by extending the present model for multi-items.

Some significant managerial insights can be highlighted from the proposed work for a sensible decision-making process:

From Tables 2, 3, the profit earned is the maximum for Pattern I. Also, the planning horizon is small as compared to Patterns II and III, therefore the manager should sell the entire stock of products with Pattern I carbon emission faster than the products with Pattern II and Pattern III. The green investment is almost similar for all the carbon emission patterns.

From Tables 5, 6, when γ follows a uniform distribution, profit earned is the smallest when the supplier sells the products with Pattern II carbon emission. However, the supplier gets enough time to sell the product. Green investment is almost similar, and preservation technology cost is minimum for Pattern III. Suppliers earn maximum profit by selling the product with Pattern I carbon emission policy, although the preservation cost is maximum for Patterns II and III.

From Tables 8, 9, when γ follows triangular distribution, the planning horizon and green investment are similar to the uniform distribution, although the profit in each pattern under triangular distribution reduces as compared to the profit in the corresponding patterns under the uniform distribution.

From Tables 11–12, when γfollows beta distribution, profit earned is minimum for Pattern III. In this case, also the green investment is almost similar as compared to other cases. The profit earned is the maximum for Pattern I.

Suppliers earn a maximum profit on selling the product with the Pattern I carbon emission. However, the profit maximizes when the product follows the beta distribution.

From Tables 14–16, the initial stock has a positive impact on profit. Therefore, storing a large quantity of product for sale is beneficial for the supplier. Moreover, increasing the instantaneous replenishment will increase the profit of the suppliers.

From Table 26 it can be observed that profit earnings increase with the decrease in the rate of carbon emission. Therefore, the manager should select the products with the smallest rate of carbon emission to maximise the profit except for the case of Pattern III. Also, form Fig. 3, the profit is maximum between emission Pattern I and Pattern II which $0 < \gamma < 0.4$ and for Pattern III profit is maximum when $\gamma \geq 0.6$.

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Appendix 1. Algorithms

Particle swarm optimisation (PSO)

In the group of swarm intelligence, particle swarm optimisation (PSO) is one of the popular algorithms used by many researchers to solve highly non-linear equations with multiple numbers of variables. In this present model, we apply weighted PSO [37, eq. 20.5], where a small inertial weight is attached with the velocity vector. Let $x(i, j)$ be a jth solution at the ith position and $v(i, j)$ be the velocity of ith particle in jth the position then the standard velocity equation is given by:

$$v(i,\ j+1) = wv(i,\ j) + \alpha u_1 \left(c\,\text{best} - x(i,\ j)\right) + \left(g\,\text{best} - x(i,\ j)\right)$$

where w – inertial weight α and β are accelerating factors, u_1 and u_2 be two random numbers in (0, 1). $c$best and $g$best are the current and global best.

Pseudo-algorithm
Initialized the population
Initialize the global best solutiom and
for $i = 1: m$
for $j = 1: n$
$x(i, j + 1) = x(i, j) + v(i, j)$
end
end
Compute the fitness of profit function with the solution 1st position
update the current best
update the global best.
stop

We consider the values of the parameters associated with the PSO
Population size $N = 100$
Number of iterations $I = 500$
Maximum inertial weight $w\text{max} = 0.9$
Minimum inertial weight $w\text{min} = 0.4$
Coefficients of acceleration $\alpha = 2$ and $\beta = 2$

Genetic algorithm (GA)

In a genetic algorithm [37], there are four steps to optimise a function these steps defined as follows:

Selection. In the selection process, we select a population of finite size randomly from the search boundary. There are various methods of selecting the population. In this algorithm, the elitist selection method is chosen for selection purposes.

Crossover. The crossover operation usually depends on the problem to optimise. Due to the continuous objective function of real decision variables, the crossover operation is defined as follows.

function [$y1$ $y2$] = crossover ($x1$, $x2$)
w = rand () //random uniform number from 0 to 1
$y1 = r \times x1 + (1 - r) \times x2$
$y2 = r \times x2 + (1 - r) \times x2$
end

Mutation. The purpose of mutation operation is to retard the rate of convergence. The probability of mutation is usually small and included in the algorithm to protect the evolution of the solution from being trapped in local optimum. In this GA coding, we consider the mutation operation as a minute change in the decision variable and it is defined as follows

function$y$ = mut(x)
r = rand () //random number from 0 to 1
$y = x + r$
end

Stopping criteria. We provide a valid condition to terminate the iteration process. The process terminates if the number of iteration exceeds N (maximum iteration number) and a solution is accepted if the standard deviation of global best solutions is less than 10^{-5}.

Pseudo-algorithm
Initialize the population (chromosome) x
Define the fitness function $f(x)$
Define crossover function using equation
Define mutation function using equation
Compute the fitness of each chromosome
Compute the fitness probability

$$p(x) = \frac{f(x)}{\sum f(x)}$$

Initialize the global best at the iteration ite = 0
Initialize the algorithm
Ite = ite + 1
if $p(x) < m$ then
Call the mutation function mut(x)
Number of chromosome selected for mating $M = N \times c$
Select two individual for mating
  Ch(1) = rand(1, M)
  Ch(2) = rand(1, M)
Call the crossover function crossover(Ch(1), Ch(2))
Compute the fitness $f(x)$ with new individual
Replace the worst chromosome from the population with the new individual (if any)
Update the global best
Stopping criterion

To perform soft computing, we assume the following values of the parameters of GA.
Population size $N = 100$
Crossover probability $c = 0.7$

Mutation probability $m = 0.1$
Maximum iteration Gen = 500

Appendix 2. Sensitivity analysis

The sensitivity analysis of weighted parameters of particle swarm optimisation (W-PSO) and genetic algorithm (GA) has been performed to observe any change in the optimal value of profit function obtained in Table 2 and Table 3. We made the change from –20% to 20% in each parameter.

Sensitivity analysis of W-PSO

Table A1. Sensitivity analysis of N

Parameter	Per cent change	Profit		
		Pattern I	Pattern II	Pattern III
N	20	537 494.41	283 979.21	283 979.21
	10	537 494.41	283 979.21	283 979.21
	–10	537 494.41	283 979.21	283 979.21
	–20	537 494.41	283 979.21	283 979.21

Table A2. Sensitivity analysis of I

Parameter	Per cent change	Profit		
		Pattern I	Pattern II	Pattern III
I	20	537 494.41	283 979.21	283 979.21
	10	537 494.41	283 979.21	283 979.21
	–10	537 494.41	283 979.21	283 979.21
	–20	537 494.41	283 979.21	283 979.21

Table A3. Sensitivity analysis R

Parameter	Per cent change	Profit		
		Pattern I	Pattern II	Pattern III
R	20	537 494.41	283 979.21	283 979.21
	10	537 494.41	283 979.21	283 979.21
	–10	537 494.41	283 979.21	283 979.21
	–20	537 494.41	283 979.21	283 979.21

Table A4. Sensitivity analysis w_{max}

Parameter	Per cent change	Profit		
		Pattern I	Pattern II	Pattern III
w_{max}	20	537 494.41	283 979.21	283 979.21
	10	537 494.41	283 979.21	283 979.21
	–10	537 494.41	283 979.21	283 979.21
	–20	537 494.41	283 979.21	283 979.21

Table A5. Sensitivity analysis of $w_{\min}$

Parameter	Per cent change	Profit		
		Pattern I	Pattern II	Pattern III
$w_{\min}$	20	537 494.41	283 979.21	283 979.21
	10	537 494.41	283 979.21	283 979.21
	–10	537 494.41	283 979.21	283 979.21
	–20	537 494.41	283 979.21	283 979.21

Table A6. Sensitivity analysis of α

Parameter	Per cent change	Profit		
		Pattern I	Pattern II	Pattern III
α	20	537 494.41	283 979.21	283 979.21
	10	537 494.41	283 979.21	283 979.21
	–10	537 494.41	283 979.21	283 979.21
	–20	537 494.41	283 979.21	283 979.21

Table A7. Sensitivity analysis of β

Parameter	Per cent change	Profit		
		Pattern I	Pattern II	Pattern III
β	20	537 494.41	283 979.21	283 979.21
	10	537 494.41	283 979.21	283 979.21
	–10	537 494.41	283 979.21	283 979.21
	–20	537 494.41	283 979.21	283 979.21

Sensitivity analysis of GA

Table A8. Sensitivity analysis of N

Parameter	Per cent change	Profit		
		Pattern I	Pattern II	Pattern III
N	20	537 494.41	283 979.21	283 979.21
	10	537 494.41	283 979.21	283 979.21
	–10	537 494.41	283 979.21	283 979.21
	–20	537 494.41	283 979.21	283 979.21

Table A9. Sensitivity analysis of Gen

Parameter	Per cent change	Profit		
		Pattern I	Pattern II	Pattern III
Gen	20	537 494.41	283 979.21	283 979.21
	10	537 494.41	283 979.21	283 979.21
	–10	537 494.41	283 979.21	283 979.21
	–20	5374 94.41	283 979.21	283 979.21

Table A10. Sensitivity analysis of *c*

Parameter	Per cent change	Profit		
		Pattern I	Pattern II	Pattern III
c	20	537 494.41	283 979.21	283 979.21
	10	537 494.41	283 979.21	283 979.21
	–10	537 494.41	283 979.21	283 979.21
	–20	537 494.41	283 979.21	283 979.21

Table A11. Sensitivity analysis *m*

Parameter	Per cent change	Profit		
		Pattern I	Pattern II	Pattern III
m	20	537 494.41	283 979.21	283 979.21
	10	537 494.41	283 979.21	283 979.21
	–10	537 494.41	283 979.21	283 979.21
	–20	537 494.41	283 979.21	283 979.21

From Tables A1–A7 and Tables A8–A11, it can be observed that, the change in parameters does not affect the optimal value of average profit at all. Thus we conclude that the optimal solutions are stable and does not affect by the change in the parameters of both the algorithm.

Appendix 3. Convergence characteristic curves

In this section, the convergence of both W-PSO and GA are represented pictorially for each case and all the patterns.

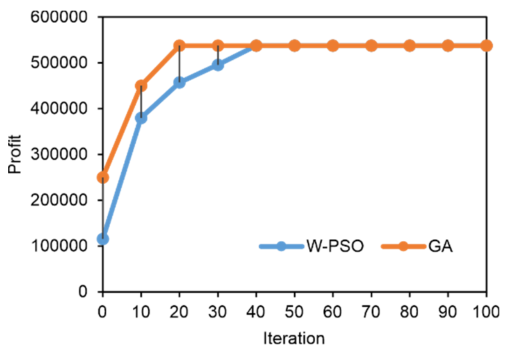

Fig. A1. Convergence curve for Pattern I

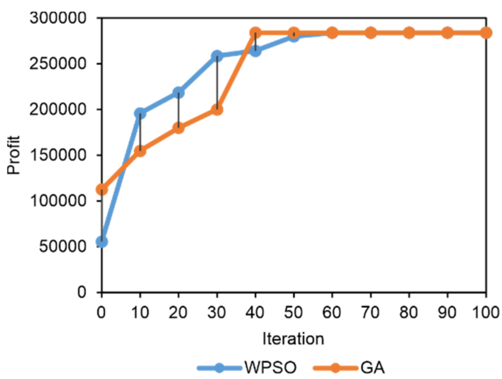

Fig. A2. Convergence curve for Pattern II

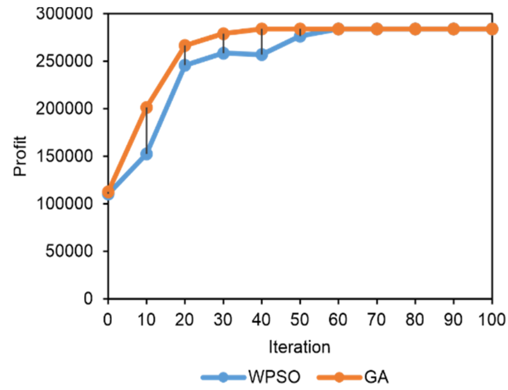

Fig. A3. Convergence curve for Pattern III

Convergence curves under uniform distribution

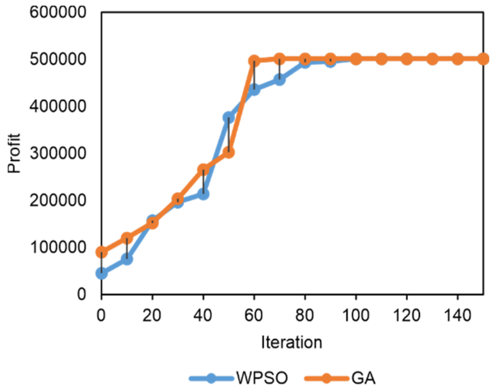

Fig. A4. Convergence curve for Pattern I under uniform distribution

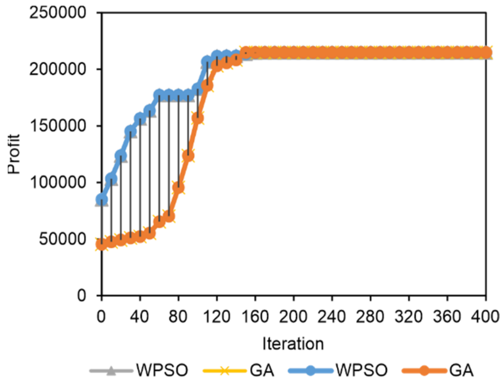

Fig. A5. Convergence curve for Pattern II under uniform distribution

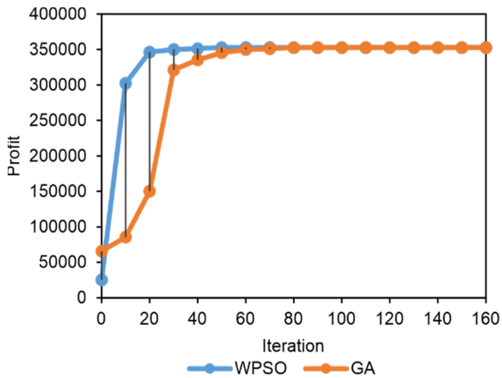

Fig. A6. Convergence curve for Pattern III under uniform distribution

Convergence curves under triangular distribution

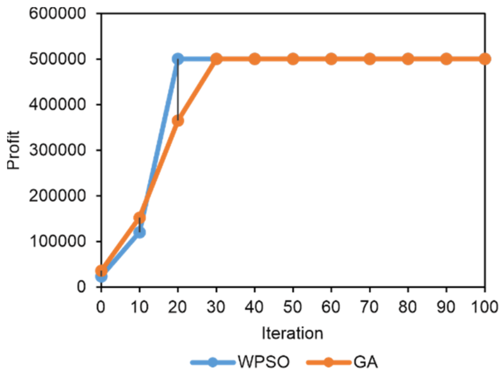

Fig. A7. Convergence curve for Pattern I under triangular distribution

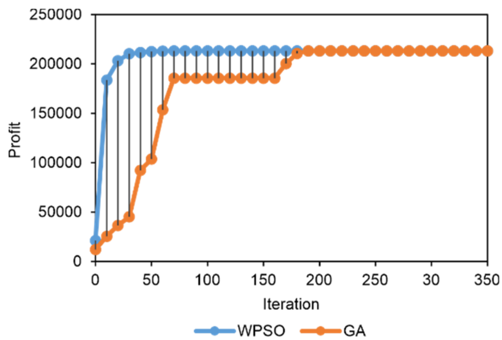

Fig. A8. Convergence curve for Pattern II under triangular distribution

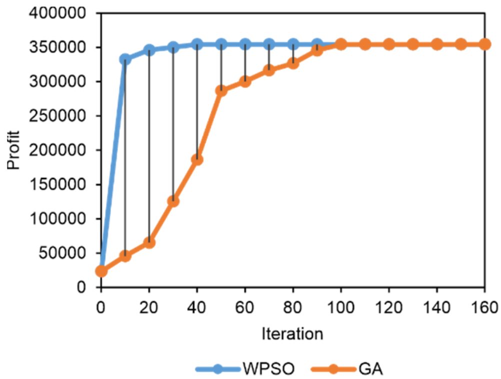

Fig. A9. Convergence curve for Pattern III under tringular distribution

Convergence curves under beta distribution

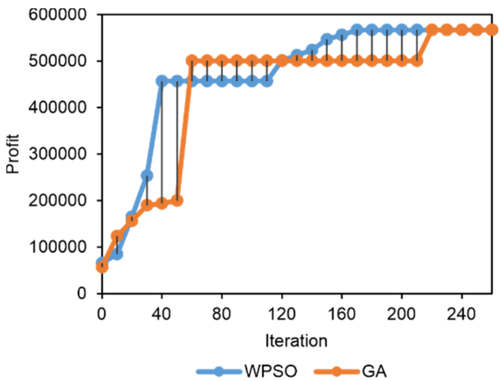

Fig. A10. Convergence curve for Pattern I under beta distribution

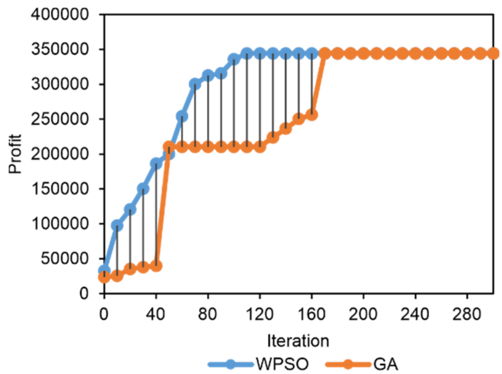

Fig. A11. Convergence curve for Pattern II under beta distribution

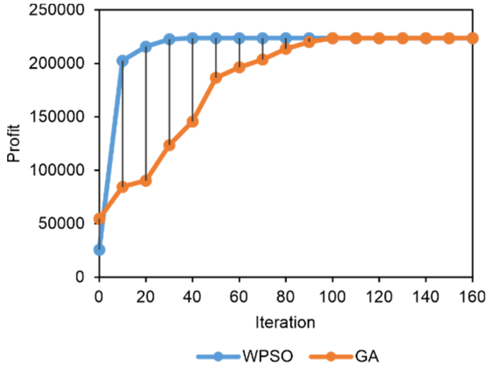

Fig. A12. Convergence curve for Pattern III under beta distribution