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Research paper

Calculation of second-order effects in columns – applications and examples

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Abstract: Standard PN-EN 1992-1-1 for designing reinforced concrete structures gives a major priority to the issues relating to second-order effects, but presents in detail only two approximate calculation methods: the nominal stiffness method and the nominal curvature method. As regards the general method, only certain requirements and suggestions are provided. In typical situations, when the appropriate assumptions are satisfied, the approximate methods yield satisfactory results. However, in engineering practice one can come across several cases (e.g. very tall columns, columns with a cantilever for a gantry girder, and floor joists) in which the approximate methods will prove unreliable. This paper presents and discusses a procedural algorithm for analysing second-order effects using the general method. The algorithm is employed to perform exemplary calculations and their results are compared with the results yielded by the approximate methods commonly used by engineers. Moreover, areas in which the approximate methods can be unreliable are indicated. The analyses have confirmed the significant advantage of the general method over the approximate methods. Therefore it is worth popularizing this method, the more so that its calculation procedures can be to a large extent automated and dedicated computer programs can be developed.

Keywords: buckling, column, eccentric tension, reinforced concrete, second-order effects

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1. Introduction

Standard [1] devotes much attention to second-order effects as applying to slender columns, and even to beams. When designing cross sections, the second-order effects can be reduced to the displacements of the axes of structural members. In classical statics the principle of solidification, which disregards the effect of load-induced strains on the internal forces, holds good. But this simplification cannot be applied to slender members, which tall columns usually are.

Even though standard [1] provides several general guidelines and requirements, in practical terms only two approximate calculation methods – the nominal stiffness method and the nominal curvature – are described. The first of the two methods is similar to the one found in old standard [2] which had been in force before Eurocodes were adopted in Poland. Owing to the above and the method's greater universality it has been more often used in engineering practice. For rather typical situations the results yielded by the two methods are satisfactory, but in more complex cases, e.g. very tall columns or stepped columns, they fail. It should be noted that there have been attempts to refine the methods proposed by standard [1]. One should mention here the method proposed by Klempka and Knauff in [3], which takes into account the main assumptions of standard [1] and is based on the incremental analysis using numerical integration. The method takes into account changes in column stiffness, resulting from strain increments.

The fact that there are no publications which would explain the principles of approximate methods poses a practical difficulty. Sometimes, especially when ready-made computer software is used, this leads to serious errors. Therefore it is essential to have good knowledge of all the assumptions and the consequent limitations lying at the basis of the simplified methods.

Nevertheless, it should be noted that there are available publications (e.g. [4]) highly attractive for practicing engineers owing to the way in which design problems are presented and solved (algorithms, nomographs, tables).

As regards the nominal stiffness method, one should bear in mind the following facts:

– One of the principal parameters is (critical) buckling force N_B defined as follows:

$$N_B = \frac{\pi^2 EI}{l_0^2} \tag{1.1}$$

where: EI – bending stiffness, l_0 – the buckling length of the column.

The buckling force has no physical sense, but only a mathematical sense. As a matter of fact, for different static systems it has a different value solely in order that the differential equation yields nonzero solutions [5]. This comes down to the condition that the determinant of the main system (the so-called indeterminate system) of equations for calculating integration constants must equal zero. Therefore in order to simplify and generalize the calculation procedures the notion of buckling length l_0 was introduced, whereby Eq. (1.1) can be used in all situations. Obviously, also the buckling length has no explicit physical interpretation (it does not stand for sections of the sinusoid!).

– The design bending moment (M_{Ed}) which takes into account the second-order effects is defined by the relation:

$$M_{Ed} = M_{0Ed}\left(1 + \frac{\beta}{\frac{N_B}{N_{Ed}} - 1}\right) \tag{1.2}$$

where: M_{0Ed} – a first-order bending moment, β – a coefficient dependent on the distribution of first-order and second-order bending moments, N_{Ed} – the design axial force.

If $\beta = 1$ (described in the standard as a rational simplification) is used in Eq. (1.2), then it becomes clear that the end value of the moment is the sum of an infinite geometric progression with ratio $q = N_{Ed}/N_B$. This progression is convergent when $q < 1$. The other possible standard values of β (from the interval of 0.82÷1.23) are used only for a modification to take into account the distribution of moments. Actually, tracing the successive increments in the moments [6] for a pinned-pinned column with a constant stiffness and a constant first-order moment along its whole length, in the middle of the column's height one gets a sequence very similar to a geometric progression. If, however, the paths of the moments and especially, the mode of support differ from the above, this sequence no longer resembles a geometric progression. Consequently, the conditions for the convergence of this sequence and the sum of the latter should be estimated in a different way.

– The previously described assumptions concerning the estimation of the increment in the moments can be treated as realistic to some degree only when it is additionally assumed that the column's stiffness is constant.

But here other difficulties arise. The stiffness of the eccentrically compressed cross section depends on the applied force and the moment. Figure 1 shows exemplary graphs of stiffness versus bending moment for selected values of axial forces and bending moments.

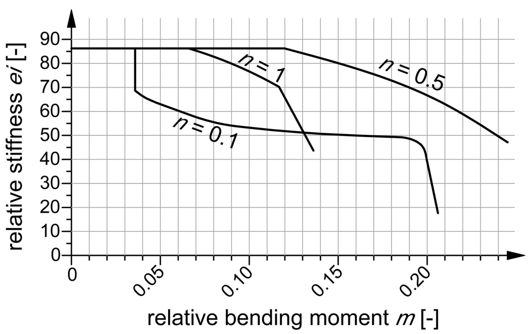


Fig. 1. Relative bending moment *m* versus relative stiffness *ei* for selected values of force *n*

In order to ensure greater generality, dimensionless coordinates were used. The relative values of the force *n*, bending moment *m* and stiffness *ei* were defined as follows:

$$n = \frac{N_{Ed}}{f_{cd}bd} \tag{1.3}$$

$$m = \frac{M_{Ed}}{f_{cd} b d^2} \tag{1.4}$$

$$ei = \frac{EI}{f_{cd} b d^3} = \frac{m}{\kappa d} \tag{1.5}$$

where: f_{cd} – the design compressive strength of the concrete, b – the width of the column cross section, d – the effective height of the column cross section, κ – the local curvature of the axis of the beam.

In the nominal stiffness method, stiffness can be assumed as constant and given by the formula:

$$EI = K_c E_{cd} I_c + E_s I_s \tag{1.6}$$

where: K_c – a coefficient dependent on the effects of cracking, creep and so on, E_{cd} – the design E-modulus of the concrete, I_c – the moment of inertia of the concrete cross section, E_s – the E-modulus of the steel, I_s – the moment of inertia of the reinforcement steel.

If the moment is invariable along the column height and the column is cracked, the above estimation, although highly conservative, can be regarded as dependable. If, however, we are dealing with a fixed column subjected to a strong axial force and to a linearly variable moment, the reliability of this estimation sharply decreases. This is illustrated in Fig. 2 [7]. The diagram was produced for overall reinforcement ratio $\rho = 2‰$. The reinforcement ratio at the less compressed edge (ρ_1) differed from the one at the more compressed edge (ρ_2).

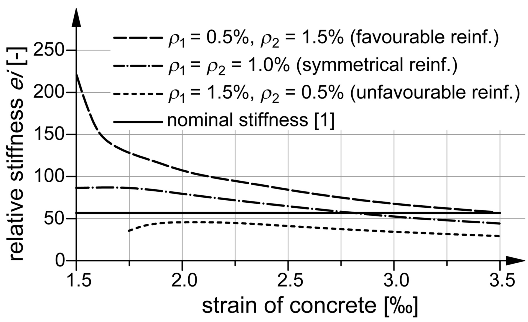


Fig. 2. Dependence between strain in more compressed concrete fibres and relative stiffness (for $\rho = \rho_1 + \rho_2 = 2‰$ and $n = 1$)

The solid line in Fig. 2 represents the relative stiffness calculated from Eq. (1.6), assuming that the coefficient k_2 dependent on the longitudinal force and slenderness, used to calculate coefficient K_c, reaches the maximum value of 0.2 [1]. The dashed lines represent changes in stiffness as a function of the strain in the concrete for three configurations of reinforcement in the cross section: *favourable configuration* – $\rho_2 = 1.5‰$ in the more compressed zone, *symmetrical configuration* – $\rho_1 = \rho_2 = 1‰$, *unfavourable configuration* – $\rho_2 = 0.5‰$ in the more compressed zone.

Two important conclusions emerge from the above. The nominal stiffness corresponds to the symmetrically reinforced cross section in which the strain in the concrete amounts to the plastic strain (along the whole length of the column). In this method the effect of the reinforcement locations is completely disregarded (the less compressed location and the more compressed one are treated equally). When most of the reinforcement is placed in the more compressed zone (*the favourable configuration*), this results in considerably greater stiffness of the cross section. In the case of the reverse reinforcement configuration, the stiffness is even lower than the one calculated from Eq. (1.6) for each stress intensity level.

The second of the approximate methods (the nominal curvature method) consists in adding up the first-order moment and the moment resulting from the appearance of second-order eccentricity e_2. The latter is originally determined as the displacement of the axis of a pinned-pinned beam in the middle of the column height (as for a constant first-order moment). Using Euler's solution, after transformations one gets the following dependence between curvature κ and the eccentricity:

$$e_2 = \kappa \frac{l^2}{\pi^2} \tag{1.7}$$

In order to generalize the dependence to cover other cases of support or moment paths it is enough to replace l with l_0 and π^2 with c – similarly as in the case of the nominal stiffness method.

Initial curvature κ_0 is determined in accordance with the assumptions illustrated in Fig. 3.

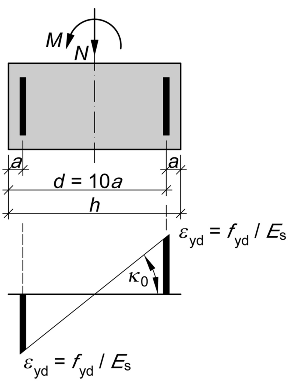


Fig. 3. Schematic of cross section and strain configuration for determining κ_0

It is assumed that one of the reinforcements is in compression while the other one is in tension and that both have just reached plastic strain ε_{yd}. It is also assumed that $a/d = 0.1$. This leads to the relation:

$$\kappa_0 = \frac{f_{yd}}{E_s} \frac{1}{0.45d} \tag{1.8}$$

After completions taking into account the effect of creep (K_φ) and that of the axial force on stiffness (K_r) one gets:

$$e_2 = \kappa_0 \frac{l_0^2}{c} K_r K_\varphi = \frac{f_{yd}}{E_s} \frac{1}{0.45d} \frac{l_0^2}{c} K_r K_\varphi \tag{1.9}$$

Coefficient K_φ is purely empirical and K_r is defined by the formula:

$$K_r = \frac{n_u - n}{n_u - n_{\text{bal}}} \leq 1 \tag{1.10}$$

Also the nominal curvature method is based on greatly simplified solutions derived from the Euler beam model. This particularly applies to the use of the notion of the design column length. The nominal curvature method to a higher degree (coefficient K_r) takes into account the effect of the force on the stiffness (curvature) of the cross section than the nominal stiffness method. According to the assumptions, the maximum curvature occurs when both the reinforcements are mobilized and it linearly decreases to zero when the whole cross section is in compression. This approximation departs from reality and in Fig. 4 is represented by the straight line. Relative forces n_{bal} and n_u, respectively, correspond to the above situations. Relative force n_{bal} corresponds to the maximum moment which the cross section can bear. It can be determined by independently plotting envelope curves (of interactions) for specific conditions. Standard [1] specifies $n_{\text{bal}} = 0.4$. If the concrete is of higher grade than C50/60, then $n_{\text{bal}} = 0.5$. As the concrete grade increases, n_{bal} decreases. Moreover, one should exercise caution when calculating n_u. The formulas given in standard [1] assume that under axial compression no reinforcement is mobilized. However, the maximum strains in the concrete when the whole cross section is in compression are limited to ε_{c3}. The latter is higher than ε_{yd} only for high concrete grades. In the case of common concrete grades, the reinforcement in this system is only partially mobilized ($\sigma_s = \varepsilon_{c3} E_s < f_{yd}$).

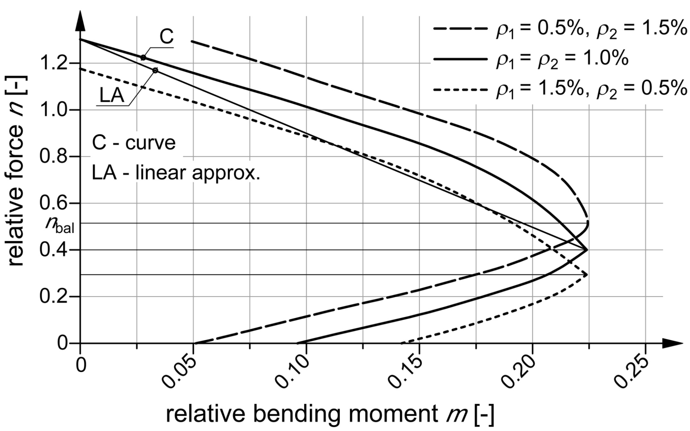


Fig. 4. Differences between envelope curves for symmetrical and asymmetrical

Summing up, it can be stated that:

- The fact that the method is based on the simplest Euler beam model and its modification is limited to the use of the fictional buckling length and the replacement of π^2 by slightly more universal coefficient c whose values range from 8 to 10 raises substantial doubts.
- The adoption of the strain system as shown in Fig. 3 for determining the maximum curvature is justified, even though there are situations when the curvature can be greater [7] (but these are extreme conditions). However, the relation expressed by Eq. (1.10) raises doubts. It implies a much quicker decrease in curvature (as the axial force increases) than it is the case in reality.
- For the already mentioned reasons the method is practically limited to symmetrically reinforced cross sections.

2. Gist of general method and relevant procedural algorithms

As opposed to the nominal stiffness and curvature methods, there are no reservations concerning the general method. However, the method is much less exhaustively described in standard [1]. Practically one can find there only general guidelines and recommendations. Neither can one find exhaustive descriptions in the available studies and other publications [8, 9]. The opinion prevails that because of its requirements the method is complicated. But this is not true. The work expenditure in this case is greater, but the results better describe the actual behaviour of the column.

The general method algorithms are to enable the solution of the basic differential equation describing the column axis displacements as a function of changing moments and the corresponding stiffness. In other words, the dependence between the local load and the local curvature must be the basis for the calculations. It must take into account the behaviour of the cross section, and so the onset of cracking and the yielding of the concrete and the steel. Another important issue is how to solve the differential equation with these dependences taken into account. A solution in the form describing the displacements of the whole axis would require highly complicated computer programs. The alternative recommended by the standard is to use the finite difference method which enables one to determine the values of displacements and moments in selected cross sections.

In the case of the general method, changes in column axis displacements are traced on a continuous basis [11]. Whereas the simplified methods limit themselves to the determination of the final state. The tracing simply consists in successive iterations which can lead to a situation in which the column (the steel or the concrete) will fail or the internal force will stabilize at a safe level. Consequently, it is not necessary to introduce assumptions about the shape of the beam's axis and the character of the sequence of displacements (a geometric progression or other) into the calculations and there are no notions of the design length or the buckling force.

As opposed to members in bending, the curvature of the axis depends on not only the moment value, but also the axial force value. This entails increased work expenditure. In practice it is worth working out a solution using the already mentioned relative dimensionless parameters: m, n and κd (bending moment, axial force and curvature, respectively), whereby it will be possible to use ready-made solutions when analysing other cases. The calculations are performed for appropriate steel and concrete models. The models can be highly advanced, but such as the ones shown in Fig. 5 are sufficient.

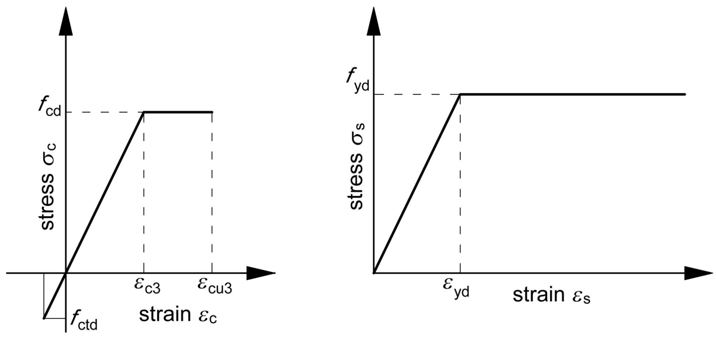


Fig. 5. Stress-strain dependences for concrete (a) and steel (b), assumed for moment-curvature dependence calculations

The m–κd dependence is generated for the assumed proper n, the specific reinforcement ratios ρ_1 and ρ_2, the concrete strength class and the a/d ratio. The variable is the strain in the edge concrete fibres in compression. Its minimal value corresponds to the axial compression of the cross section and its maximum value is equal to ε_{cu3}. Depending on the value of relative force n, one goes in this way through the different stages in the behaviour of the cross section, such as:

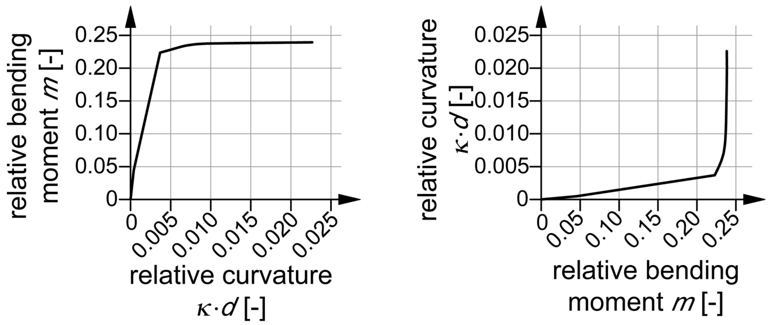
1) the whole cross section is being uniformly compressed;
2) the cross section is being compressed, but not uniformly;
3) a tension zone appears;
4) a crack appears, but the concrete and both the steels behave elastically;
5) the concrete begins yielding and both the steels remain within the elastic region;
6) the concrete continues to yield and the steel yields in the tension zone;
7) the edge concrete fibres reach strain equal to ε_{cu3}, which ends the calculations.

Obviously there are many more possible paths [7]. There can be such paths in which cracking will never occur or both the steels will yield. At each stage, using the law of flat sections and the equations of equilibrium of forces one determines the strains and forces in the concrete and in the reinforcement. On this basis one can already calculate the corresponding moment and curvature. A sample of the results of such calculations, covering the range from the compression of the entire cross section through the appearance of tension and cracking to the yielding of the steel in tension, is presented in Table 1. As the strain in the concrete at the more compressed edge (ε_c) continues to increase, the concrete yields gradually until the maximum value of m and minimal relative strength $\frac{m}{\kappa d}$ are reached. The following notations are used in Table 1: ε_{c1} – the strain in the concrete at the less compressed edge, ξ – the relative height of the compression zone, $\varepsilon_{s1}/\varepsilon_{s2}$ – the strain in the less/more compressed reinforcement, n_{c1}/n_{c2} – the relative force in the less/more compressed concrete, n_{s1}/n_{s2} – the relative force in the less/more compressed steel. Compressive actions and tensile actions were assumed to be respectively positive and negative.

Table 1. Sample results of calculations of changes in cross section stiffness

ε_c	ε_{c1}	ξ	ε_{s2}	ε_{s1}	n_{c1} n_{c2}	n_{s1} n_{s2}	m	$\kappa d \cdot 10^3$	$\frac{m}{\kappa d}$	Remarks
[‰]	[‰]	[–]	[‰]	[‰]	[–]	[–]	[–]	[–]	[–]	
0.2	0.0512		0.191	0.061	0.0312 0.045	0.006 0.018	0.014	0.139	98	
0.25	0.0012		0.234	0.017	0.0007 0.076	0.002 0.022	0.023	0.233	98	
0.26	–0.0088	1.03	0.243	0.008	–0.0001 0.077	0.001 0.023	0.025	0.252	98	
0.30	–0.0488	0.92	0.278	–0.027	–0.0021 0.079	–0.003 0.026	0.032	0.327	98	
0.35	–0.0988	0.83	0.322	–0.071	–0.0066 0.083	–0.007 0.030	0.041	0.421	98	
0.37	**–0.1170**	0.81	0.338	–0.087	–0.0086 0.085	–0.008 0.032	**0.045**	0.455	98	crack
0.39		0.73	0.350	–0.142	 0.081	–0.013 0.033		0.528	85	
0.60		0.55	0.528	–0.478	 0.095	–0.045 0.049	0.077	1.078	71	
0.80		0.49	0.692	–0.826	 0.112	–0.077 0.065	0.108	1.626	66	
1.00		0.46	0.854	–1.184	 0.131	–0.111 0.080	0.139	2.184	63	
1.20		0.44	1.017	–1.549	 0.150	–0.145 0.095	0.170	2.749	62	
1.40		0.42	1.179	–1.917	 0.169	–0.179 0.110	0.201	3.317	61	
1.54		0.42	1.292	**–2.175**	 0.182	–0.203 0.121	0.223	3.715	60	yielding

Exemplary $m - \kappa d$ and $\kappa d - m$ dependences are shown in Fig. 6. If computer software is used, one can obtain analytical forms of the relative moment-curvature dependence. The forms differ depending on the cross section strength utilization. They are different at the stage preceding cracking and different after cracking or after any of the steels yields or after the concrete yields.



(a) (b)

Fig. 6. Exemplary m–κd (a) and κd–m (b) dependences

In the next step the finite difference method is used. The basic differential equation is written as follows:

$$\kappa_i d = \frac{w_{i-1} - 2w_i + w_{i+1}}{a^2} d = m_i \frac{f_{cd} b d^3}{EI} \tag{2.1}$$

The procedural algorithm can be presented as follows:

1) divide the column axis into sections each with length a to obtain selected places where the values of displacements w_i and relative moment m_i will be determined,
2) calculate the values of the relative first-order moments in these places,
3) determine the relative curvatures corresponding to the moments on the basis of the plotted m–κd dependence,
4) calculate the displacements (w_i) of the points using appropriate dependences, the finite difference method and the boundary conditions (for a corbel column and a pinned-pinned column they are given in [7]),
5) calculate the moment increments caused by the displacements and the values of the current moments,
6) repeat the procedure for new moments,
7) perform further iterations until the differences in moment increments are negligibly small or it turns out that the load-bearing capacity has been exceeded.

The above processes can be easily automated using computer programs.

Figure 7 shows the results of consecutive iterations for a certain column [7]. Case **c** differs from case **b** in a considerable increase in the moment in the lower node. This leads to the failure of the whole column – its load capacity $m_{Rd} = 0.25$ is exceeded.

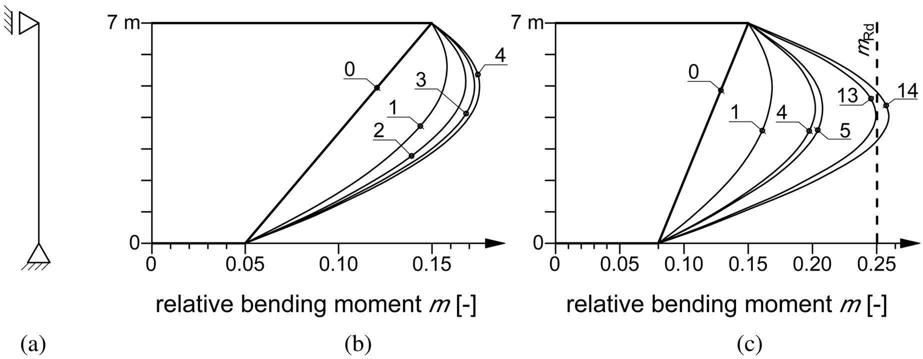


Fig. 7. Column loading diagram (a) and results of exemplary analysis for convergent sequence (b) and divergent sequence (c)

3. Examples and comparison

Exemplary analyses of the second-order effects were carried out for a tall corbel column (used in, e.g., high bay racked warehouses) and a stepped column (e.g. a column with a cantilever for a gantry girder) with the vertical force and the horizontal force on the intermediate cantilever.

3.1. Example 1 – tall column loaded with axial force in upper node and uniformly distributed horizontal force

The analysed corbel column and its cross section are shown in Fig. 8. The column was made of concrete C30/37 and reinforced with steel with characteristic yield point f_{yk} = 500 MPa. The longitudinal reinforcement is symmetrical: $\rho_1 = \rho_2 = 1.0\%$. The vertical axial load amounts to F = 200 kN and the horizontal load uniformly distributed

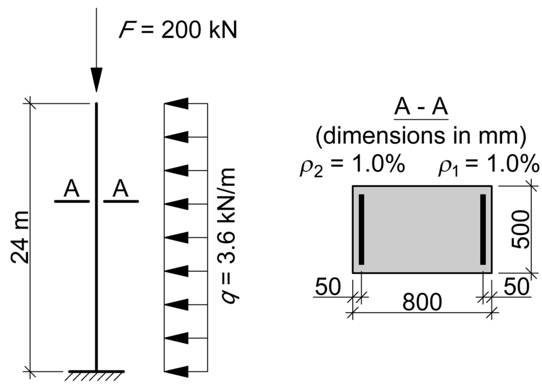


Fig. 8. Loading diagram and cross section of column analysed in example 1

along the column height amounts to $q = 3.6$ kN/m. Thus the maximum first-order moment in the column fixing is equal to:

$$M_{0Ed} = 0.5 \cdot q_d \cdot l^2 = 0.5 \cdot 3.6 \cdot 24^2 = 1036.8 \text{ kNm} \tag{3.1}$$

The column's resistance to bending is equal to $M_{Rd} = 1226.6$ kN·m.

In the considered case the second-order effects are produced solely by the vertical force acting at the eccentricity resulting from the column axis displacement caused by wind pressure. According to this paper, one can consider the use of the three methods.

3.1.1. The nominal stiffness method

This method is out of the question since the second-order effects do not apply to moments produced by wind pressure.

3.1.2. The nominal curvature method

This method can be used as the column is symmetrically reinforced. The initial curvature amounts to:

$$\kappa = \frac{1}{r_0} = \frac{f_{yd}}{E_s \cdot 0.45d} = \frac{435}{200 \cdot 0.45 \cdot 0.75} \cdot 10^{-3} = 6.44 \cdot 10^{-3} \text{ m}^{-1} \tag{3.2}$$

If the long-term effects are disregarded, one gets:

$$\frac{1}{r_0} = \frac{1}{r} = 6.44 \cdot 10^{-3} \text{ m}^{-1} \tag{3.3}$$

The deflection of the column axis end is equal to:

$$e_2 = \frac{1}{r}\frac{l_0^2}{c} = 6.44 \cdot 10^{-3} \cdot \frac{48^2}{10} = 1.48 \text{ m} \tag{3.4}$$

Thus the total moment amounts to:

$$M_{Ed} = M_{0Ed} + N_{Ed} \cdot e_2 = 1037 + 200 \cdot 1.48 = 1333 \text{ kNm} > M_{Rd} \tag{3.5}$$

Conclusion: the column will fail.

3.1.3. The general method

The graphs of the $m - \kappa d$ dependence together with the relevant equations are shown in Fig. 9, while the values of the relative moment in the column fixing and the displacements of the column's upper node are presented in Table 2.

The ultimate value of the bending moment will amount to:

$$M_{Ed} = m f_{cd} b d^2 = 0.1911 \cdot 21.4 \cdot 0.5 \cdot 0.75^2 = 1150 \text{ kNm} < M_{Rd} \tag{3.6}$$

This means that the column will bear these loads. The difference between the results yielded by the general method and the ones yielded by the nominal curvature method is not large (about 16%), but a decisive one. If, however, the axial force were stronger, the difference would sharply increase.

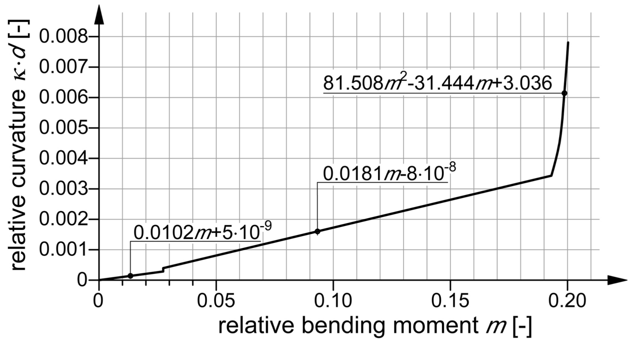


Fig. 9. Graphs of m–κd dependence

Table 2. Results of successive iterations

Iteration	Relative fixing moment m	Tip displacement w
	[–]	[m]
0	0.1720	0.000
1	0.1867	0.443
2	0.1905	0.556
3	0.1910	0.573
4	0.1911	0.576
5	0.1911	0.576

3.2. Example 2 – column with variable geometry and load

The analysed corbel column, its cross sections and graphs of axial forces N_{Ed} and first-order bending moments M_{0Ed} are shown in Fig. 10. The column was made of concrete C30/37 and reinforced with steel with characteristic yield point $f_{yk} = 500$ MPa. The longitudinal reinforcement is symmetrical: $\rho_1 = \rho_2 = 1\%$ and 0.5% respectively for the lower and upper section of the column. The load has the form of the following concentrated forces: vertical force applied to the tip $F_1 = 0.200$ MN at the eccentricity of 0.1 m, vertical force $F_2 = 0.350$ MN at the eccentricity of 0.2 m and horizontal force $H_2 = 0.035$ MN applied to the cantilever. The column was divided into 2.0 m long sections. Cross section no. 6 was doubled as an abrupt change in curvature (due to a change in cross section and load) takes place there. First the $m - \kappa d$ dependence was plotted for both parts of the column. The results of successive iterations for relative moment m and displacement w are compiled in Table 3.

Practically already after four iterations both the moments and the displacements stabilized. The relative moment in the fixing (cross section 0) increased from 0.133 to 0.151, i.e. by 13.5%. The change in moment in the upper part of the column is minimal.

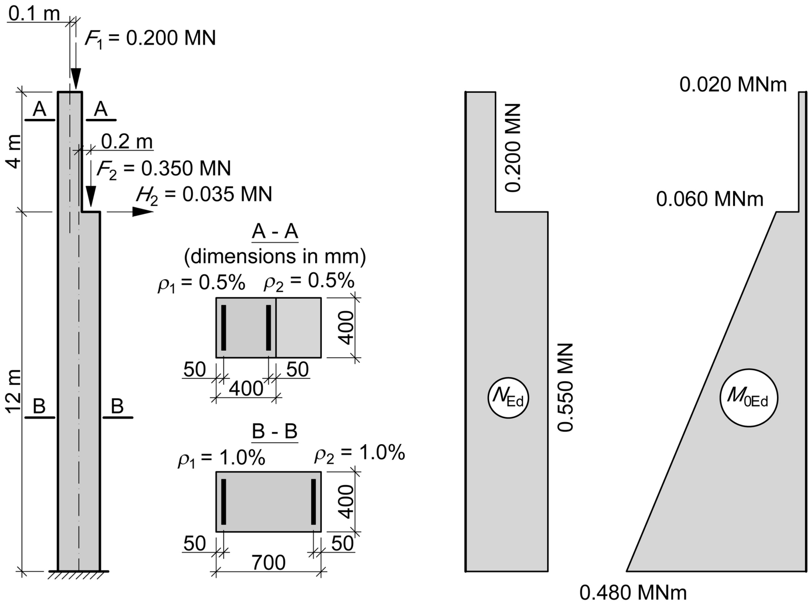


Fig. 10. Loading diagram and cross sections of column analysed in example 2 together with graphs of axial forces N_{Ed} and first-order bending moments M_{0Ed}

Table 3. Results of successive iterations

Cross section	Iteration								
	0		1		2		3		4
	m	w	m	$+w$	m	w	m	w	m
	[–]	[m]	[–]	[m]	[–]	[m]	[–]	[m]	[–]
0	0.133	0.000	0.148	0.000	0.151	0.000	0.151	0.000	0.151
1	0.113	0.006	0.128	0.007	0.131	0.007	0.131	0.007	0.131
2	0.094	0.024	0.107	0.027	0.110	0.028	0.110	0.028	0.110
3	0.075	0.049	0.086	0.057	0.087	0.058	0.088	0.058	0.088
4	0.055	0.081	0.063	0.094	0.064	0.096	0.064	0.096	0.064
5	0.036	0.118	0.040	0.137	0.041	0.140	0.041	0.140	0.041
6	0.019	0.156	0.020	0.182	0.020	0.186	0.020	0.186	0.020
7	0.019	0.159	0.019	0.184	0.019	0.188	0.019	0.189	0.019
8	0.019	0.164	0.019	0.189	0.019	0.193	0.019	0.194	0.019

If this column were analysed using the method proposed in [11], one would have to check the second-order effects only in the column's upper part, which turns out to be false.

4. Conclusions

In Poland the approximate methods are currently most often used in practice to determine second-order effects. However, the knowledge of their assumptions, theoretical foundations and limitations leaves a lot to be desired, which often leads to serious errors. This also applies to the available relevant computer programs. Nevertheless, one can say that in typical situations they perform satisfactorily.

However, in more complex cases it is necessary to use the general methods. They are then somewhat more labour-intensive, but indispensable. Therefore they need to be better popularized among designers and students.

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Obliczanie efektów II rzędu w słupach – zastosowania i przykłady

Słowa kluczowe: efekty 2. rzędu, mimośrodowe ściskanie, słup, wyboczenie, żelbet

Streszczenie:

W aktualnej normie do projektowania konstrukcji żelbetowych PN-EN 1992-1-1 problemom związanym z efektami drugiego rzędu nadano znaczącą rangę, ale ograniczono się do szczegółowego omówienia jedynie dwóch przybliżonych metod obliczeniowych – nominalnej sztywności i nominalnej krzywizny. W odniesieniu do metody ogólnej przedstawiono jedynie pewne wymagania i sugestie. W typowych sytuacjach, gdy spełnione są odpowiednie założenia, metody przybliżone dają zadawalające rezultaty. W praktyce inżynierskiej można jednak napotkać szereg przypadków, w których metody przybliżone będą zawodne – np. bardzo wysokie słupy i słupy ze wspornikami pod belki podsuwnicowe lub belki stropowe. W artykule przedstawiono i omówiono algorytm postępowania dla analizy problemu metodą ogólną. Według tego algorytmu wykonano przykładowe obliczenia a ich rezultaty porównano z rezultatami uzyskanymi z wykorzystaniem powszechnie stosowanych przez inżynierów metod przybliżonych. Wskazano ponadto obszary, w których metody przybliżone mogą zawodzić. Przeprowadzone analizy potwierdziły istotną przewagę metody ogólnej nad przybliżonymi. Warto ją propagować, tym bardziej że można jej procedury obliczeniowe zautomatyzować i opracować programy komputerowe.

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