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## **Spatial contagion between stock markets in Central Europe\*\***

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### **1. Introduction**

The process of globalization has enabled investors to invest in financial markets all over the world. However, the appearance of global investors has tightened relationships between financial markets in different parts of the world. This, in turns, has made international portfolio diversification a very difficult task. Hence, a deeper analysis of the existence and strength of relationships between markets for risk management and optimal portfolio allocation has become important as never before.

In recent years, a large amount of financial and econometric literature has been devoted to analyze various kinds of short- and long-term linkages between stock markets. Their aim has been to give a better description of information flow. Initially, the largest part of the literature about co-movement and interdependencies has concentrated on developed markets (e.g., Hamao et al., 1990; Cappiello et al., 2006). Subsequently, the linkages between developed and emerging markets have also been examined (see, for example, Chen et al., 2002; Kim et al., 2005; Syllignakis and Kouretas, 2011). Only a small part of the literature describes such relationships between stock markets in Europe. Moreover, the conclusions from empirical research are not always consistent.

Long-range dependencies between European markets have been analyzed by Voronkova (2004), Černý and Koblás (2005), Égert and Kočenda (2007), Syriopoulos (2007), Wójtowicz (2015), among others. The majority of these indicate the existence of long-term relationships (cointegration, fractional cointegration) between daily data of European stock markets (both developed

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and emerging). On the other hand, analogous intraday relationships have not been confirmed.

More common results can be observed when short-term relations are investigated. Hanousek et al. (2009) showed significant spillover effects on the stock markets in Prague, Budapest, and Warsaw. Moreover, the situation in CEE markets is significantly influenced by the situation of the stock market in Frankfurt. This impact is even stronger than the impact of CEE markets themselves. Similar results were obtained by Černý and Koblas (2005). The important impact of developed European markets on CEE emerging markets was also indicated by Égert and Kočenda (2007), who showed significant intraday causalities between returns of CEE markets and causal relationships from developed to emerging markets. On the other hand, Égert and Kočenda (2011) found very few positive time-varying correlations between the intraday returns of BUX, PX50, and WIG20.

The existence of a high or very low correlation between stock markets is not the only factor that should be taken into account in a portfolio-diversification strategy. Changes in the strengths of linkages between markets during calm and turbulent periods seem to be even more important. The existence of a low correlation during a calm period encourages diversification; however, when correlation between markets increases during a crisis, then a loss on one of them is accompanied by a loss on the others. This leads to the issue of contagion between stock markets that also should be taken into account when building a well-diversified portfolio. However, the literature lacks the one common formal definition of contagion for financial markets; thus, there is no common measure of contagion. There are several different approaches to this issue. In general, we can say that contagion occurs when interdependencies between markets are higher during turbulent times than in tranquil times (see, for example, Forbes and Rigobon, 2002). Thus, there is a difference between “strong interdependency between markets” and “contagion”. Contagion does not exist when two markets are highly correlated during both calm and turbulent times. It occurs when markets are more-tightly connected in turbulent times than they are during calm periods. Hence, in this paper, we say that there is contagion between two markets when a significant shift in correlation is observed during a turbulent period.

The majority of empirical studies on contagion is based on data concerning periods around financial crises. A natural approach is to compare correlations between markets before and after a crisis. Such comparisons have also been done by applying conditional correlation (CC) models. Using Constant Conditional Correlation (CCC) and Smooth Transition Conditional Correlation (STCC) models, Savva and Aslanidis (2010) showed that markets in the Czech Republic, Hungary, and Poland revealed stronger correlations with the Euro area than

smaller CEE markets like Slovenia and Slovakia. Using the Dynamic Conditional Correlation (DCC) GARCH models, Syllignakis and Kouretas (2011) showed that the 2007–2009 global financial crisis significantly shifted conditional correlation between the developed markets (Germany and US) and emerging CEE markets. On the other hand, Baruník and Vácha (2013) studied contagion between CEE markets using the wavelet approach and confirmed contagion only between the stock markets in the Czech Republic and Germany.

The analysis of contagion based on a comparison of correlation before and after a crisis has a considerable drawback associated with the significance test for the shift in correlation during these two periods. This is caused by different properties of financial time series during tranquil and turbulent times. Hence, Durante and Jaworski (2010) (see also Durante et al., 2013; Durante and Foscolo, 2013) considered an alternative approach to analyze changes in stock market co-movements. They introduced the notion of spatial contagion based on the copula approach. Instead of comparing correlation before and after a given crisis, they proposed a comparison of correlation for very low returns (in the left tail of returns distribution) with correlation around the median (in the central part of the distribution). This definition of contagion does not depend on the choice of a particular crisis, but all cases of severe losses on both markets are taken into account. Such an approach is in line with the results of Longin and Solnik (2001), who applied the extreme value theory to show an increased correlation of large negative returns.

In this paper, we study contagion among three European stock markets; namely, the stock exchanges in Frankfurt (FSE), Vienna (VSE), and Warsaw (WSE). They are specially selected stock markets because they differ considerably but also share some similarities. The Frankfurt Stock Exchange is an example of a large developed market. It is one of the largest and the most-important stock markets in Europe. Its capitalization is about eighteen times greater than that of the Vienna Stock Exchange (VSE) and about eleven times greater than that of the Warsaw Stock Exchange (WSE)<sup>1</sup>. The Vienna Stock Exchange (somehow smaller than the FSE) is also a developed market. On the other hand, the stock exchange in Warsaw is still seen as an emerging market. Despite these differences, both the VSE and WSE are among the largest stock markets in Central and Eastern Europe. Moreover, both the VSE and WSE ensure enough liquidity to be taken into account in a global diversification issue. Hence, in this paper, we analyze contagion between stock markets by taking into account their sizes as well as their degrees of development. This

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<sup>1</sup> At the end of July 2015, capitalization of the FSE was at the level of €1.65 trillion compared to €147 billion of capitalization of the WSE and €90 billion capitalization of the VSE [source: Federation of European Securities Exchanges; [www.fese.eu](http://www.fese.eu)].

will show how the similarities and differences between markets under study are reflected in contagion effects.

The main part of the analysis of contagion between the abovementioned stock markets is performed on the basis of the daily returns of their main indices. However, due to the very fast and significant reaction of stock markets to important publicly available information<sup>2</sup>, we also perform an analysis on the basis of intraday data. The application of data sampled with different frequency will show how the speed of information flow influences the strength of the relationships between the markets. To study the relationships among the stock markets, we apply the spatial contagion measure of Durante and Jaworski (2010). We also propose its modification (called the conditional contagion measure) that takes into account the impact of the state of a third market on a contagion measure between two given markets. This enables us to verify whether contagion between markets is induced by external factors such as other stock markets.

The rest of the paper is organized as follows. In the next section, we provide a short description of the spatial contagion measure applied in the paper. We also propose a conditional contagion measure to analyze the impact of one stock market to the contagion between the other markets. In Section 3, we present and analyze in detail the data that we use in the empirical study in Section 4. A brief summary concludes the paper.

## 2. Spatial contagion measure

### 2.1. Contagion measure between two markets

According to the description presented in the introduction, we say that there is contagion between two markets if the correlation between returns of their indices during a turbulent time is stronger than during a tranquil time. In turbulent times or during crises, large decreases of stock prices are observed, and negative returns dominate. On the other hand, returns vary around zero in tranquil times. Hence, contagion may be understood as the difference between the correlation of very low negative index returns (in the lower tail of the bivariate distribution of returns) and that of returns from around their medians (in the central part of the return distribution). This is the basic idea of the spatial contagion measure proposed by Durante and Jaworski (2010) (see also Durante et al., 2014).

Let  $X$  and  $Y$  be the random variables that represent the returns of two stock market indices, and let the dependence between them be described by means of

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<sup>2</sup> For example, Gurgul and Wójtowicz (2014, 2015) showed that the stock markets in Vienna and Warsaw react to US macroeconomic news in just the first minutes after news announcements.

copula C. For  $\alpha_1, \alpha_2, \beta_1, \beta_2 \in [0, 1]$ , consider the two following sets of  $\mathbb{R}^2$ : *tail* set  $T_{\alpha_1, \alpha_2}$  and *central* set  $M_{\beta_1, \beta_2}$ , given by formulas:

$$T_{\alpha_1, \alpha_2} = [-\infty, q_X(\alpha_1)] \times [-\infty, q_Y(\alpha_2)] \quad (1)$$

$$M_{\beta_1, \beta_2} = [q_X(\beta_1), q_X(1-\beta_1)] \times [q_Y(\beta_2), q_Y(1-\beta_2)] \quad (2)$$

Where  $q_X$  and  $q_Y$  are the quintile functions associated with random variables  $X$  and  $Y$ , respectively.

Tail set represents negative returns during a turbulent time on both markets. These returns are smaller than the given thresholds  $q_X(\alpha_1)$  or  $q_Y(\alpha_2)$ . On the other hand, set corresponds to a tranquil time and describes returns in the central part of their joint distribution.

Following Durante and Jaworski (2010) and Durante et al. (2014), we say that there is *symmetric contagion* between  $X$  and  $Y$  at given threshold level  $\alpha \in (0, 0.5)$  if the Spearman correlation in the tail is significantly greater than that in the central part of the distribution; i.e.,

$$\rho(T_\alpha) > \rho(M_\alpha) \quad (3)$$

where  $\rho(T_\alpha) = \rho(X, Y | (X, Y) \in T_{\alpha, \alpha})$  is the Spearman correlation between  $X$  and  $Y$  on tail set  $T_{\alpha, \alpha}$  and  $\rho(M_\alpha) = \rho(X, Y | (X, Y) \in M_{\alpha, \alpha})$  is the Spearman correlation between  $X$  and  $Y$  on central set  $M_{\alpha, \alpha}$ , and the inequality (3) is statistically significant.

In the above definition of contagion, however, the choice of threshold  $\alpha$  may influence the final result. To avoid the problem caused by an arbitrary choice of threshold  $\alpha$ , Durante et al. (2014) defined the *symmetric contagion measure* between  $X$  and  $Y$  by the following formula:

$$\gamma(X, Y) = \frac{1}{\lambda(L)} \lambda(\{\alpha \in L | \rho_{T, \alpha} > \rho_{M, \alpha}\}) \quad (4)$$

where  $L \subset [0, 0.5]$  is a connected set of possible values of thresholds  $\alpha$ ,  $\lambda$  is the Lebesgue measure on  $[0, 1]$ . The above contagion measure simply counts how many times correlation  $\rho_{T, \alpha}$  between  $X$  and  $Y$  in tail set  $T_{\alpha, \alpha}$  is significantly greater than correlation computed in central set  $M_{\alpha, \alpha}$ .

In order to determine the estimates of coefficients  $\rho_{T, \alpha}$  and  $\rho_{M, \alpha}$ , the threshold copula approach could be applied. However, to overcome difficulty with assumption about a proper copula function, an empirical copula can be taken into consideration. The correctness of such an approach follows from the properties of empirical copulas discussed by Schmid and Schmidt (2007).

Hence, the procedure of spatial contagion measure calculation is as follows (for details, see Durante et al., 2014):

1. Univariate return series are filtered via the appropriate AR-GARCH models.
2. The empirical cumulative distribution function of residuals is computed.
3. Interval is equally divided into a finite number of equidistant points  $\alpha_i$ .
4. For each threshold  $\alpha = \alpha_i$  ( $i = 1, \dots, n$ ), tail set  $T_{\alpha,\alpha}$  and central set  $M_{\alpha,\alpha}$  are determined and Spearman correlations  $\rho_{T,\alpha}$  and are computed.
5. For each  $\alpha_i$ , the null hypothesis that  $\rho(T_{\alpha}) = \rho(M_{\alpha})$  against  $\rho(T_{\alpha}) > \rho(M_{\alpha})$  is tested<sup>3</sup>.
6. The contagion measure is then computed as the percentage of significant inequalities  $\rho(T_{\alpha}) > \rho(M_{\alpha})$ ; that is:

$$\hat{\gamma}(X, Y) = \frac{\#\{i : \rho(T_{\alpha_i}) > \rho(M_{\alpha_i})\}}{n}$$

## 2.2. Conditional contagion measure

It is well-known that relationships between two stock markets are also influenced by their relationships with other markets. To take this fact into account, bivariate relations are frequently studied by means of multivariate models with additional exogenous variables. Hence, we propose a generalization of the definition of the contagion measure above between two return series  $X$  and  $Y$  to the case when information about third return series  $Z$  is also taken into account. To do this, we consider a copula describing the joint distribution of trivariate return series  $(X, Y, Z)$ . Let:

$$\rho(T_{\alpha}, T_{\alpha_0}) = \rho(X, Y | (X, Y) \in T_{\alpha,\alpha}, Z \in T_{\alpha_0}) \quad (5)$$

$$\rho(M_{\alpha}, T_{\alpha_0}) = \rho(X, Y | (X, Y) \in M_{\alpha,\alpha}, Z \in T_{\alpha_0}) \quad (6)$$

be the Spearman correlations between return series  $X$  and  $Y$  when  $(X, Y, Z) \in T_{\alpha,\alpha} \times T_{\alpha_0}$  or when  $(X, Y, Z) \in M_{\alpha,\alpha} \times T_{\alpha_0}$ , respectively.

There is contagion between  $X$  and  $Y$  conditional on  $Z \in T_{\alpha_0}$  when

$$\rho(T_{\alpha}, T_{\alpha_0}) > \rho(M_{\alpha}, T_{\alpha_0}) \quad (7)$$

and the inequality is significant.

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<sup>3</sup> This hypothesis (or 'these hypotheses') can be verified by bootstrap methods (see Durante et al., 2014).

Analogously, we define contagion between  $X$  and  $Y$  if  $Z$  is in central set  $M_{\alpha_0}$  as:

$$\rho(T_\alpha, M_{\alpha_0}) > \rho(M_\alpha, M_{\alpha_0}) \quad (8)$$

and the inequality is significant.

Formulas (7) and (8) describe contagion between  $X$  and  $Y$  on given level  $\alpha$  when  $Z$  is in lower tail  $T_{\alpha_0}$  (and negative returns of  $Z$  are observed) or when  $Z$  is in central set  $M_{\alpha_0}$  (and returns around the median are observed), respectively. Conditional contagion between  $X$  and  $Y$  exists when the change in strength of the relationship between  $X$  and  $Y$  is significant during a turbulent time for (Formula [7]) or during a tranquil time for (Formula [8]). Let us note that, in the above definitions, we allow for different values of thresholds  $\alpha$  and  $\alpha_0$  for  $(X, Y)$  and  $Z$ .

The algorithm of the computation of the conditional contagion measure is similar to the algorithm for the contagion measure described in the previous subsection; however, it is based on a three-dimensional empirical copula, and the classification to the tail and central sets is performed conditionally on the values of  $Z$ .

In Step 5 of the algorithm, the existence of conditional contagion can be examined by testing the significance of inequality (7) (or [8]). Additionally, we can test the significance of the impact of variable  $Z$  on contagion between  $X$  and  $Y$ ; i.e., whether restriction of  $Z$  to tail set strengthens contagion between  $X$  and  $Y$ . It is equivalent to test:

$$H_0 : \rho(T_\alpha) - \rho(M_\alpha) = \rho(T_\alpha, T_{\alpha_0}) - \rho(M_\alpha, T_{\alpha_0}) \quad (9)$$

against

$$H_1 : \rho(T_\alpha) - \rho(M_\alpha) < \rho(T_\alpha, T_{\alpha_0}) - \rho(M_\alpha, T_{\alpha_0}) \quad (10)$$

Alternative hypothesis (10) means that the difference between the correlations of  $X$  and  $Y$  in their tail and central sets is larger when we take into account information about the values of  $Z$ . An analogous test can be defined for  $Z \in M_{\alpha_0}$  to verify whether restriction to a calm period for significantly impacts contagion between  $X$  and  $Y$ <sup>4</sup>.

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<sup>4</sup> All of these tests can be performed by the bootstrap method.

### 3. Data

The analysis presented in this paper is based mainly on the daily log-returns of the main indices of stock exchanges in Frankfurt, Vienna, and Warsaw; namely, DAX, ATX, and WIG20<sup>5</sup>. Daily index returns cover the period from January 4, 2000, to December 31, 2014. This 15-year period contains phases of bull as well as bear markets; in particular, it includes the period of the recent global financial crisis (2007–2009). This allows us to study contagion not only when stock prices increase but also when they fall. Thus, we will be able to not only describe the impact of the financial crisis, but it will also ensure the robustness of the contagion measures. To model the daily data, we will apply AR(1)-GARCH(1,1) models with skewed Student's  $t$ -distribution.

Additionally, we study the contagion effect on the basis of 5-minute log-returns. The intraday data covers the rather-stable market period between March 22, 2013, and July 31, 2014. During this period, DAX increased by about 17% while ATX and WIG20 decreased by about 5% and 2%, respectively. Application of the intraday data from this quite calm period will show how the relationships between the stock markets under study changed in response to local and short-lived negative impulses that could not be recorded in the daily data.

In an analysis of the intraday data, the trading hours of the stock markets must be taken into account, because they were open at different hours of the day in the period under study<sup>6</sup>. Due to the differences in trading hours on the markets and to the fact that first 5-minute intraday return is observed at 9:05, intraday relations are analyzed only during the common periods between 9:05 and 16:50. When modeling intraday data, it is a well-known fact that intraday volatility usually increases at the beginnings and ends of trading sessions; this should be taken into account. Restricting the analysis to the period of 9:05–16:50 does not completely remove the periodic pattern from volatility series. Figure 1 shows a U-shaped pattern observed in intraday return volatility<sup>7</sup>. Additionally, in 5-minute return volatility, we can observe the very strong impact of US macroeconomic news (usually announced at 14:30). This strong impact of various US macroeconomic news announcements on the European stock market is widely confirmed by empirical works (see; e.g., Harju and Hussain, 2011; Gurgul and Wójtowicz, 2015).

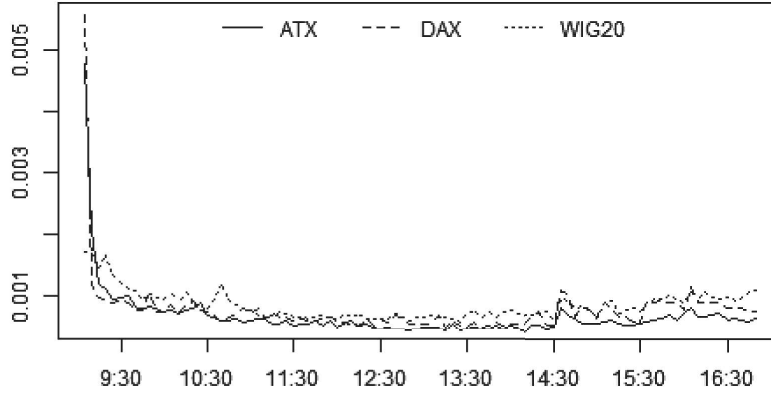
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<sup>5</sup> Data comes from Bloomberg, the Vienna Stock Exchange, and Warsaw Stock Exchange, respectively.

<sup>6</sup> In 2013 and 2014, continuous trading started at 8:55 on the VSE and at 9:00 on the FSE and WSE. It ended at 16:50 (WSE), 17:30 (FSE), and at 17:35 (VSE). Moreover, on the FSE and VSE, there were intraday auctions at 13:00 and 12:00, respectively.

<sup>7</sup> Because we trim the period under study at 16:50, the increase of returns volatility at the end of the trading session is not visible in Figure 1.





**Figure 1.** Cross-sectional standard deviations of 5-minute returns of ATX, DAX, and WIG20 during period of March 22, 2013 – July 31, 2014

Source: Authors' calculation

To deal with the periodic pattern in volatility and with the impact of US news announcements, we apply a method of Flexible Fourier Form (FFF) adopted to intraday data by Andersen and Bollerslev (1997). Specifically, we decompose 5-min returns  $R_{t,n}$  at time  $n$  on day as:

$$R_{t,n} - E(R_{t,n}) = s_{t,n} \sigma_{t,n} Z_{t,n} \quad (11)$$

where  $Z_{t,n}$  is i.i.d(0, 1) and  $\sigma_{t,n}$  is the daily volatility factor that can be approximated by volatility forecasts from the appropriate GARCH model with skewed Student's  $t$ -distribution constructed for daily returns.  $s_{t,n}$  is an intraday (diurnal) seasonal component such that  $\ln(s_{t,n}^2)$  can be estimated from the following FFF regression:

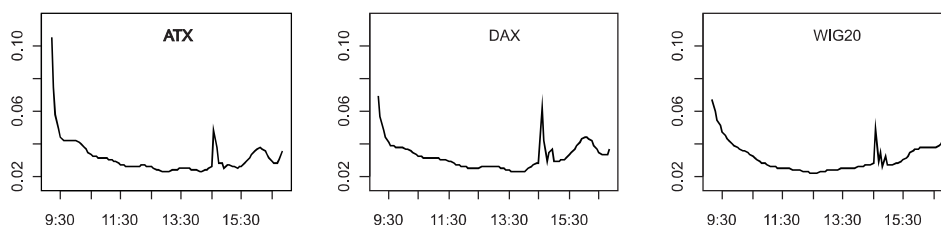
$$2 \ln \frac{|R_{t,n} - \bar{R}|}{(\bar{\sigma}_t N^{0.5})} = c + \sum_{k=1}^D \lambda_k I_k(t, n) + \delta_1 \frac{n}{N_1} + \delta_2 \frac{n^2}{N_2} + \sum_{p=1}^P \left( \delta_{c,p} \cos\left(\frac{2\pi p}{N} n\right) + \delta_{s,p} \sin\left(\frac{2\pi p}{N} n\right) \right) + \varepsilon_{t,n} \quad (12)$$

where  $N$  refers to the number of returns per day (here,  $N = 94$ ),  $N_1 = \frac{N+1}{2}$ ,  $N_2 = \frac{(N+1)(N+2)}{6}$ ,  $I_k(t, n)$  is a dummy variable related to weekdays as well as

US macroeconomic news announcements. On the basis of the literature (see; e.g., Nikkinen et al., 2006; Harju and Hussain, 2011; Gurgul and Wójtowicz, 2014, 2015), we include regression dummy variables in the FFF describing the impact

of announcements of the following US macroeconomic indicators: Consumer Price Index (CPI), Producer Price Index (PPI), Industrial Production (IP), Retail Sales (RS), Durable Goods Orders (DGO), Nonfarm Payrolls (NFP), Existing Home Sales (EHS), Housing Starts (HS), and New Home Sales (NHS).

Application of FFF confirms the conclusions from Figure 1 regarding the very-high variance of returns at the beginning of the trading session. It also indicates the strong and significant impact of US macroeconomic news announcements on intraday volatility. This is clearly visible in Figure 2, where we present examples of intraday volatility components for the days with US announcements at 14:30. After removing the daily and intraday seasonality components of volatility, we filter the 5-min returns out with AR(10)-GARCH(1,1) models with conditional skewed Student's  $t$ -distribution.



**Figure 2.** Intraday seasonal component of ATX, DAX, and WIG20 on days with US macroeconomic news announcements at 14:30

Source: Authors' calculation

## 4. Contagion – empirical results

### 4.1. Correlations

We start the analysis by describing the correlations between index returns. This will give us our very-first insight into the relationships between the stock markets. To compare the results for the data sampled with a different frequency, we first analyze the correlations between the daily returns and then between the 5-min returns. Table 1 contains Spearman correlations computed for daily returns during the whole period of 2000–2014 and also for two sub-periods of equal length: from January 4, 2000, to July 2, 2007, and from July 3, 2007, to December 31, 2014. Partition of the data and the analysis in these sub-periods allows us to compare the strength of the relationships between the markets before and after the global financial crisis of 2007–2009. All of the correlations reported in Table 1 are significant at the 1% level; hence, we can simply conclude that the relationships observed between the stock markets under study are significantly positive. The strongest correlations are between both of the developed markets

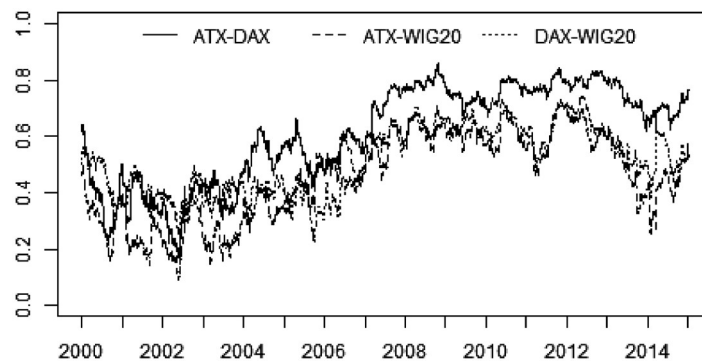
in Frankfurt and Vienna, while the weakest relation is between the VSE and WSE. These results are observed in each of the periods; however, the higher values of correlations in the second sub-period indicate a positive impact of the global crisis on the relationships between the markets. This is in line with the empirical literature that correlation between returns is not constant but varies over time.

**Table 1**  
Spearman correlations between daily returns of ATX, DAX, and WIG20

Period	ATX-DAX	ATX-WIG20	DAX-WIG20
Jan 4, 2000 – Dec 31, 2014	0.592	0.446	0.489
Jan 4, 2000 – Jul 2, 2007	0.403	0.301	0.376
Jul 3, 2007 – Dec 31, 2014	0.758	0.576	0.610

Source: Authors' calculation

To illustrate time-variation in the correlations between the markets, we apply the trivariate DCC-GARCH model of Engle (2002) with multivariate normal distribution. To model the conditional variance of the univariate returns, we apply AR(1)-GARCH(1,1) models with skewed Student's *t*-distribution. These models adequately capture the autocorrelation and heteroscedasticity of daily returns. The time-varying conditional correlations presented in Figure 3 confirm the strong impact of the 2007–2009 crisis on the relationships between the stock markets. During the period before the crisis, the correlations between the index returns varied at around 0.4. After 2008, a shift in correlations to about 0.6–0.8 is observed. At this level, the correlations remained until 2012–2013, when they started to decrease; but even after that, they still remain above the pre-crisis level. As shown in Table 1, the strongest correlations are observed between ATX and DAX for nearly the entire period.



**Figure 3.** Conditional correlations between ATX, DAX, and WIG20 returns

Source: Authors' calculation

The time-varying correlations presented above between the daily returns of ATX, DAX, and WIG20 give very general information about the strength of the relationships between the indices. The values of both conditional and unconditional correlations indicate rather-strong interdependencies, particularly between the stock markets in Frankfurt and Vienna. As we mentioned in the introduction, there is a fine line between interdependence and the contagion effect, and even a very-strong correlation does not indicate contagion. However, an increase in the correlations induced by the financial crisis may be regarded as an argument in favor of contagion on the daily level.

Analysis of the intraday data leads to similar conclusions about the relationships between the markets. All of the rank correlations of the 5-minute returns reported in Table 2 are significant, varying from 0.173 for ATX-WIG20 to 0.421 for ATX-DAX. Correlations computed on the basis of the intraday data are lower than the correlations of the daily returns. This is once again in line with the literature. As in the case of the daily data, the strongest co-movement is observed between both developed markets while the weakest intraday interrelationships are between them and the WSE. Comparing the results in Table 1 and 2 suggests the similar nature of relationships between the markets irrespective of the frequency of data applied.

**Table 2**

Spearman correlations between intraday returns of ATX, DAX, and WIG20

ATX-DAX	ATX-WIG20	DAX-WIG20
0.421	0.173	0.292

Source: Authors' calculation

## 4.2. Spatial contagion

For each pairing of the indices under study, we compute the spatial contagion measure described in Section 2 on the basis of daily data. We restrict the analysis to interval  $L = [0.05, 0.3]$ . By dividing  $L$  equally into 50 subintervals of length 0.005, we consider 51 thresholds  $\alpha = 0.05, 0.055, \dots, 0.3$ . We do not consider lower  $\alpha$  to ensure that tail set  $T_\alpha$  is non-empty. Analysis of the contagion is performed on the basis of standardized residuals from AR(1)-GARCH(1,1) models with conditional skewed Student's  $t$ -distribution computed for daily returns.

The results presented in Table 3 indicate a very-strong contagion between ATX and WIG20, a moderate contagion between ATX and DAX, and a very weak contagion between DAX and WIG20 during the whole period of 2000–2014. To help interpret the values of the contagion measures in Figure 4, we present correlations in tail sets  $T_\alpha$  (solid lines) and in central sets  $M_\alpha$  (dashed lines) for different  $\alpha$  from interval  $L$ . For the lowest threshold  $\alpha = 0.05$ , central set  $M_\alpha$  covers almost all of the

data except the most-extreme cases. Thus, the first correlation coefficient on  $M_\alpha$  is usually close to the value of the respective Spearman correlation reported in Table 1.

**Table 3**  
Spatial contagion measures between daily returns

Period	ATX-DAX	ATX-WIG20	DAX-WIG20
Jan 4, 2000 – Dec 31, 2014	0.510	0.804	0.294
Jan 4, 2000 – Jul 2, 2007	0.808	0.962	0.385
Jul 3, 2007 – Dec 31, 2014	0.385	0.462	0.269

Source: Authors' calculation

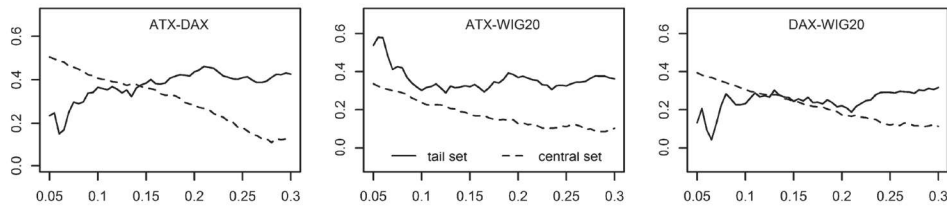
Value 0.804 of the spatial contagion measure for the ATX-WIG20 pair during the whole period means that correlation  $\rho(T_\alpha)$  in tail set  $T_\alpha$  is significantly greater than correlation  $\rho(M_\alpha)$  in central set  $M_\alpha$  in about 80% (i.e., 41 out of 51) of threshold values from interval  $L$ . These differences between  $\rho(T_\alpha)$  and  $\rho(M_\alpha)$  are visible in the central graph of Panel A in Figure 3 for very-small  $\alpha$  and  $\alpha > 0.15$ .

For the data from the whole period, the behavior of the correlations between each pairing of indices in the central sets is similar. For each pair of indices,  $\rho(M_\alpha)$  decreases from 0.4–0.5 to about 0.1–0.2 as  $\alpha$  increases, and sets  $M_\alpha$  concentrate around the medians. Differences occur when analyzing the correlations in tail sets  $T_\alpha$ . Only in the case of ATX-WIG20 correlations are  $\rho(T_\alpha)$  greater than  $\rho(M_\alpha)$  for the smallest thresholds  $\alpha$ , while the largest drops in ATX or WIG20 are only weakly correlated with very-negative changes in DAX; thus, for these pairs,  $\rho(M_\alpha)$  is greater than  $\rho(T_\alpha)$  for very small  $\alpha$ . This means that only between ATX and WIG20 are correlations for the extreme negative returns (correlations in the left tails) significantly greater than the correlations of the returns around zero. Hence, ATX and WIG20 are more-strongly tied during turbulent times than during calm periods.

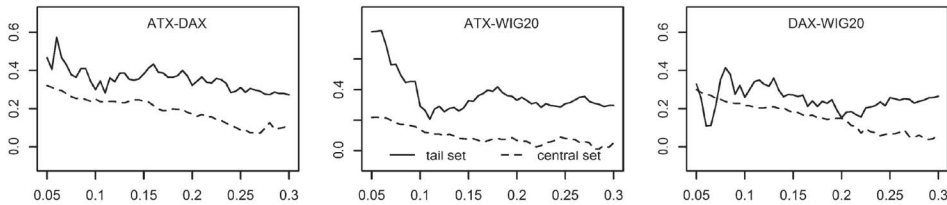
From Table 1 and Figure 3, it follows that interrelations between the markets have strengthened since 2007. To analyze the impact of the financial crisis on contagion, we repeat the above computations in the sub-periods before and after July 2007. When we restrict our attention to the period before the crisis (January 2000 – July 2007; Panel B in Figure 4), we can notice that contagion measures are higher than in the whole period for each pairing of indices. These differences are mainly due to smaller correlations in the central parts of the bivariate distributions of the returns. This is particularly visible when we compare correlations in the central sets on the graphs in Panels A and B in Figure 4. However, in the cases of ATX-DAX and ATX-WIG20, changes in  $\rho(M_\alpha)$  are also accompanied by much-higher correlation in the tails, particularly for very-small  $\alpha$ . Hence, we can conclude that, before the crisis, there was a contagion effect between ATX and DAX as well as between ATX and WIG20.

The bankruptcy of Lehmann Brothers and the global financial crisis raised correlations between the indices by about 0.1–0.2 (see Table 1). From Panels B and C in Figure 4, it follows that these changes were caused by increased correlations in the central sets and decreased correlations in the tails of return distributions. The change in the dependency between extremely negative returns after 2007 is most-pronounced for ATX-DAX, where the correlation  $\rho(T_\alpha)$  for  $\alpha = 0.05$  decreases from 0.47 (before 2007) to 0.13 (after 2007). As a result of these shifts in correlations, we observe rather-low values of contagion measures in the period of July 2007 to December 2014. A detailed analysis of the graphs in Panel C in Figure 4 indicates that, for  $\alpha$  smaller than 0.15–0.2, correlations in left tail  $T_\alpha$  are not significantly greater than correlations in the central sets. Actually, they are even smaller than the respective  $\rho(M_\alpha)$ , and the values of the contagion measures after the crisis are mainly due to the significance of  $\rho(T_\alpha) - \rho(M_\alpha)$  for the largest  $\alpha$  from the right-hand side of interval  $L$ . Hence, we can conclude that there has been no significant contagion between the markets since the crisis.

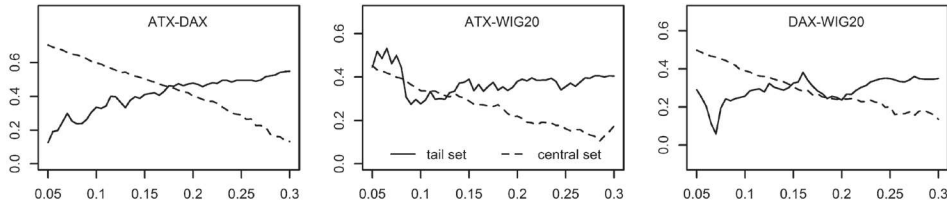
Panel A: January 4, 2000 – December 31, 2014



Panel B: January 4, 2000 – July 2, 2007



Panel C: July 3, 2007 – December 31, 2014



**Figure 4.** Correlations between ATX, DAX, and WIG20 in tails and central parts of their distributions

Source: Authors' calculation

From the above description, it also follows that the observed differences between the correlations before and after the crisis in the whole sample (and also in large central sets) explain the changes in contagion measures between these two periods. When returns are highly correlated (as in the period after the crisis), there is not enough space for further significant increases in correlation when bad news reaches the markets. This shows that the high correlation of returns and contagion are two distinct notions; in fact, the presence of high correlation actually hinders contagion.

The results from the analysis of changes in intraday co-movement of the markets are reported in Table 4. The values of the spatial contagion measures indicate the existence of strong contagion, particularly between DAX and the other indices. However, a comparison of the graphs in Figure 5 indicates differences in these relations. Values of the correlations between the returns of ATX and DAX in the left tails of their distributions are stable regardless of the value of threshold  $\alpha$ , while the correlations between extreme changes of DAX and WIG20 are very low. They are even smaller than the respective correlations in the central sets. In the case of both pairings with DAX, difference  $\rho(T_\alpha) - \rho(M_\alpha)$  is insignificant for the smallest thresholds  $\alpha$ ; i.e., for the most-extreme price declines. These differences become significant for larger mainly due to decreasing  $\rho(M_\alpha)$ . Hence, the quite-high values of the contagion measures in Table 4 are somehow virtual, because shift in correlation is not observed for the very-extreme changes in the indices but rather for moderate drops.

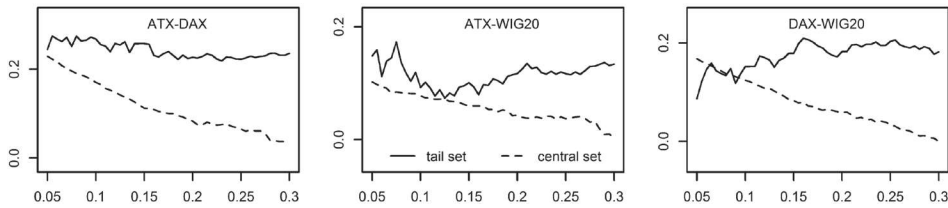
**Table 4**  
Spatial contagion measures between intraday returns

ATX-DAX	ATX-WIG20	DAX-WIG20
0.902	0.649	0.745

Source: Authors' calculation

A comparison of Table 4 with the results for daily data after the crisis (in Table 3) reveals further differences between contagion measured on the daily and intraday horizon. For example, in the case of ATX-WIG20, the strongest contagion on the daily level is accompanied by the weakest contagion on the intraday level. On the other hand, the strongest contagion between 5-minute data is observed between ATX and DAX, but they show only a moderate contagion on the daily level. This phenomenon is due to the aggregation of information during a trading session. Daily returns are the sum of intraday returns; thus, extreme changes in a very short horizon (caused, for example, by important news) do not necessarily lead to equally strong changes in the daily data. On one market, such an impulse can lead to permanent change in prices (and impact the daily returns) while it may simply disappear on the other

market and leave daily returns unaffected. This shows the difference between the analysis of contagion on the basis of data recorded with different frequency and explains why there is no contagion in the daily data but there is in the intraday returns. The analysis of contagion strongly depends on the investment horizon.



**Figure 5.** Correlations between 5-min returns in tails and central parts of bivariate distributions  
Source: Authors' calculation

### 4.3. Conditional contagion

To illustrate how linkages between stock markets impact the contagion measure for each pairing, we estimate conditional contagion measures that take into account the situation in the third market. Estimation results for the same set of  $\alpha$  as in the previous subsection and for  $\alpha_0 = 0.25$  are reported in Table 5<sup>8</sup>. In Panel A, we present the conditional contagion measures between each pairing when the returns of the third market are in tail set  $T_{\alpha_0}$ . This corresponds to Formula (7). In Panel B, we report conditional contagion measures when returns of the third market are in their central set  $M_{\alpha_0}$ . This corresponds to Formula (8). Additionally, we report (in parentheses) the percentage of rejection of the null hypothesis in tests (9)–(10) that conditional contagion is stronger than the unconditional contagion in Table 3. This allows us to evaluate the impact of the restriction of the data according to the third variable.

First of all, the very-strong impact of DAX on the contagion between ATX and WIG20 should be noted. When we intersect  $M_{\alpha}$  and  $T_{\alpha}$  with the extremely negative returns of DAX, then for the all  $\alpha$  form the interval  $L$  the correlation between the extremely negative returns of ATX and WIG20 is larger than that of the central set of their distribution. Moreover, in about 63% of the cases, the shift in these correlations in the presence of the extreme values of DAX is significantly greater than without such restrictions. This means that, during a turbulent time on the FSE, the strength of relationships between the extremely low returns of ATX and WIG20 increases more than usual.

<sup>8</sup> We chose this because it is small enough to treat as a tail set. On the other hand, it ensures a sufficient amount of data in the majority of cases.



**Table 5**  
Conditional contagion measures between daily returns

<b>Panel A: Conditional contagion measures when third variable is in tail</b>		
<b>ATX-DAX</b>	<b>ATX-WIG20</b>	<b>DAX-WIG20</b>
0.510 (0.529)	1 (0.627)	0.078 (0.078)
<b>Panel B: Conditional contagion measures when third variable is in center</b>		
<b>ATX-DAX</b>	<b>ATX-WIG20</b>	<b>DAX-WIG20</b>
0.274 (0.059)	0.667 (0.235)	0.549 (0.314)

Source: Authors' calculation

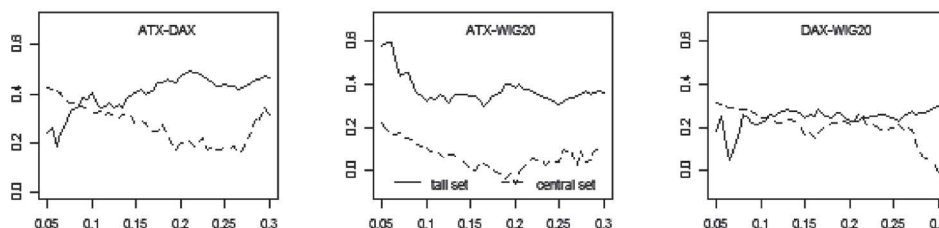
It seems that restricting the analysis to the left tail of WIG20 return distribution has no impact on the contagion measure between ATX and DAX. It is close to the value in Table 3 for the whole sample without restrictions. Also, there is a visible similarity between the left upper graphs in Figures 4 and 6. However, restricting the analysis to very-low WIG20 returns significantly increases the differences between ATX-DAX correlations  $\rho(T_\alpha)$  and  $\rho(M_\alpha)$  for about 53% of the threshold values; however, as in the case of the whole period, the correlations between the very-extreme losses of ATX and DAX are smaller than the correlations in the respective central sets. This means that the relationships between the two markets weaken during very-turbulent trading sessions.

Very low conditional contagion between DAX and WIG20 indicates that the relationships of returns in the tail and central part of their bivariate distribution are similar when restricted to the very-low ATX returns. There are no visible differences between the correlations of very-negative returns and returns close to the medians. Thus, big losses on the VSE do not affect the relationships between DAX and WIG20.

Panel B of Table 3 reports conditional contagion measures when returns of the third index are in their central set. These results are heavily biased by the very-small number of observations in the tail sets for very low  $\alpha$  (Panel B of Figure 6).

For each pairing, the size of  $T_\alpha$  is less than 50 observations for  $\alpha < 0.13$ . When we consider only those thresholds for which the number of elements in  $T_\alpha$  is greater than 50, the conditional contagion measures are equal to 0.324, 0.358, and 0.429, respectively, and the impact of the third variable on  $\rho(T_\alpha) - \rho(M_\alpha)$  becomes insignificant – the null hypothesis in tests analogous to (9)–(10) is not rejected for each  $\alpha$  from  $L$ .

Panel A: Third variable is in tail set



Panel B: Third variable is in central set

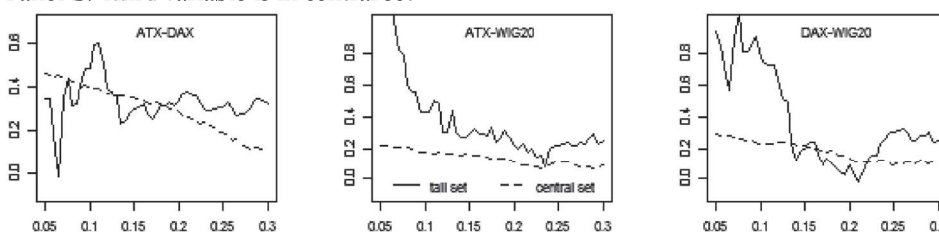


Figure 6. Correlations between daily returns in tails and central parts of their distributions when third variable is also restricted

Source: Authors' calculation

Table 6

Conditional contagion measures between intraday returns

Panel A: Conditional contagion measures when third variable is in tail		
ATX-DAX	ATX-WIG20	DAX-WIG20
0.823 (0.745)	0.765 (0.137)	0.843 (0.745)
Panel B: Conditional contagion measures when third variable is in central set		
ATX-DAX	ATX-WIG20	DAX-WIG20
0.235 (0)	0.157 (0)	0.601 (0)

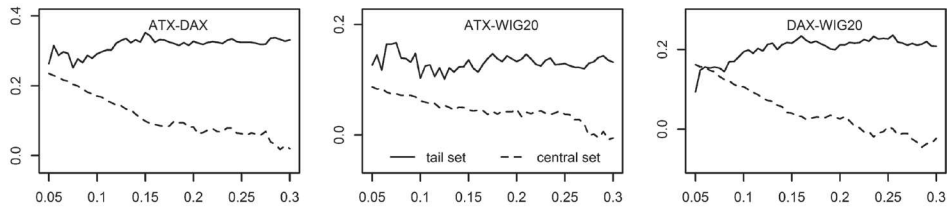
Source: Authors' calculation

Now, let us turn to the conditional measure calculation based on the intraday data. From Table 6, it follows that the intersections of  $T_\alpha$  and  $M_\alpha$  with the lower tail of the third index rather do not influence the contagion measures. The conditional contagion measures in Panel A of Table 6 are quite close to the spatial contagion measures in Table 4. Also, the graphs in Figures 5 and 7 look very similar. It follows that changes in correlations between each pairing when the analysis

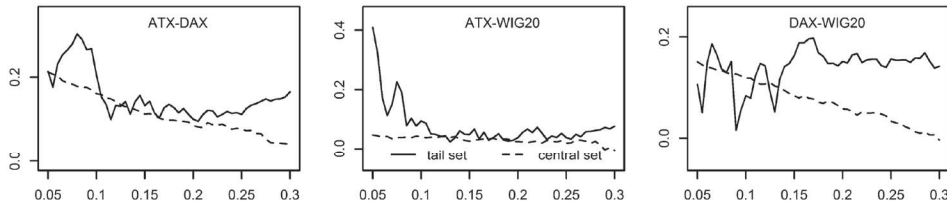
is restricted to the very-low returns of the third index are similar to the changes when the whole sample of the data is taken into account. However, as indicated by the numbers in parentheses in Panel A of Table 6, information about the value of the third index impacts correlations  $\rho(T_\omega)$  and  $\rho(M_\omega)$ . The restrictions affect the size of the shifts in the correlations. For example, in the case of ATX and DAX, the null hypothesis in Test (9)–(10) is rejected for about of the thresholds. This means that, for these thresholds when ATX and DAX switch from calm to turbulent times, changes in their correlations are larger when this shift is accompanied by drops in WIG20. A similar result is observed when we consider changes in DAX and WIG20 during a turbulent time on the VSE. In contrast to the results above, information about DAX returns has only a little additional impact on the nature of relationships between the ATX and WIG20 returns. The null hypothesis in Test (9)–(10) is rejected only in about 14% cases for this pair.

In contrast to the results above, restricting the analysis of correlations between each pairing of indices to the central part of the third index leads to weaker dependencies in the tail sets. As a result, there is no visible contagion between the restricted data despite the moderate values of the conditional contagion measures in Panel B. For each pairing, shifts in correlations do not increase significantly when we take into account only data from the third market during calm periods. The null hypothesis in the tests analogous to (9)–(10) is not rejected for any threshold.

Panel A: Third variable is in tail set



Panel B: Third variable is in central set



**Figure 7.** Correlations between 5-min returns in tails and central parts of their distributions

Source: Authors' calculation

## 5. Conclusions

In this paper, we analyze and compare the relationships between stock markets in Frankfurt, Vienna, and Warsaw. The analysis is performed on the basis of daily data from the period of January 2000 – December 2014 as well as on the basis of 5-minute returns from the period of March 22, 2013 – July 31, 2014. Contagion between these stock markets is examined by means of a spatial contagion measure (Durante and Jaworski, 2010). To describe the impact of each stock market on the relationships between the other two markets, we propose a conditional contagion measure.

Results of the empirical study on the daily basis show a strong correlation (both conditional and unconditional) between the indices under study. When the intraday data from the post-crisis period are considered, correlations are weaker but still significant. The strongest correlation is observed between the indices of the both developed markets in Frankfurt and Vienna irrespective of data frequency.

A further analysis of daily returns indicates strong contagion between both of the smaller markets; namely, VSE and WSE. There is a significant difference in correlations in the left tail and central part of the bivariate distribution of ATX and WIG20 returns. Contagion between the stock markets in Vienna and Warsaw is even more pronounced during turbulent times on the stock exchange in Frankfurt. In the case of contagion with the FSE, no shift in correlations between the central part of the return distribution and its tail is observed for very-extreme losses. This is only significant for larger values of thresholds. The analysis also reveals the significant impact that the 2007–2009 crisis had on contagion. Increased correlations after the crisis reduced contagion between the markets and hindered international diversification.

Contagion between intraday data differs considerable from the contagion for daily returns. For 5 min returns, the difference between correlations in the tails and central parts of the index returns is significant for larger sets of admissible threshold values. Stronger contagion is observed between the ATX and DAX returns. This means that (on the intraday investment horizon) very-bad news implies contemporaneous reactions of a similar strength on both of the developed markets. Additionally, applying the conditional contagion measure shows that the interrelations between each pairing depend on the state of the third market. During a turbulent time on one of the markets, shifts in correlations between the other two are significantly higher than in the whole sample.

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