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MULTI-CRITERIA OPTIMISATION OF MULTI-STAGE POSITIONAL GAME OF VESSELS

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ABSTRACT

The paper presents a mathematical model of a positional game of the safe control of a vessel in collision situations at sea, containing a description of control, state variables and state constraints as well as sets of acceptable ship strategies, as a multi-criteria optimisation task. The three possible tasks of multi-criteria optimisation were formulated in the form of non-cooperative and cooperative multi-stage positional games as well as optimal non-game controls. The multi-criteria control algorithms corresponding to these tasks were subjected to computer simulation in Matlab/Simulink software based on the example of the real navigational situation of the passing of one's own vessel with eighteen objects encountered in the North Sea.

Keywords: maritime transport, optimisation, game theory, ship control, computer simulation

INTRODUCTION

The tasks of controlling the optimal transport and logistics processes can be divided into three groups, for the first of which the cost of the process is an unambiguous function of control, for the second the cost depends on the control method and on some accidental event with a known statistical description, and for the third the cost is determined by the choice of the control method and a certain indefinite factor. The third group of optimal control tasks concerns transport and logistic game processes, whose synthesis is carried out with the use of game theory methods [1]. The reconstructing nature of marine transport and logistics processes results from the participation of many control objects, imperfections of the COLREGs maritime route regulations, the high impact of hydro-meteorological conditions and the navigator's subjectivity in making manoeuvring decisions [2, 3].

Game theory is a branch of modern mathematics that covers the theory of conflict situations and the construction and analysis of these models. The conflict may be military, social, or economic, with the influence of the natural, in the implementation of the control process during interferences by disturbances or other control objects [4].

The game in the concept of control theory is a process consisting of several transport objects remaining in a conflict situation that results from excessive proximity and collision risk, or a process with undefined disturbances or without full information.

The players are transport objects that participate with their strategies in a conflict situation. The strategy is a set of player control rules that cannot change an opponent's or nature's actions. Strategies can be pure, like elements of a set of admissible strategies, or mixed, like a probability distribution on a set of pure strategies. The game ends with a pay-out, which is the result of the game in the form of winning or losing or in the form of a probability of transport or logistics [5, 6].

The largest class of games that can be used in the game control of technical processes, and among them the control of maritime transport and logistics processes, represents fairly complex dynamic differential games [7, 8].

In practice, for the synthesis of game control algorithms in real-time objects, the differential game is simplified and reduced to a matrix or positional kinematic games, and the object's dynamics are taken into account by the advance time of the manoeuvre. The basic game control systems are positioning systems for the positional control of objects, and thus feedback systems representing position games, such as the safe steering of a vessel in collision situations at sea [8].

The aim of the paper is to present a new, detailed mathematical model of a positional game of many objects and the synthesis of a multi-stage optimisation algorithm for a multi-criteria positioning game, taking into account the degree of cooperation of objects [9, 10].

MATHEMATICAL MODEL OF THE POSITIONAL GAME

The basis for the formulation of the positional game is the dependence of the own vessel strategy on the positions of the objects encountered at the current stage of movement. In this way, possible changes in the course and speed of the objects encountered during the control implementation are taken into account in the process model [11] (Fig. 1).

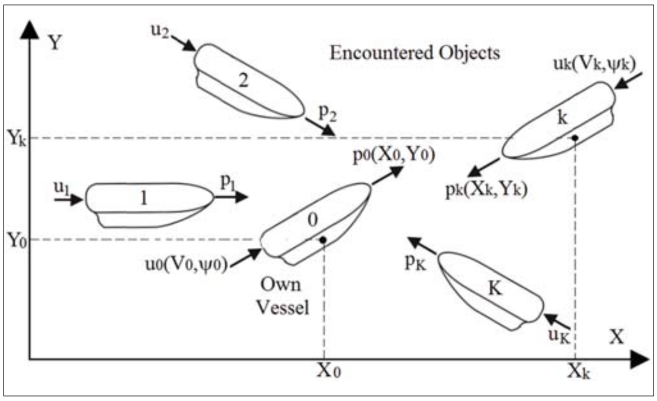


Fig. 1. The positional game of the own vessel – located in position $p_0(X_0, Y_0)$ and control u_0 by changing speed V_0 and ψ_0 course, with the encountered k object – located in the position $p_k(X_k, Y_k)$ and control u_k by changing speed V_k and the course ψ_k, k = 1, 2, …, K

THE CONTROL AND STATE VARIABLES

The process of preventing collisions of ships is controlled by changes in the ψ_0 course and V_0 speed of the own vessel, as well as the ψ_k courses and V_k speeds of the $k = 1, 2, …, K$ objects encountered.

The state of the process is determined by the own vessel position p_0 and the $k = 1, 2, …, K$ positions p_k of the encountered objects. The source of information on the state of the process is the on-board ARPA anti-collision system, which also allows assessment of the distance D_k and bearing N_k to the k object encountered and the collision risk parameters in the form of the DCPA distance of the closest point of approach and the TCPA time to the closest point of approach.

Process control constraints take into account the dynamic properties of the objects and the condition of maintaining a safe passing distance of the D_s objects and the recommendation of the right of the COLREGs sea path.

THE SETS OF ACCEPTABLE STRATEGIES

Steering to avoid collisions with objects is the player's safe strategy. In practice, there are many possible strategies to avoid collisions, which form sets of acceptable strategies, from which the best solution is selected, i.e. the optimal one according to the previously accepted criterion of optimality. As a criterion, one can consider the minimum of road losses for safe passing of objects, i.e. at a distance not less than the previously assumed safe value D_s [12, 13].

The method of the geometrical determination of sets of acceptable strategies of the encountered k-th object, which is to pass the own vessel at a distance not less than value D_s, is shown in Fig. 2.

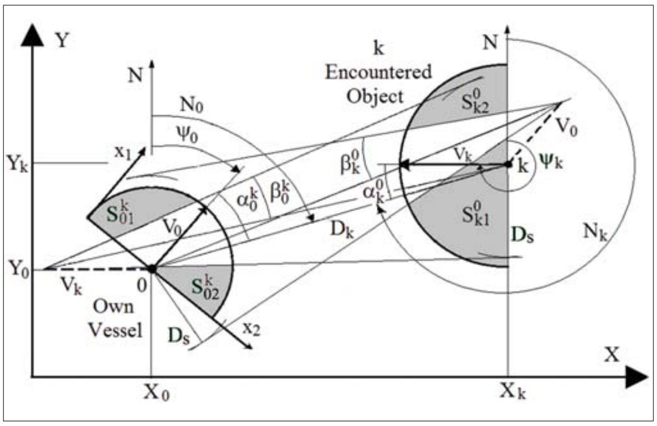


Fig. 2. The method of determining the acceptable strategy sets of the own ship $S_0^k = S_{01}^k \cup S_{012}^k$ and the k-th encountered vessel $S_k^0 = S_{k1}^0 \cup S_{k2}^0$, k = 1, 2, …, K

The set of acceptable own vessel strategies S_0^k, which consists of the subsets S_{0ps}^k and S_{0ss}^k, is described by the following inequalities:

$$m_0^k \; V_{0x1} + n_0^k \; V_{0x2} \le o_0^k \qquad V_{0x1}^2 + V_{0x2}^2 \le V_0^2 \tag{1}$$

$$\vec{u}_0 = \vec{V}_0(V_{0x1'}, V_{0x2})$$

$$m_0^k = -\, z_0^k \cos(\alpha_0^k + z_0^k \beta_0^k)$$

$$n_0^k = z_0^k \sin(\alpha_0^k + z_0^k \beta_0^k) \tag{2}$$

$$o_0^k = -\, z_0^k \, [V_k \sin(\alpha_0^k + z_0^k \beta_0^k) + V_0 \cos(\alpha_0^k + z_0^k \beta_0^k)]$$

$$z_0^k = \begin{cases} -1 \;\; for \; S_{01}^k & (PORT\ SIDE) \\ +1 \;\; for \; S_{02}^k & (STARBOARD\ SIDE) \end{cases}$$

where V_{0x1}, V_{0x2} are components of the V_0 velocity of the own vessel in the coordinate system (x_1, x_2).

Similarly, a set of acceptable strategies of the k-th object encountered against the own vessel is determined according to

$$
\begin{aligned}
m_k^0 \; V_{kx1} + n_k^0 \; V_{kx2} &\leq o_k^0 \\
V_{kx1}^2 + V_{kx2}^2 &\leq V_k^2
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
&\vec{u}_k = \vec{V}_k(V_{kx1'}, V_{kx2}) \\
&m_k^0 = -\, z_k^0 \cos(\alpha_k^0 + z_k^0 \beta_k^0) \\
&n_k^0 = z_k^0 \sin(\alpha_k^0 + z_k^0 \beta_k^0) \\
&o_k^0 = -\, z_k^0 \; [V_k \sin(\alpha_k^0 + z_k^0 \beta_k^0) + V_0 \cos(\alpha_k^0 + z_k^0 \beta_k^0)] \\
&z_k^0 = \begin{cases} -\,1 \;\; for\; S_{k1}^0 & (PORT\ SIDE) \\ +\,1 \;\; for\; S_{k2}^0 & (STARBOARD\ SIDE) \end{cases}
\end{aligned}
\tag{4}
$$

where V_{kx1}, V_{kx2} are components of the V_k velocity of the k-th encountered object in the coordinate system (x_1, x_2).

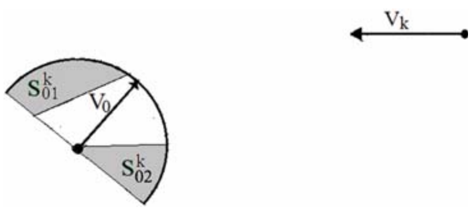
The values of the logic functions z_0^K and z_k^0 are determined on the basis of a semantic interpretation of the legal rules COLREGs of manoeuvring ships in collision situations. The safe and optimal manoeuvre of the own vessel is determined from the following set of combined acceptable strategies for all encountered objects [14, 15]:

$$
S_0 = \bigcap_{k=1}^{K} S_0^k \qquad k = 1, 2, \ldots, K \tag{5}
$$

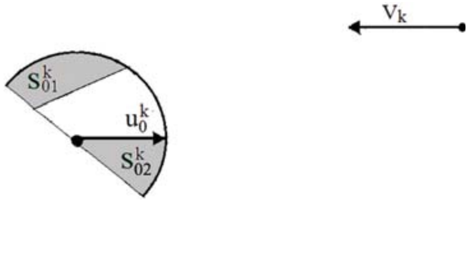
MULTI-CRITERIA OPTIMISATION OF THE POSITIONAL GAME

Synthesis of the own ship's optimal control u_0* is achieved by determining successively:

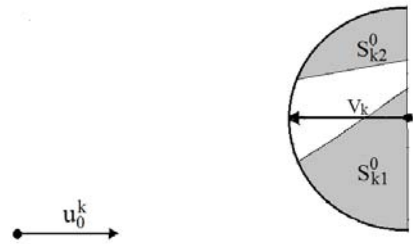
1. the $S_0^k\{S_{01}^k, S_{02}^k\}$ set of the own ship's acceptable strategies with respect to each of the k encountered vessels:



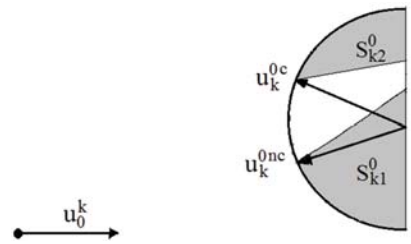
2. the own ship's control u_0^k, for each encountered vessel k, providing the shortest path L_0 to the point of return of the set motion trajectory – min $L_0 = L_0$*:



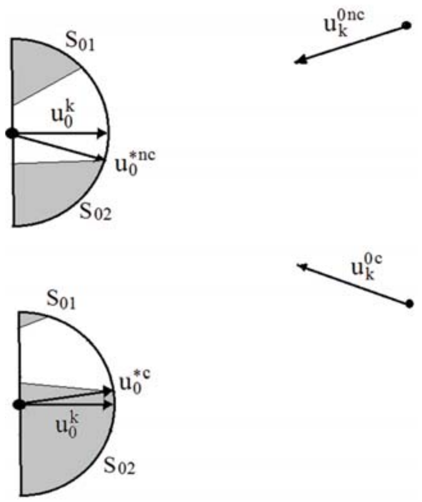
3. the $S_k^0\{S_{k1}^0, S_{k2}^0\}$ set of each encountered vessel's acceptable strategies with respect to the own ship:



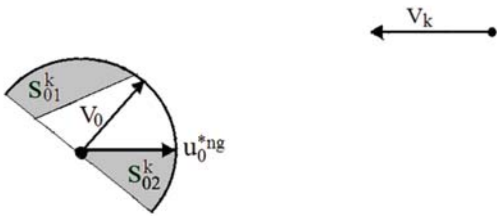
4. the k encountered vessel's control u_k^0, for the own ship's control u_0^k, from the set S_{k1}^0 or set S_{k2}^0, providing the longest path L_0 of the own ship to the point of return of the set motion trajectory – max $L_0 = L_0^{*nc}$ (u_k^{0nc} with S_{k1}^0 subset for a non-cooperative game) or the shortest path L_0 of the own ship to the point of return of the set motion trajectory – min $L_0 = L_0^{*c}$ (u_k^{0c} with S_{k2}^0 subset for a cooperative game):



5. the $S_0\{S_{01}, S_{02}\}$ set of acceptable own ship's strategies, in relation to all K encountered vessels,
6. the own ship's optimal control u_0^*, in relation to all K encountered vessels, providing the shortest path L_0 to the point of return of the set motion trajectory – min $L_0 = L_0^*$, for a non-cooperative game as u_0^{*nc} and for a cooperative game as u_0^{*c}:



7. the own ship's optimal control u_0^{*ng}, in relation to all K encountered vessels, providing the shortest path L_0 to the point of return of the set motion trajectory – min $L_0 = L_0^*$, for non-game control, i.e. the assumption that the encountered vessels move at a constant rate and speed:



MIN-MAX-MIN CRITERION OF NON-COOPERATIVE GAME

The algorithm of the multi-stage non-cooperative positional game *MPG_nc* uses the *min-max-min* form of multi-criteria optimisation of the own ship's way while safely passing the encountered vessels, using the linear programming method three times at each stage:

$$I^*_{nc} = \min_{u_0^* \in S_0 = \bigcap_{k=1}^{K} S_0^k} \left\{ \max_{u_k^0 \in S_k^0} \min_{u_0^k \in S_0^k} L[p_0(t), L_0] \right\} = L_0^{*nc}, \quad k = 1,2, \ldots, K \tag{6}$$

L_0 is the distance of the own ship to the nearest point of return on the reference cruise route.

First, the control of the own ship is determined to ensure the shortest trajectory of the change, i.e. the smallest loss of the road (*min* condition) for non-cooperative control of every vessel encountered, contributing to the largest extension of the own ship's trajectory (*max* condition). At the end, from the acceptable set of the own ship's control to the particular *k* encountered vessel, the own ship's control is selected in relation to all *K* encountered vessels, ensuring the smallest loss of the road (condition *min*).

According to the three optimisation conditions (*min-max-min*), the linear programming method is used to solve the game, obtaining the optimal values of the course and the speed of the own ship. The smallest road losses are achieved for the maximum projection of the own ship's speed vector on the direction of the reference course. Optimal control is calculated many times at each discrete stage of motion using the Simplex method to solve the triple linear programming problem for variables in the form of components of the own ship's speed vector [16, 17].

MIN-MIN-MIN CRITERION OF COOPERATIVE GAME

The algorithm of the multi-stage cooperative positional game *MPG_c* uses the *min-min-min* form of multi-criteria optimisation of the own ship's way while safely passing the encountered vessels, using the linear programming method three times at each stage:

$$I^*_{c} = \min_{u_0^* \in S_0 = \bigcap_{k=1}^{K} S_0^k} \left\{ \min_{u_k^0 \in S_k^0} \min_{u_0^k \in S_0^k} L[p_0(t), L_0] \right\} = L_0^{*c}, \quad k = 1,2, \ldots, K \tag{7}$$

The difference with the *MPG_nc* algorithm results from the use of cooperative action between ships in order to avoid collision by all the encountered vessels and to replace the second *max* condition by the *min* condition.

MIN CRITERION OF NON-GAME CONTROL

The algorithm of multi-stage non-game control *MC_ng* uses the *min* form of one-criterion optimisation of the own ship's way while safely passing the encountered vessels, using the linear programming method once at each stage:

$$I^*_{ng} = \min_{u_0^* \in S_0 = \bigcap_{k=1}^{K} S_0^k} \left\{ L[p_0(t), L_0] \right\} = L_0^{*ng}, \quad k = 1,2, \ldots, K \tag{8}$$

The selection of the own ship's optimal trajectory according to criteria (6), (7) and (8) boils down to determining its course and speeds so as to ensure the smallest loss of the path for safe passing of the encountered vessels, which is not less than the assumed value of safe distance for passing ships D_s, taking into account the dynamics of the ship in the form of the time to overtake manoeuvre. The smallest road losses are achieved for the maximum projection of the own ship's speed vector on the direction of the reference course. The object's dynamics are taken into account by the advance time of the manoeuvre. The time of advance of the manoeuvre consists of the time of advance of the change of course and the time of advance of the change of the own ship's speed.

The *MPG_nc*, *MPG_c* and *MC_ng* algorithms for determining a safe trajectory for the own ship in a collision situation were developed using the *lp* – linear programming function of the Matlab/Simulink Optimization Toolbox software.

VISUALISATION OF ALGORITHMS

The method of entering the initial data for calculations describing the navigational situation is shown in Fig. 3, and the form of the results of calculations of the own ship's safe trajectory is illustrated in Fig. 4.

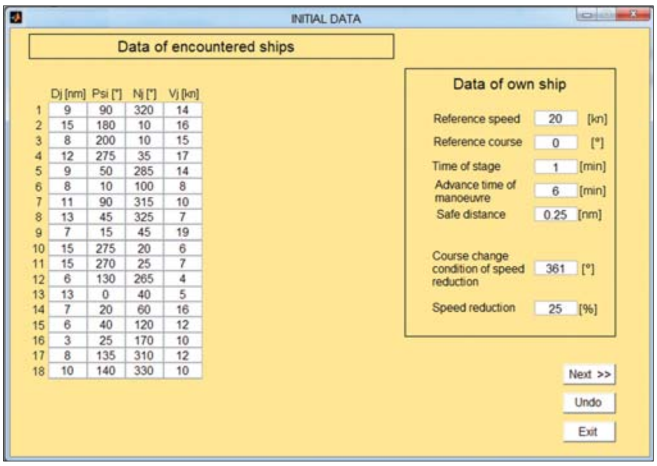


Fig. 3. Algorithm window with initial data of the navigational situation

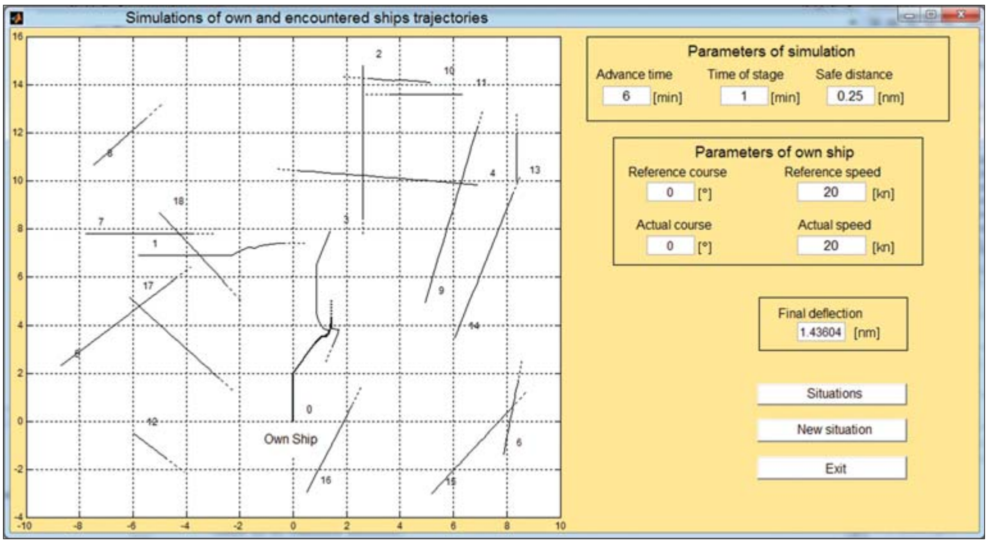


Fig. 4. The algorithm window with the results of calculations of the own ship's safe trajectory

COMPUTER SIMULATION

The own ship's safe trajectories in the situation of 18 ships in the Kattegat Strait, in conditions of (rv) – restricted visibility at sea at D_s = 2.0 nm and (gv) – good visibility at sea at D_s = 0.5 nm, determined according to the algorithms of multi-criteria optimisation: *MPG_nc*, *MPG_c* and *MC_ng*, are shown in Figs. 5, 6 and 7.

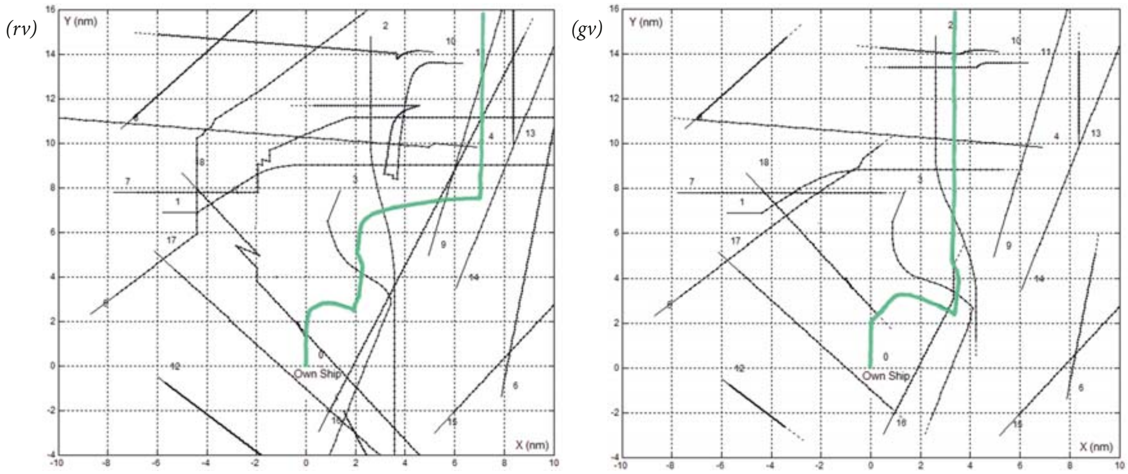


Fig. 5. Safe trajectory of the own ship in a non-cooperative positional game

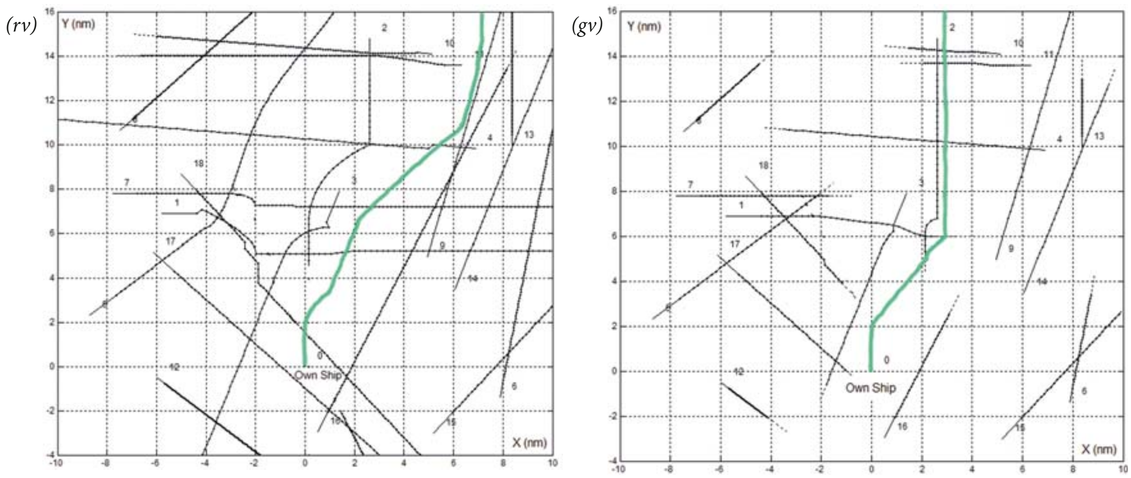


Fig. 6. Safe trajectory of the own ship in a cooperative positional game

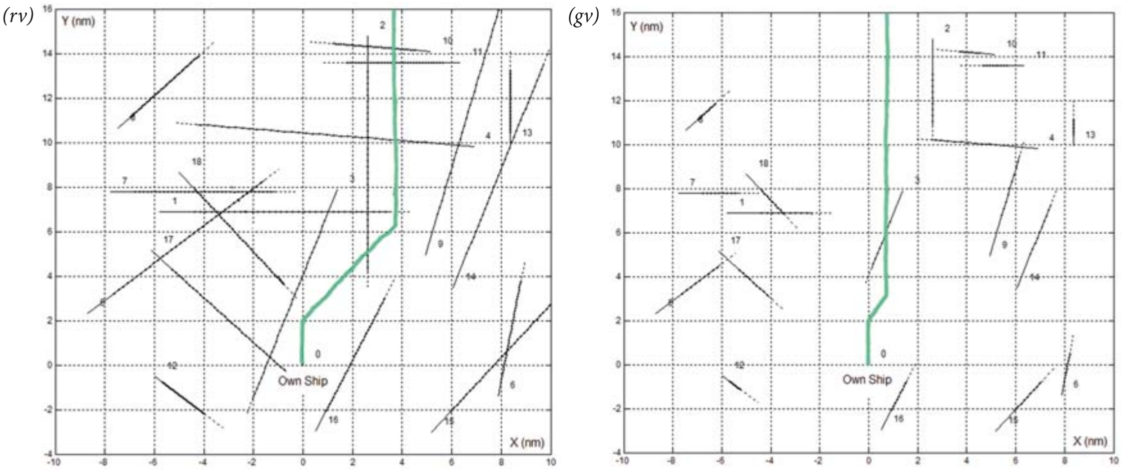


Fig. 7. Safe trajectory of the own ship during non-game control

CONCLUSIONS

The formulation of a mathematical model of the process of safely guiding a ship's movement while passing more ships as a positional game model makes it possible to take into account the indeterminacy of the navigational situation caused by the imperfection of the law of the sea route and the subjectivity of the navigator making the decision to avoid collision.

A multi-criteria approach to the task of safe control optimisation allows the development of appropriate non-cooperative, cooperative, and non-game control algorithms. The safe own ship's trajectories obtained differ primarily in the value of the final deviation from the reference trajectory of the movement.

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