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Derivative Free Optimal Thrust Allocation in Ship Dynamic Positioning Based on Direct Search Algorithms

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ABSTRACT: In dynamic positioning systems, nonlinear cost functions, as well as nonlinear equality and inequality constraints within optimal thrust allocation procedures cannot be handled directly by means of the solvers like industry-standardized quadratic programing (QP), at least not without appropriate linearization technique applied, which can be computationally very expensive. Thus, if optimization requirements are strict, and problem should be solved for nonlinear objective function with nonlinear equality and inequality constraints, than one should use some appropriate nonlinear optimization technique. The current state-of-the-art in nonlinear optimization for gradient-based algorithms is surely the sequential quadratic programing (SQP), both for general applications and specific thrust allocation problems. On the other hand, in recent time, one can also notice the increased applications of gradient-free optimization methods in various engineering problems. In this context, the implementation of selected derivative free direct search algorithms in optimal thrust allocation is proposed and discussed in this paper, and avenues for future research are provided.

1 INTRODUCTION

As far as the propulsion systems of marine vessels are concerned, a number of thruster types have been developed up to date for the requirements of the maritime industry, but when it comes to their application in dynamic positioning systems, the non-retractable and retractable azimuth thrusters, in eventual combination with tunnel thrusters, are mostly used. This choice is quite understandable, and is justified by the fact that due to the different directions of external disturbances, thrusters should be able to operate in all 360° at all times. Somewhat less common application is the Voith Schneider cycloid propulsor and combinations that include main propeller(s) with rudder(s).

The orientation, i.e. the angle or azimuth of each thruster, as well as the required thrust it generates, is determined by the control logic of dynamic positioning systems. This whole process is called the thrust allocation and represents a very complex mapping of the previously calculated or estimated environmental forces and moment in the set of referent states of the available thrusters.

Since dynamically positioned vessels usually have fixed and azimuth thrusters, the vector of design variables must have one variable for fixed (e.g. tunnel) thrusters and two for azimuth thrusters. The basic constraints on the objective function are (at least) three equalities stemming from the fact that the generated thrust forces in surge and sway and moment in yaw should be equal with environmental loads for all three horizontal degrees of freedom. This simplified optimization approach can be reduced on finding the conditional extremes of the Lagrange's objective function. If additional constraints, usually expressed by the matrix inequalities (e.g. thruster saturation, thruster efficiency, electrical power limitation, etc.), are added to the basic constraints, the optimization task becomes considerably more complex and is usually solved by the quadratic programming algorithms or so-called QP solvers (Jenssen and Realfsen, 2006).

From a theoretical point of view, the problem of the thrust allocation could be solved by linear programming (LP solvers), but due to the approximation of the relation between the electrical power consumption (kW) and the generated thrust (kN) by the quadratic function, some of the variants of the quadratic programming are usually used (Snijders, 2005; Wit, 2009).

If the problem of the quadratic programming of the thrust allocation is set correctly, it can be explicitly solved, i.e. it is possible to determine the global minimum (Leavitt, 2008). In general, the QP consists of the quadratic objective function and linear equalities and inequalities representing the conditions, i.e. the constraints. In addition to this, Sørdalen (1997) has shown that the constraints on azimuth thrusters can lead to singular configurations, which he solved using the method of singular values decomposition (SVD). This approach provided significantly lower power consumption, effectively eliminated the issue of the forbidden zones, reduced tear and wear of thrusters. With the application of so-called logical inequalities and Moore-Penrose pseudo-inverse matrix (SVD method), it is possible to directly determine the vector of demanded forces and moment (Gierusz and Tomera, 2006; Yang et al., 2011b).

If the constraints in the quadratic optimization task become nonlinear, it is no longer possible to use the QP solvers directly. One of the possible solutions to this problem is the application of the so-called sequential quadratic programming (SQP) technique that is generally used to minimize an arbitrarily selected objective function regarding the nonlinear constraint set in the form of equalities and inequalities. The possible applications of SQP approach in optimum thrust allocation were investigated by Liang and Cheng (2004) and Johansen et al. (2004). Although tested only on simulation models, the obtained results (Liang and Cheng, 2004) indicate very good capabilities of the SQP solver which in a computational sense can execute the allocation very fast with a small thrusters' azimuth change. Johansen et al. (2004) have further expanded the application possibilities of the SQP approach with the emphasis on avoiding possible singularities that are unacceptable in control sense.

In addition to the SQP approach for solving the problem of nonlinear constraints of the optimization task, most recently the genetic algorithms (GA) have been increasingly used as a robust solution that ensures a good convergence of the global optimization process (Yang et al., 2011a; Zhao et al., 2010). The tests that have been carried out by Yang et al. (2011a) indicate the promising results on using these algorithms, although the authors point out the problem of possible application of GA in thrust allocation regarding the slow convergence.

In order to recap, one should notice that the current state-of-the-art in nonlinear optimization for gradient-based algorithms is surely the sequential quadratic programing (SQP), both for general applications and specific thrust allocation problems. In comparison with e.g. Lagrangian multiplier method (LMM) or pure QP algorithms, which are both appropriate solutions for optimization problems with linear equality and linear inequality constraints, SQP approach is superior when dealing with problems that have significant nonlinearities within their constraints.

On the other hand, and in comparison with the gradient-based optimization methods, derivative free optimization methods usually does not need any particular information about the gradient or Hessian matrix of the objective function. Moreover, derivative free methods can be applied even for objective functions that are not continues nor differentiable, which makes them particularly convenient in cases when the objective function is not explicitly defined, when evaluation of the objective function and/or its derivatives is too much time consuming and thus not acceptable, when objective function is noisy and derivatives or finite difference approximations are not reliable nor acceptable for further analysis, etc.

Although the field of derivative free optimization is usually extended, or at least coupled with the so-called black-box optimization methods, the focus in this work is placed only to the family of the so-called direct search (DS), or pattern search (PS) algorithms. The main reason for this choice is that direct search algorithms are much better supported and have very detailed literature background on rigorous convergence analysis (Audet and Hare, 2017; Conn et al., 1997, 1991; Torczon, 1997). Therefore, the applicability and implementation issues of selected derivative free direct search algorithms in optimal thrust allocation problems have been analysed and discussed in this paper, and avenues for future research are emphasized as well.

2 METHODOLOGY

2.1 *General considerations on direct search algorithms*

Direct or pattern search algorithms are based on a common idea by which a sequence of points is determined with the property that in each successive point the value of objective function decreases. As already mentioned, this sequence of points, which defines directions from one point to another, is not calculated by means of function gradient, but is rather based on a set of points around the current point, in which the objective function is evaluated. These surrounding points are determined by polling and thus they create a so-called poll set that presents a mesh, i.e. all possible vector directions by which one can shift from the current point to any other point from the poll set. If some point within the poll set is found that sufficiently decreases the objective function at the current step, than that point becomes a new current point for the next iteration. Otherwise, the mesh should be redefined so the algorithm could try to find a new direction on a smaller scale. In general, one can differ three main direct search algorithms as follows:

– generalized pattern search (GPS) algorithm,

- generating set search (GSS) algorithm,
- mesh adaptive direct search (MADS) algorithm.

Besides the all other significant properties, generally speaking, the main difference between GPS, GSS and MADS is the number of directions from the current point to any other point from the poll set, as well as direction geometrical characteristics. Other important properties and differences are mostly related to handling of linear and nonlinear constraints.

2.2 *Generating set search algorithm with augmented Lagrangian*

The generating set search (GSS) algorithm is very similar to GPS, particularly for the problems of unconstrained optimization in which their patterns are identical. The main difference between GPS and GSS is related to constrained optimization problems. GSS, as an extension of GPS, is well suited for bounds and linear constraints, where directions in positive spanning set $\mathbb{D}$ are determined using the nearby active constraints from the working set (Kolda et al., 2006, 2003). In other words, GSS is more efficient in comparison with GPS for linearly constrained optimization problems.

When it comes to nonlinear constraints, GPS is not well suited, but with implementation of augmented Lagrangian within the GSS algorithm, which was introduced by Kolda et al. (2006), GSS can handle optimization problems with both linear and nonlinear constraints. However, this approach has been analysed under the assumption that the objective function and constraints should be twice continuously differentiable, which is typical required property for gradient-based methods.

The augmented Lagrangian pattern search (ALPS) algorithm is primarily used for solving optimization problems with nonlinear equality and inequality constraints, which means that bounds and linear constraints are handled differently, usually with nearby active constraints strategy (Kolda et al., 2006, 2003). For some general optimization problem with nonlinear equality and inequality constraints of the following form

$$\left.\begin{aligned} &\min_{\boldsymbol{x}\in R^n} f(\boldsymbol{x}) \\ &\text{s.t.}\quad \boldsymbol{c}_i(\boldsymbol{x})=0, \quad i\in E \\ &\qquad\ \boldsymbol{c}_i(\boldsymbol{x})\geq 0, \quad i\in I \end{aligned}\right\} \tag{1}$$

that is considered to be solved by some pattern search algorithm, associated sub-problem based on augmented Lagrangian should be formed as follows

$$\begin{aligned} L(\boldsymbol{x},\boldsymbol{\lambda},\boldsymbol{s},\boldsymbol{\rho}) &= f(\boldsymbol{x}) - \sum_{i\in I}\boldsymbol{\lambda}_i \boldsymbol{s}_i \log(\boldsymbol{s}_i + \boldsymbol{c}_i(\boldsymbol{x})) \\ &+ \sum_{i\in E}\boldsymbol{\lambda}_i \boldsymbol{c}_i(\boldsymbol{x}) + \frac{\rho}{2}\sum_{i\in E}\boldsymbol{c}_i^2(\boldsymbol{x}) \end{aligned} \tag{2}$$

where $\boldsymbol{\lambda}_i \geq 0$ are Lagrangian multipliers, $\boldsymbol{s}_i \geq 0$ are slack variables, and ρ is positive penalty parameter. One should notice that each sub-problem (2) presents one iterative step, which makes this approach with nonlinear constraints highly computationally expensive. During each iteration, values of $\boldsymbol{\lambda}$, $\boldsymbol{s}$, and $\boldsymbol{\rho}$ are kept constant, until the sub-problem (2) is minimized, whereupon are all updated. Otherwise, the penalty parameter ρ is increased and a new sub-problem is formed. These steps are repeated until the termination, which is based on some predefined stopping criteria.

2.3 *Mesh adaptive direct search algorithm*

The MADS algorithm, which was introduced by Audet and Dennis (2006), primarily as direct search algorithm for solving constrained optimization problems of the general form

$$\min_{\boldsymbol{x}\in\Omega} f(\boldsymbol{x}) \tag{3}$$

does not require any assumptions related to the smoothness of objective function nor to constraints that could be either linear, or nonlinear, or both. If $\Omega = R$ in (3), than the previous optimization problem becomes unconstrained.

General constraints with MADS algorithm are usually handled by the so-called extreme barrier strategy (Audet and Hare, 2017), which is based on extreme barrier function $f_\Omega : R^n \to R \cup \{\infty\}$ defined as

$$f_\Omega = \begin{cases} f(\boldsymbol{x}), & \text{if } \boldsymbol{x}\in\Omega \\ \infty, & \text{if } \boldsymbol{x}\notin\Omega. \end{cases} \tag{4}$$

The associated principle is very simple and is based on the fact that optimization is performed using the barrier function $f_\Omega(\boldsymbol{x})$ as the objective, rather than $f(\boldsymbol{x})$. More advanced approaches can take into account two-phase extreme barrier strategy, filter methods (Audet and Dennis, 2004; Dennis Jr. et al., 2004), progressive barrier strategy (Audet and Dennis, 2009; Le Digabel, 2011) or mixture between extreme barrier and progressive barrier called progressive-to-extreme barrier strategy (Audet et al., 2010).

However, the MADS algorithm is primarily orientated to inequality constraints with bounds, which means that equality constraints could be challenging. For this purpose, one can substitute one equality constraint with two equivalent inequality constraints, although this approach could be cumbersome in some optimization problems and algorithm efficiency could be questionable or even not acceptable. This issue is also related to the complexity of equality constraints and to the selection of initial point $\boldsymbol{x}_0$.

Possible alternatives for handling this issues could be closely related to approaches introduced with GPS and GSS algorithms, i.e. to equality constraint handling by means of the nearby active constraints or augmented Lagrangian method (Kolda et al., 2006, 2003). Recent research directions are also aimed towards the combining of gradient-based methods, like sequential quadratic programing, and derivative free optimization with equality constraints (Tröltzsch, 2016).

3 OPTIMAL THRUST ALLOCATION

3.1 *Problem definition*

From the optimization point of view, thrust allocation problem usually comes down to the minimization of total power consumption or some other appropriate objective function $f: X \to \mathbb{R}$, in terms of thrust $\boldsymbol{x}$. Hence, this optimization task can be defined as follows

$$\left.\begin{aligned} &\boldsymbol{x}^* = \arg\min_{\boldsymbol{x}\in R^n}(f) \\ &\text{s.t.}\quad h_i(\boldsymbol{x}) = 0 \\ &\qquad\ g_j(\boldsymbol{x}) \le 0 \\ &\qquad\ \boldsymbol{l}_B \le \boldsymbol{x} \le \boldsymbol{u}_B \end{aligned}\right\} \tag{5}$$

where $\boldsymbol{x} \in X$ and $X = \{\boldsymbol{x} \in \mathbb{R}^n \mid \boldsymbol{h}_i(\boldsymbol{x}) = 0,\ i = 1,2,\ldots,p;\ \boldsymbol{g}_j(\boldsymbol{x}) \le 0,\ j = 1,2,\ldots,q\}$ presents a set of feasible thrust region that depends on equality and inequality constraints $\boldsymbol{h}(\boldsymbol{x}) = [h_1(\boldsymbol{x}), h_2(\boldsymbol{x}) \ldots h_p(\boldsymbol{x})]^{\mathrm{T}}$, and $\boldsymbol{g}(\boldsymbol{x}) = [g_1(\boldsymbol{x}),\ g_2(\boldsymbol{x}), \ldots\ g_q(\boldsymbol{x})]^{\mathrm{T}}$, respectively, $\boldsymbol{l}_B$ and $\boldsymbol{u}_B$ are lower and upper boundaries of $\boldsymbol{x}$, and power function f is commonly assumed to be twice-continuously differentiable, i.e. sufficiently smooth.

Therefore, nonlinear optimization problem (5) for thrust allocation can be redefined in the following form

$$\left.\begin{aligned} &\boldsymbol{u}^* = \arg\min_{\boldsymbol{u}\in R^{2r}}\left(\sum_{i=1}^{r}\frac{P_{i,\max}}{(T_{i,\max})^{m_i}}(u_{ix}^2+u_{iy}^2)^{\frac{m_i}{2}}\right) \\ &\text{s.t.}\quad \boldsymbol{b}_{\mathrm{eq}} - \boldsymbol{A}_{\mathrm{eq}}\boldsymbol{u} = 0 \\ &\qquad\quad \boldsymbol{A}_i^{\mathrm{FZ}_{ik}}\boldsymbol{u}_i \le \boldsymbol{0}_i^{\mathrm{FZ}_{ik}} \\ &\qquad\quad u_{ix}^2 + u_{iy}^2 \le T_{i,\max}^2 \end{aligned}\right\} \tag{6}$$

where objective function is total delivered power for *r* thrusters (Leavitt, 2008; Wit, 2009), defined in terms of individual thrust components u_{ix} and u_{iy} of resultant individual thrust T_i of each thruster in body reference frame {*b*}, $P_{i,\max}$ and $T_{i,\max}$ indicate maximum power and maximum thrust for any *i*-th thruster, respectively, $1 < m_i \le 2$, and $\boldsymbol{u}$ presents the space of thruster states, which is for *r* thrusters defined as

$$\boldsymbol{u} = [u_{1x}, u_{1y}, u_{2x}, u_{2y}, \ldots, u_{ix}, u_{iy}, \ldots, u_{rx}, u_{ry}]^{\mathrm{T}} \in \mathbb{R}^{2r}. \tag{7}$$

Matrices $\boldsymbol{A}_{\mathrm{eq}}$ and $\boldsymbol{b}_{\mathrm{eq}}$ in (6) are defined as $\boldsymbol{A}_{\mathrm{eq}} = [\boldsymbol{B}, \boldsymbol{C}_{\mathrm{eq}}]^{\mathrm{T}}$ and $\boldsymbol{b}_{\mathrm{eq}} = -[\boldsymbol{\tau}, \boldsymbol{0}_{\mathrm{eq}}]^{\mathrm{T}}$, which take into account thrust allocation problem $\boldsymbol{Bu} = -\boldsymbol{\tau}$ and additional equality constraints for tunnel thrusters in form of $\boldsymbol{C}_{\mathrm{eq}}\boldsymbol{u} = \boldsymbol{0}_{\mathrm{eq}}$, if there are any. Matrix $\boldsymbol{B} \in R^{3\times 2r}$ is a well-known configuration matrix and although $\boldsymbol{\tau}$ is usually a control vector calculated by a DP controller, in this quasi- static analysis it presents a vector of environmental loads in the horizontal plane that are calculated on the basis of model tests according to the usual design recommendations. Matrices $\boldsymbol{A}_i^{\mathrm{FZ}_{ik}}$ and $\boldsymbol{0}_i^{\mathrm{FZ}_{ik}}$ are defined as

$$\boldsymbol{A}_i^{\mathrm{FZ}_{ik}} = \begin{bmatrix} -\cos\varphi_{\mathrm{start}}^{\mathrm{FS}_{ik}} & \sin\varphi_{\mathrm{start}}^{\mathrm{FS}_{ik}} \\ \cos\varphi_{\mathrm{end}}^{\mathrm{FS}_{ik}} & -\sin\varphi_{\mathrm{end}}^{\mathrm{FS}_{ik}} \end{bmatrix} \in R^{2\times 2},\quad \boldsymbol{0}_i^{\mathrm{FZ}_{ik}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \in R^2, \tag{8}$$

and are basis for modelling forbidden zones in terms of circle sectors bounded with two radii at angles $\varphi_{\mathrm{start}}^{\mathrm{FS}_{ik}}$ and $\varphi_{\mathrm{end}}^{\mathrm{FS}_{ik}}$, where *k* indicates what feasible set FS_{ik} is selected according to some predefined criteria. Final inequality equations $u_{ix}^2 + u_{iy}^2 \le T_{i,\max}^2$ in (6) are related to saturation of thrusters and in this form they also present nonlinear thrust regions for each thruster.

Alternative approach, for $m_i = 1.5$, in which the relationship between delivered power and generated thrust is based on thrust and torque coefficients K_T and K_Q, is very similar to (6). The only difference is in the form of the objective function, while all other constraints are the same. In this case, objective function of nonlinear optimization problem can be defined as

$$\boldsymbol{u}^* = \arg\min_{\boldsymbol{u}\in R^{2r}}\left(\sum_{i=1}^{r}\frac{2\pi K_{Q0,i}}{D_i\, K_{T0,i}\sqrt{\rho K_{T0,i}}}(u_{ix}^2+u_{iy}^2)^{\frac{3}{4}}\right) \tag{9}$$

where $K_{T0,i}$ and $K_{Q0,i}$ are thrust and torque coefficients at bollard pull conditions, respectively, D_i is propeller diameter, ρ is (sea) water density, and $i = 1,2,\ldots,r$.

3.2 *Numerical example and analysis of results*

Straight applications of direct search algorithms in optimal thrust allocation do not require any additional transformation of optimization tasks. In other words, optimization tasks like (6) or (9) are already fully prepared in order to be solved by means of pattern search algorithms, which is very convenient, particularly if one wants to perform appropriate comparisons between these algorithms and any other algorithm that could be of interest, like QP, SQP, etc.

Table 1. Thruster configuration with basic data

#	Thruster	l_x (m)	l_y (m)	D (m)	P/D	n (min^{-1})	$T_{\max}$ (kN)	$T_{\min}$ (kN)	$P_{\max}$ (kW)
1	Tunnel	82.0	0.0	2.0	1.2	330.0	165.0	-165.0	±1200.0
2	Azimuth	57.0	4.5	2.5	1.2	900.0	390.0	0.0	2400.0
3	Azimuth	52.0	-4.5	2.5	1.2	900.0	390.0	0.0	2400.0
4	Azimuth	28.0	-15.0	2.5	1.2	900.0	390.0	0.0	2400.0
5	Azimuth	-22.0	15.0	2.5	1.2	900.0	390.0	0.0	2400.0
6	Azimuth	-60.0	15.0	3.6	1.2	630.0	760.0	0.0	4500.0
7	Azimuth	-60.0	-15.0	3.6	1.2	630.0	760.0	0.0	4500.0

For the purpose of this paper, a simple example is provided for heavy lift DP vessel Saipem 3000, which was selected as a reference vessel with the length overall of $L_{oa} = 162.0$ m, beam $B = 38.0$ m, displacement $\Delta = 24000$ t, and is equipped with seven thrusters, one of which is a bow tunnel thruster and the rest are azimuth thrusters. Their approximate positions on the hull regarding the body reference frame are given in Table 1, together with basic thruster data.

In order to illustrate the problem, only the allocation results with environmental loads at wind speed of $v_{wind} = 10$ m/s, significant wave height $H_s = 3.21$ m, wave peak period $T = 8.41$ s and sea current velocity $v_c = 0.5$ m/s are presented. The angle of attack was the same for all disturbances at any time and varied from $\gamma = 0°$ to $360°$ with the rate of $10°$. Environmental loads were calculated on the basis of model tests according to usual design recommendations. Finally, the vector $\tau = [F_{x,loads}\ F_{y,loads}\ N_{z,loads}]^{\mathrm{T}}$, which represents the total environment load for some angle of attack γ, was calculated quasi-statically in order to be used in (6).

The optimal thrust allocation is performed by MADS algorithm with augmented Lagrangian and obtained results with MADS are compared with results obtained by SQP. For this analysis, results of SQP algorithm served as reference values, and comparisons were made in terms of average RMSE of optimal solutions between MADS and SQP, as well as in terms of average convergence time for these two approaches. The target PC configuration was based on Intel(R) Core (TM) i5-7500 CPU @ 3.40 GHz, 16 GB RAM, x64-based processor, 64-bit operating systems (MS Windows 10). Optimization was performed using optimization task (6) for $m_i = 1.5$ and with MathWorks MATLAB R2017b as a support software. In order to additionally simplify this analysis, forbidden zones were not included, so the problem of non-convex thrust regions could be omitted.

Hence, after all 36 optimization tasks had been solved, the average convergence time for MADS was 0.587459 s, while for SQP average convergence time was 0.020125 s. RMSE between optimal solutions $\boldsymbol{u}^*$ for MADS and SQP was equal to $4.0748 \cdot 10^{-4}$. These results clearly indicate that there are no significant differences between optimal solutions obtained with MADS and SQP, but one can notice that SQP is relatively much faster. However, the reason for this is also in relative simplicity of optimization task (6), particularly in this case, i.e. without forbidden zones included and without some additional nonlinear constraints. Moreover, objective function in this case is also relatively simple and convex, which presents favourable conditions for gradient-based algorithm like SQP. Nonetheless, MADS showed overall very promising results, especially in comparison with other derivative free algorithms like genetic algorithms (GA) for which convergence time is usually the biggest issue (Yang et al., 2011a; Zhao et al., 2010).

4 CONCLUSION

Although the results of optimal thrust allocation problem obtained with direct search algorithm are more than satisfactory, particularly in comparison with state-of-the-art algorithm like SQP, it should be pointed out that the gradient-based algorithms could and probably should be a better choice for optimization problems where the gradient and/or Hessian of the objective function is known or at least it can be obtained in sufficient amount of time. However, this will probably be true for most unconstrained optimization problems, but on the other hand, when a large number of nonlinear constraints is added into the optimization task, and even more when the objective function is constantly changing due to some external disturbances, then the calculation of associated Lagrangian and its gradient, Hessian or Hessian approximation could be very demanding within gradient-based methods. Thus, in these cases direct search algorithms could present a good or even better alternative. In order to better evaluate the possibilities of direct search algorithms in optimal thrust allocation, future analyses and comparisons should take into account forbidden zones, i.e. non-convex thrust regions, thrust loss effects and complete environmental envelope. Moreover, additional procedures in order to enable faster convergence of direct search algorithms should be identified as well.

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