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## RESEARCH ON METAL FATIGUE OF RAIL VEHICLE WHEEL CONSIDERING THE WEAR INTENSITY OF ROLLING SURFACE

### BADANIE ZMĘCZENIA METALU KOŁA POJAZDU SZYNOWEGO Z UWZGLĘDNIENIEM INTENSYWNOŚCI ZUŻYCIA POWIERZCHNI TOCZNEJ

*The article overviews scientific research studies that examine the interaction between railway vehicle wheel and rail, and the phenomena of wear on wheel rolling surface. Unique experimental research has been conducted, in which regularity of weariness on rolling surface of exploitable locomotive wheel and phenomena of metal fatigue on wheel were researched. A hypothesis is made, that according to the differences in weariness intensity of wheel rolling surface it is possible to determine the start of metal fatigue. The inequality of wear intensity of different locomotive wheels is assessed by the Sharpe ratio, adapting it to describe the wheel wear intensity criteria. Based on the results of research, the authors propose a simplified and reliable methodology for determining metal fatigue on locomotive wheels at initial stages. The uneven wear on rolling surface of different wheels of wheelset inevitably changes the values of Sharpe ratio, which can accurately describe the conditions in which the critical metal fatigue on wheels begins to emerge.*

**Keywords:** rail vehicle, wheel-set, wheel rolling surface, wear intensity, metal fatigue, Sharpe ratio.

*W artykule omówiono badania naukowe dotyczące wzajemnych oddziaływań między kołem pojazdu szynowego a szyną oraz zjawiska zużycia powierzchni tocznej kół. Przeprowadzono nowatorskie badania eksperymentalne, w których zbadano prawidłowości dotyczące zużywania się powierzchni tocznej eksploatowanego koła lokomotywy oraz zjawisko zmęczenia metalu koła. Złożono hipotezę, że na podstawie różnic w intensywności zużycia powierzchni tocznej kół można określić początek procesu zmęczenia metalu. Różnice w intensywności zużycia różnych kół lokomotywy oceniano na podstawie współczynnika Sharpe'a, dostosowując go do opisu kryteriów intensywności zużycia kół. Na podstawie wyników badań autorzy zaproponowali uproszczoną, rzetelną metodologię określania zmęczenia metalu kół lokomotywy na początkowym etapie tego procesu. Nierównomierne zużycie powierzchni tocznej różnych kół zestawu kołowego zmienia wartości współczynnika Sharpe'a, które można wykorzystać do precyzyjnego opisu warunków, w jakich dochodzi do krytycznego zmęczenia metalu kół.*

**Słowa kluczowe:** pojazd szynowy, zestaw kołowy, powierzchnia toczna koła, intensywność zużycia, zmęczenie metalu, współczynnik Sharpe'a

#### 1. Introduction

One of the most commonly analysed technical issue related to the safety usage of railway vehicle is the interaction between wheel and rail. Rolling stability, train traffic safety and continuity of wagons depend on technical condition of wheelset. During the exploitation of railway vehicles, one of the rolling stability and safety ensuring parameter, which must be constantly monitored and controlled, is the weariness of wheel rolling surface. Two parts in the interaction between railway vehicle wheel and rail wear the most: the rolling surface and the flange. Weariness gets particularly intensive when vehicle is rolling on track curves or railroad switches, when wheel and rail is in contact with both rolling surface and flange. The regularity of weariness on these contacting surfaces are constantly researched by scientists Nielsen [8], Shackleton and Iwnicki [16], Polách [10]. It should be noted that evaluations of scientists are different from the approach carried in practice, which states that the contact between wheel and rail is only in one or two points. Researchers Ren, Iwnicki and Xie [11] examine the regularity of linear and “stepped” load distribution on different wheel/rail contact areas. The same assumption, that the load in the wheel-rail contact is evenly distributed on rail-wheel surface area, is made by scientists Sichani [17] and Nielsen [8]. It should

be noted, that scientists in these articles are analysing only vertical forces occurring in wheel-rail interaction. The variation of roughness/ unevenness of the rail must be evaluated in order to assess the interaction between wheel and rail more accurately [22].

Exploitation of railway vehicle and intense weariness of wheel rolling surface lead to radical changes in geometrical parameters of the wheel. When the lowest critical values of these parameters are reached, the wheelset rolling imbalance increases due to worse wheel/rail contact conditions and the derailment threat increases [15]. It should be noted that, wheel dimensions are regularly and carefully monitored and registered during the vehicle exploitation [20-21]. A very significant technological dimension is the thickness of wheel flange. When it decreases to a critical value, the threat of wheel derailment increases, so it is important to turn new wheel rolling surface. Another important parameter for locomotives is the thickness of rim. The rim that is too thin heats up and expands considerably, during locomotive's traction or braking, when wheels are towing or sliding on rails. The loose rim skids on wheel disc and the locomotive no longer generates the required traction or braking force. Other critical damages of wheelset wheel are the rolling surface damages, such as cracks, scaling and splits, which are caused by metal fatigue.

The detection of metal fatigue is a particularly challenging task at the initial stage [6].

The condition of vehicle wheel rolling surface (whether there are no rolling surface damages) is controlled by trackside automated control equipment (hereinafter TACE), which determines the values of vertical wheel impact on rail [1, 23]. The devices show the critical values that exceed vertical impact forces, when there are wheel damages. The size of wheel impact force is completely stochastic, so the specialists of railway vehicle exploitation are faced with problems of repeatability and reliability of measuring equipment readings [3, 19]. The problem is that dynamic measuring equipment (TACE) shows different vertical impact forces, after passing with the same damaged wheel through the same rail point.

Another important aspect of interaction between wheel and rail is the provision of wheel's adhesion with rail during the traction for locomotives [7] and braking for the train [2]. The quality adhesion between wheel and rail is significantly dependant on the accuracy of geometrical wheel/rail rolling surface, level of weariness and the continuation of interacting surfaces.

A vertical static load of more than 180 kN is present on one of its wheels while the locomotive is still, and it can grow up to 2-3 times when rolling on rails. In addition, horizontally (longitudinal and transverse) dynamic forces appear on wheel/rail contact, when the vehicle is rolling, due to the wheelset oscillation process and the geometric inequalities of the railway track [14]. Due to these dynamic creepage loads, both the rolling surface of the wheel and the rail surface are heavily worn. It is noteworthy, that wheel metal fatigue appears due the cyclical of these relative slip loads. The study of Shackleton & Iwnicki [16] is valuable because the authors describe multiple methodologies used for examination of interaction between wheel and rail surfaces, and provide examples of their implementation in different software packages. One of the results of this study is the regularity of the displacement of the wheel / rail contact surface. Wheel - rail interaction is explored similarly in the article by researchers Ferrara, Leonardi & Jourdan [4]. The authors analyse the regularity of the variation of the rail oscillation acceleration. During the exploitation, the major factor, which determines wear in railway vehicle wheel, is the elastic and plastic deformation of the rolling surface. It causes metal fatigue and changes in physical properties. The phenomenon of wheel and rail surface deformation has been thoroughly examined by scientists Sebès, Chevalier, Ayasse & Chollet [12]. This article presents examples of modelling of wheel and rail contact. It should be noted that most of the consequences of the primary metal fatigue (cracks, scaling and splits) is rubbed with pads, when the locomotives are braked by pressing the brake pads against the rolling surface of the wheel. Thus, the effects of fatigue during the exploitation of the vehicle are eliminated or significantly reduced.

In the last century, scientific researches on the development of railway vehicle wear reduction measures and evaluation of their effectiveness have been intensively carried out, which laid the foundations for the breakthrough of modern scientific and technological achievements in this area. From the scientific point of view, researches on individual cases of wheel and rail interaction, for example, in railway switches, are innovative. The interaction between wheel and rail is highly influenced by design of railway switches. The results of such studies are presented in the Pålsson's article [9]. The author proposed a methodology for modelling profile of railway switch cross in order to minimize the dynamic load on the wheel.

The variety and complexity of the factors that cause the fatigue to the vehicle wheel rolling surface listed above, reduce the chances of vehicle exploitation specialist success on determining wheel metal fatigue in the early stages. In solving this problem, the regularity of the wear of the wheel surface was studied in detail, considering metal fatigue phenomena. Different wear in wheels of the same bogie are found between the wheel casts, with the increase in the mileage (pe-

riodicity) of the vehicles. This different wheel wear appears not only due to the wear of the rolling surface, but also due to metal fatigue. In this article, the authors analyse the possibilities of applying more abstract interdisciplinary research methods, by using knowledge and methods of related fields of science.

New methods must be applied in order to forecast intensity of wheelset wheel wear and metal fatigue, due to increasingly sophisticated trouble-shooting equipment. During vehicle exploitation, it would be rational to evaluate both the average intensity of wheel wear and the discretion of this wear depending on the location of wheelset in bogie. Such evaluation may be a criterion for the design of a running gear in the future, to ensure the even wear of wheels during exploitation. The problem gets an aspect of comparison of changes in element value (quality). In this article, the authors assume that, it is possible to use methods for comparing change of value (quality) that are used in other fields of science, in order to examine and forecast vehicle wheel wear. The following article describes the adaptation of the Sharpe ratio in order evaluate intensity of rolling surface weariness distribution on different wheels [13]. In this research, the Sharpe ratio is considered as a complex indicator of the technical condition (quality) of wheelsets, adapting it to describe the wear intensity parameters.

## 2. Methodology for the assessment of wheelset wheel weariness and metal fatigue

One of the most researched and methodically applied qualitative methods in the financial sciences is the evaluation of changes in the investment value (for example, stock portfolio). This qualitative change in value is perceived as the return on investment. Mathematically, both of the processes mentioned above can be examined as identical, just in opposite directions. One of the best-performing indicators that evaluates (compares) the return on investment is Sharpe ratio [5]. The higher the Sharpe ratio is; the sooner the fund compensates for the risks involved [18]. The Sharpe ratio is calculated from the investment return rate minus the risk-free rate of return and dividing the result by the average standard deviation of the investment risk:

$$SR = \frac{R_f - R_n}{\sigma}; \quad (1)$$

where:  $R_f$  – the average annual rate of return on the investment fund;  $R_n$  – rate of return on risk-free investment;  $\sigma$  – average standard deviation of the average annual rate of return fund.

The risk-free return rate on investment  $R_f$  is defined as the average return on investment over a long period (about 10 years). Typically, all of these values are measured in monetary terms, and their ratio in the formula (1) is dimensionless. This is one of the essential features of similarity criteria. Thus, this attribute of the Sharpe ratio allows it to be adapted to control the change of value (quality) in other areas of activity. When solving engineering problems, it is often necessary to compare the changes in technical condition of wearing parts and value (residual resources or other qualitative indicators). When examining the wear of wheelset wheels, it is assumed that weariness of wheel rolling surface reduces the qualitative value of the wheels.

## 3. Analysis of vehicle wheel weariness and metal fatigue processes

Vehicle wheel weariness can be distinguished in two main directions: wear of rolling surfaces and metal fatigue. Wheel wear researches can be divided in wheel rolling surface weariness and wheel flange weariness.



Fig. 1. Visual attributes of metal fatigue on rolling surface of wheel

The visual attributes of metal fatigue on rolling surface of wheel are shown in Figure 1.

The defective wheel rolling surface is covered with a special high-penetration pigment. If the surface of the wheel is intact, the pigment cleanses away from it, and if it contains cracks, the pigment penetrates into them and does not come off. This can be used to detect cracks that usually occur due to metal fatigue. These defects appear, when the wheel has minor wear and after the wheel surface turning the vehicle has driven through a mileage of 150 – 200 thousand kilometres. Defects are usually overlooked, when wheel surface wears more intensively and after the wheel surface turning the vehicle has driven through a mileage of 100 – 150 thousand kilometres. Metal fatigue layer is cut off from the wheel rolling surface. Therefore, if the wheel reaches such a mileage that metal fatigue appears, this is considered to be a result of proper operating conditions in the aspect of wheel wear. Such defects in locomotive wheels were observed on Lithuanian railways, when the new generation of German locomotives SIEMENS ER20CF were put into operation in 2008 instead of Russian freight locomotives, and travelled a respective mileage. Lithuanian scientists Jastremskas et al describe this more thoroughly in the article (2010). These new generation reliable locomotives do not always reach more than 150,000 kilometres without casting wheels. The wear intensity of wheel surface is determined not only by the quality of the locomotive (wheel slip control system, accuracy of parts, wheel flange lubrication system), but also by other conditions during exploitation (locomotive control quality, railroad curves and switches, working culture). Traction and braking forces determine the wear of wheel rolling surface. These forces depend on the train's acceleration and mass. The wear of the flange forms on the railroad curves and switches. Sometimes the wheel is in contact with the rail by both the rolling surface and the flange. These two surfaces rotate at the same angular speed when the wheel rolls, but the linear speed differs. As a result, inevitably one of them slips (skids) and, of course, gets damage. This process is illustrated in Figure 2.

Rail contact with wheel rolling surface is marked by symbol B and contact with flange is marked by symbol A.  $\beta$  is the angle be-

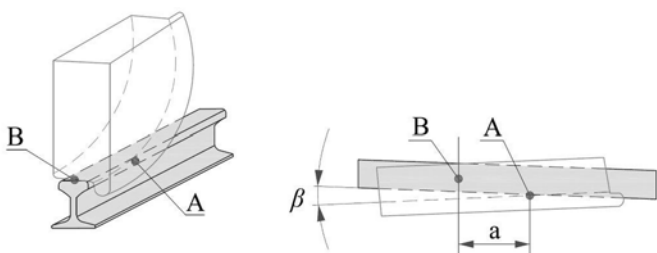


Fig. 2. Rail in contact with wheel rolling surface and flange

tween wheel-rail rolling surface,  $a$  – the distance between points of rail contact with rolling surface and rail contact with flange. In proper vehicle operating conditions, wheel flange does not lean on rail at point A and due to that it does not wear. Two wheels are fastened on the axle, both of which rotate at the same angular speed with the axle. The linear velocity (along the train's direction) of each wheel centre is proportional to the diameter of rolling wheel surface. Wheel usually does not climb on the rail if one of the wheelset wheel is getting closer to the rail and the other one is moving away from it. It starts to roll at a larger diameter of the surface cone and other wheelset wheel – at a smaller diameter. The linear speed of its centre begins to increase as compared to the linear speed of another wheel. The wheelset then turns in the direction of the decelerating wheel. If another wheelset wheel is approaching to the rail, the same happens in the opposite direction. This way the direction of the rolling wheelset on rails is continuously adjusted and the wheelset tilts in the track gauge. This adjustment is called wheelset oscillation.

Due to the oscillation phenomena, longitudinal and transverse forces of relative slip arise between wheel and rail. These cyclic nature forces cause metal fatigue in contacting surfaces. In straight railway track, oscillation is approximately symmetrical, in the curved track – the required ratio of rolling wheel diameters is adjusted. Due to this phenomenon, the wheel does not lean on the rail with the flange. However, if the wheel systematically leans on the rail with the flange (and therefore flange wears), this is considered that the exploitation is not completely normal. It is true that this happens in some of the more primitive structures or lower production culture vehicles. The wheel flange on modern and properly operated locomotives is only a fuse for special occasions for example, when curve radii are smaller than foreseen or when traveling through railroad switches. For such cases, a flange lubrication system is also provided. As a result, the wear is systematically found in the rolling surface of the wheel, but not in the wheel flange. During exploitation, it is noticeable that the weariness on rolling surfaces of the wheels on the same locomotive varies. The difference in intensity of rolling surface wear in different wheelset wheels was researched in Lithuanian railways. Four SIEMENS “ER20CF” freight locomotives were used for the research. The main technical data of the mentioned locomotives is given in Table 1.

Table 1. The main technical data of the SIEMENS “ER20CF” diesel locomotive (<http://www.siemens.fi/pool/lithuania>)

Parameter	Parameter value
Wheel arrangement	Co'Co'
Track gauge, mm	1520
Weight, kN (tf)	1354 (138)
Axle load, kN (tf)	225 (23)
Length over couples, mm	22850
Wheel diameter (new / worn), mm	1100 / 1020
Maximum speed, km/h	120
Diesel engine power, kW	2000
Power at the wheel rim, kW	1600
Starting tractive effort, kN	450
Electric braking power, kW	1600 kW / with self-loading capability
Ambient temperature range, °C	from -34 up to +40

Locomotive SIEMENS “ER20CF” has two bogies, each of which has three driven wheelsets. The numeration of wheelsets starts from the beginning of the locomotive. During this study, weariness of the vehicle wheelset wheel's rolling surface has been periodically meas-

ured. Changes of wheelset wheel wear measured are different mileages are shown in Figure 3.

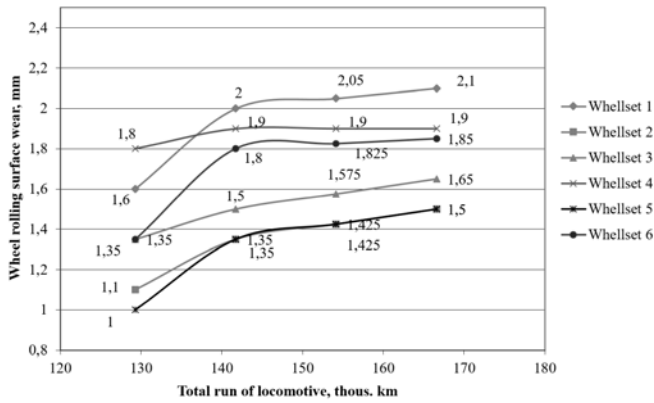


Fig. 3. Wheel rolling surface wear of different wheelset wheels

The wear of each locomotive wheel is measured three times (in different locations, every 120°) and the average for both wheelset wheels is calculated. From Figure 3 it can be seen that 1<sup>st</sup>, 3<sup>rd</sup>, 4<sup>th</sup>, 6<sup>th</sup> wheelset wheel rolling surfaces are getting the most weariness and the middle wheelsets of the bogie 2<sup>nd</sup>, 5<sup>th</sup> - wear the least. This happens, because the wheelsets of triaxle bogies, which are located on the edges (1<sup>st</sup>, 3<sup>rd</sup>, 4<sup>th</sup> and 6<sup>th</sup> wheelsets) get the most load, when rolling into track curves, railway switches and when braking.

#### 4. Applying the sharpe ratio to forecast metal fatigue

To use the Sharpe ratio for assessment of wear unevenness on locomotive wheel rolling surface, the following measures for evaluation of wear are taken into account instead of normal structure of the formula (1):

$$SR_i^w = \frac{W_i - \bar{W}}{\sigma_w}; \quad (2)$$

where:  $SR_i^w$  – Sharpe ratio, used to evaluate the intensity of wear on wheel of  $i$ -th wheelset;  $W_i$  – average of wear on wheel of  $i$ -th wheelset, mm;  $\bar{W}$  – average wear of locomotive wheels, mm;  $\sigma_w$  – average standard deviation of wear on locomotive wheel rolling surface.

After evaluation of wheel wear intensity data of four Siemens "ER20CF" locomotives obtained over mileage of 165 thousand kilometres and by using the same data, which has been used in Figure 3, the Sharpe ratio distribution on wheelset diagram has been created

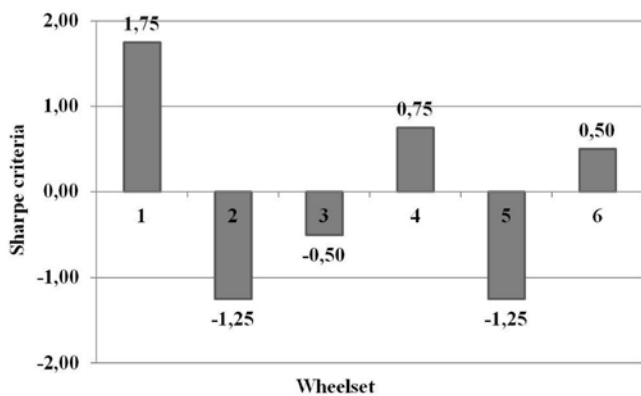


Fig. 4. Evaluation of weariness on the wheel rolling surface using the Sharpe ratio

(the wheelsets are numbered from the front of locomotive). The diagram is shown in Figure 4.

Results of analysis, obtained by using different methods, but using the same data, are provided in Figure 3 and Figure 4. Results of regression analysis presented in Figure 3 reflect the general tendencies of weariness of locomotive wheel rolling surface. The differences in the intensity of wear on the locomotive wheels are shown in Figure 4, in 165 thousand kilometres within the mileage range, which means that Sharpe ratio highlights the level of wear on the wheel rolling surface. Sharpe ratio value that is greater than 1.0 or less than -1.0 indicates that the difference between the average wear  $W_i$  on wheel rolling surface of  $i$ -th wheelset and average wear  $\bar{W}$  on locomotive wheels, exceeds the standard deviation  $\sigma_w$ . The diagrams in Figure 3 show that the wear on the surface of the locomotive wheels is not equal throughout the mileage interval. It is noteworthy that when the mileage is 140 thousand kilometres this intensity lowers. This means that the intensity of the wear of the different axle wheels should be compared in different mileage intervals. Based on data in Figure 3, it is convenient to distinguish these mileage intervals: 0–130, 130–140, 140–150 and 150–160 thousand kilometres. Sharpe ratio for these mileage intervals is shown in Figure 5. The end of interval is shown

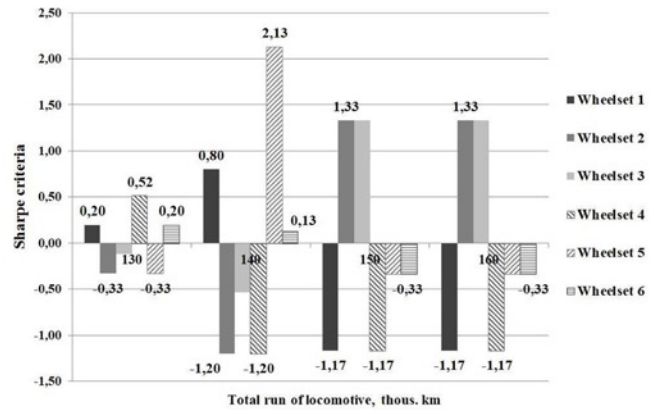


Fig. 5. Assessment of difference in wear of wheel rolling surface in mileage interval, by using Sharpe ratio

on abscissa axis for example: 130 means mileage interval from 0 to 130 thousand kilometres.

After looking at data in Figure 3, it was expected that the indicators of wheel wear may change after a mileage of 130 thousand kilometres. From the data of Figure 5, it is necessary to realize that the changes of distribution of wear intensity between different wheelset wheels is as important as changes in average wear intensity. Differences in wear, which are significantly (up to 2 times) greater than the average deviation of wear intensity at the mileage interval, emerge after 130 thousand kilometres of mileage. This is indicated by the value of the Sharpe ratio in Figure 5. The increased values of Sharpe ratio determine the mileage in which metal fatigue in rolling surface begins to appear.

From the described facts, it is reasonable to raise the hypothesis that by the values of Sharpe ratio it is possible to determine the possible start of metal fatigue in wheel rolling surface. The intensity on wheel wear, the size of the wear and the regularity of the wear dependence on the mileage may vary, depending on the exploitation conditions of vehicle. The wear of wheels (the size of wheel wear per unit of mileage) is determined by the railway track curves, pulled train weight, the vibrations and accelerations of the vehicle elements, the braking intensity and the climatic conditions. The authors of the study suggest that regardless of the impact of exploitation on locomotive wheel parameters, it is possible to determine the

start of metal fatigue on the rolling surface of a locomotive based on the Sharpe ratio values.

## 5. Conclusions

1. A large number of scientific publications that examine the interaction between the railway vehicle wheel and rail do not eliminate the large gap between theoretical research and the objectives of practical research. It is necessary to more effectively apply results of theoretical research in order to solve relevant exploitation problems of railway vehicle, such as forecasting the metal fatigue on railway vehicle wheels by using the simplified method.
2. Problem of vehicle wheel flange wear is significantly reduced by using vehicle wheel flange lubrication systems, compared to the problem of wheel rolling surface wear and metal fatigue. Currently, one of the most pressing problems is the metal fatigue on the rolling surface of the wheel and its timely detection.
3. It is easy to calculate the uneven wheel wear on different wheels by using the dimensions (wear data) of railway vehicle wheel rolling surface during exploitation. According to the critical values of this dimension, the initial formation of metal fatigue on rolling surface of the wheel can be determined. Thus, by the values of Sharpe ratio it is possible to determine the possible start of metal fatigue in locomotive wheel rolling surface.
4. The probability of fatigue on wheel rolling surface appears when deviations in wear intensity of wheel rolling surface are significantly larger or smaller than average deviation of wear intensity of bogie wheelset wheels.
5. The results of experimental research carried out by the authors show that the signs of primary metal fatigue on the rolling wheel surface appear when the values of the Sharpe ratio calculated for the bogie wheelset wheels are greater than 1.0 or less than minus 1.0.

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