JAISCR, 2021, Vol. 11, No. 3, pp. 217 – 227
10.2478/jaiscr-2021-0013

A NEW APPROACH TO DETECTION OF CHANGES IN MULTIDIMENSIONAL PATTERNS - PART II

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Submitted: 23rd August 2020; Accepted: 12th April 2021

Abstract

In the paper we develop an algorithm based on the Parzen kernel estimate for detection of sudden changes in 3-dimensional shapes which happen along the edge curves. Such problems commonly arise in various areas of computer vision, e.g., in edge detection, bioinformatics and processing of satellite imagery. In many engineering problems abrupt change detection may help in fault protection e.g. the jump detection in functions describing the static and dynamic properties of the objects in mechanical systems. We developed an algorithm for detecting abrupt changes which is nonparametric in nature and utilizes Parzen regression estimates of multivariate functions and their derivatives. In tests we apply this method, particularly but not exclusively, to the functions of two variables.

Keywords: edge curve detection, regression function, nonparametric estimation

1 Introduction

Sudden changes are not common in nature but once they happen they may indicate the onset of important events with far reaching and often catastrophic consequences, e.g., earthquakes, tsunami waves, heart attacks, stock market crashes, etc. In engineering and signal processing it is important to predict and detect sudden changes. From the mathematical standpoint, change detection problem is equivalent to the detection of discontinuity. Another question is how to properly qualify or classify the observed change. In, e.g. [9] one may find simple classification of basic change types:

– anomalies (or glitches) - accidental, often single, outlier aberrations or errors. Typically they are less important and can be ignored and easily removed by filtering or correction;

– trends, drifts and gradual changes that cannot be easily observed. They usually require long term observations and are rarely analyzed in the current literature;

– narrow, steep or abrupt changes, called edges.

They represent significant aberrations or deviations from the steady state observed thus far. They are usually important and require the proper attention (e.g. in medicine, seismology, weather forecasting, stock market, network security and others);
The problem of change detection in multidimensional space typically requires discovering the curves defined by multidimensional functions. This is more complex task, which requires significant computer resources like memory, processor power, and efficient (and often parallel) computations. In the article we develop a novel algorithm for detection of discontinuities of multidimensional functions based on Parzen kernel estimation of functions and their derivatives. We describe in detail how the proposed algorithm can be used to recovery of the edge curves on 3-dimensional surfaces.

2 Brief review of edge detection research

Edge detection term commonly refers to one-dimensional case of detection of an abrupt, narrow or steep change, i.e., when function value suddenly changes resulting in jump discontinuity at the jump point in the plot of the function. The problem of determining where or when the change occurred is equivalent to finding this jump point. In case of multi-dimensional functions the jump edge becomes a spatial curve. We can either estimate this curve (or its scatter plot) or its projection on a subspace.

There are known several solutions for narrow changes detection problem. For a survey of a plethora of edge detection techniques in computer vision we refer the reader to, e.g., [23, 4, 46]. Next we discuss only the most common ones.
First-order methods based on gradients computations include Sobel, Prewitt, Robert's [37] and Canny [6] algorithms. Another approach is based on detection of zero-crossing of the second-order derivative of the image smoothed by Laplacian or Gaussian filtering [36]. Note that, in case of digital images the design points typically form uniform grid. This condition is difficult to fulfill in some other applications. It is not easy to generalize such methods to situations with more general design points [40]. One can use neighboring observations and approximate the derivatives by computing their differences.

In case of time series data the most common approaches are based on density or distribution estimation [9].

Change detection can also be accomplished by means of more general criterion such as mean square error, Kolmogorov-Smirnov or Wilcoxon tests (see e.g. [7]). The main idea behind the statistical tests is to form a special function of the data called test statistic which is sensitive to significant changes in the data. If data changes lead to distributional changes they can be detected by tracking the distance between distributions and relative entropies commonly called the Kullback-Leibler distance [33]. These techniques are applicable for moderate data volumes, and they are often applicable off-line, however they are not applicable directly to data streams. A popular technique for detecting change in data streams is likelihood tracking in the adjacent sliding time windows. An interesting idea for detecting change in data streams has been proposed in [16]. The data in the neighboring time windows are clustered by k-means algorithm and discrete distributions for each cluster are estimated. The the Kullback-Leibler divergence between these distributions is tracked and sudden change is detected when the divergence approaches value 1.

A compromise semi-parametric approach falling between parametric Hoteling detector and non-parametric Kulback-Leibler divergence approach was also investigated in [16], where the authors used Mahalanobis distance and Gaussian mixtures in their log-likelihood detector.

An entirely new approach for edge detection has been presented in [41]. It is based on nonparametric regression estimation by radial basis functions (RBF). It uses the scalable radial kernels $K(\mathbf{x},\mathbf{y}) := \Phi(\mathbf{x}-\mathbf{y})$ where Φ is a radial function, defined on R^d. Since K is a symmetric kernel it can be replaced by $\Phi(r)$, where $r = \|\cdot\|$ is the distance norm and $\Phi : [0,\infty) \to \Re$ is a scalar function of a single non-negative real variable. In [41] were used the Wendland kernels of polynomials with even order of smoothness. Kernels on R^d can be scaled by the positive factor delta in the following way: $K(\mathbf{x},\mathbf{y},\delta) := K\left(\frac{\mathbf{x}}{\delta},\frac{\mathbf{y}}{\delta}\right),\ \forall \mathbf{x},\mathbf{y} \in R^d$. The RBF approach belongs to the kernel-type methods.

The shape parameter δ controlling interpolation accuracy and stability of the algorithm can be adjusted experimentally. The main idea behind the kernel approach is to interpolate the data with radial kernels and then estimate the coefficients of the interpolation using some cardinal functions. In Fourier series analysis a well-known phenomenon called Gibbs phenomenon happens when jump discontinuity of the approximated function gives rise to persistent high frequency oscillations in the Fourier series near the jump point. Fourier coefficients corresponding to these large frequencies take large absolute values and a suitable thresholding strategy could be used to detect the jump.

Kernel methods belong to the class of nonparametric approaches used when the functional form of underlying distributions or densities are unknown.

The approach based on regression analysis has evolved over the years to become a popular tool in classification and modelling of objects, forecasting of phenomena, and in machine learning, where neural networks, fuzzy sets and genetic algorithms (e.g. [44]) dominate the field. Edge detection techniques based on kernel regression estimation have also been studied by Qiu in [39, 40]. The methodology described in this article is applicable in diverse applications such as classification, computer vision, diagnostics, etc. (see e.g. [25, 26, 27, 34]. Numerous regression models applied to stream data are described in [12, 13, 14, 30].

In this paper, we introduce an original approach for the challenging problem of abrupt change detection in shapes defined by multidimensional functions, namely multi-dimensional edge detection problem. The algorithms are described in details and are applicable to two-dimensional functions. For the sake of better exposition of the proposed approach we restricted our considerations to three-dimensional space, but its extension to d-dimensional space seems obvious.

We adopt the nonparametric Parzen kernel method for estimation of unknown multidimensional functions and their derivatives which lead the novel algorithms for jump detection from noisy measurements.

The proposed methodology allows to compute edge spatial curves along which the sudden changes happen and to track function changes along the edges. In the course of estimating the location of abrupt changes we utilize kernel derivatives which are easy to compute and our algorithms are validated in simulation experiments. Finally, our approach scales up easily and does not require uniform distribution of sampling points.
The problem of edge localization has been thoroughly investigated over several decades. There is vast literature on the topic, e.g., [1, 2, 5, 11, 29, 31, 32, 35, 38, 47, 51, 53]. Preliminary study concerning application of Parzen kernels to detecting sudden changes has been presented in [20].

3 New approach for multidimensional edge curve detection

Due to the ignorance of precise mathematical equations numerous phenomena can be described by regression models. Also the abrupt change detection problem can be solved by the regression analysis approach for multidimensional case. Then the sequence of points of probable abrupt changes can take form of a curve along which regression function $R(\cdot)$ is discontinuous. High-dimensional space in obvious way is computationally very demanding and appropriate algorithms are complex.

The aim of our work is to derive a new, simple method of detecting step changes of multivariate functions based on kernel-type estimators for functions and their derivatives. We used multivariate Parzen kernel algorithms applied to a set of noisy measurements. Theoretical analysis of the Parzen method in so-called random design case and of other nonparametric regression estimates techniques, see, e.g., [28, 15].

We consider the model in the form:

$$y_i = R(\mathbf{x}_i) + \varepsilon_i, \quad i = 1, ..., n \tag{1}$$

where $\mathbf{x}_i$'s are deterministic input vectors, $\mathbf{x}_i \in R^d$, y_i is the scalar random output, and ε_i is an additive random noise with zero mean and bounded variance. Our problem at hand is the fixed-design regression problem, see, e.g., [15]. Note that mathematical form of function $R(\cdot)$ is entirely unknown so, the problem of finding its value at any point $\mathbf{x}$ is more difficult than in parametric estimation problem.

As the estimator of unknown function $R(\cdot)$ at point $\mathbf{x}$ we use the Parzen kernel based algorithm of the integral type

$$\hat{R}(\mathbf{x}) = {h_n}^{-d} \sum_{i=1}^{n} y_i \int_{D_i} \mathbf{K}\left(\frac{\|\mathbf{x}-\mathbf{u}\|}{h_n}\right) d\mathbf{u} \tag{2}$$

where $\|\mathbf{x}-\mathbf{u}\|$ denotes a norm or the distance function in d-dimensional space defined for points $\mathbf{x}$ and $\mathbf{u}$. So called smoothing parameter or bandwidth is denoted as h_n and it depends on the number of observations.
Next we partition the domain D of R into n disjunctive nonempty sets D_i and the measurements $\mathbf{x}_i$ are chosen from D_i, i.e.: $\mathbf{x}_i \in D_i$.

As an example consider one-dimensional case $D = [h_n, 1-h_n]$. Then $\cup D_i = [0,1]$, $D_i \cap D_j = \emptyset$ for $i \neq j$, the points x_i are chosen from D_i, i.e.: $x_i \in D_i$. In particular, for uniform partition $D_i = [h_n + (i-1)h, h_n + ih], i = 1, ..., n$, where $h = \frac{1-2h_n}{n}$. The set of input values $\mathbf{x}_i$ is selected during the experiment planning and data collection phase. This can be, for instance, stock data for a certain period of time, equally distant samples of a recorded ECG signal, or internet activity on a specific TCP/IP port on the server logs. A balanced representation of R functions in domain D should be provided.

When mathematical forms describing object are known the problem is to determine a set of unknown parameters. Examples are, for instance, linear regression and/or splines method and we say that is the parametric approach. The nonparametric is applicable when no assumption on the mathematical form of unknown function is imposed. But then the experimenter should try to select the measurement points in such a way as to represent the tested function as accurately as possible. This is possible if we follow the assumptions stated in the convergence theorems especially condition imposed on the maximum diameter of D_i. It has to converge to zero whenever the number of observations n tends to infinity (see e.g. [21, 17, 18]). Then we may presume that the essential properties of $R(\cdot)$ are in some sense embedded in the set of pairs $(\mathbf{x}_i, y_i)$. $K(\cdot)$ is the kernel function satisfying the conditions:

$$\begin{aligned} &K(t) = 0 \qquad \text{for} \quad t \notin (-\tau, \tau), \tau > 0 \\ &\int_{-\tau}^{\tau} K(t)dt = 1 \\ &\sup_t |K(t)| < \infty. \end{aligned} \tag{3}$$

Without losing the generality of considerations we choose the cosine kernel defined by:

$$K(t) = \begin{cases} \frac{\pi}{4}\cos\left(\frac{\pi}{2}t\right) & for \quad t \in (-1,1) \\ 0 & otherwise. \end{cases} \tag{4}$$

The derivatives of order k are estimated by using kernel derivatives. Note, that the cosine kernel (4) is k times continuously differentiable function. In one-dimensional case the k-th derivative estimate in the fixed point x can be estimated by the algorithm:

$$\hat{R}^{(k)}(x) = {h_n}^{-1} \sum_{i=1}^{n} y_i \int_{D_i} K^{(k)}\left(\frac{x-u}{h_n}\right) du. \tag{5}$$

In one-dimensional case for research of nonparametric procedures in similar applications to ours refer to [22, 19].

The main concept is to infer change dynamics of an unknown regression function by analyzing the form of the first derivative estimated from the sample. The rule is simple: the higher the first derivative the steeper the slope at a given point. Following this idea we propose the estimator of the derivatives described previously as a detector of abrupt changes. The choice of the parameter h_n plays a vital role in algorithm performance and interpretation of results. When we choose the bigger the h_n then the level of smoothing is stronger, but then the detection of the point becomes more difficult or impossible. Contrary, too small h_n causes high oscillations and consequently, the numerous sharp peaks

in the estimates. Then deciding which peaks correspond to real jump may be difficult. Good selection of bandwidth parameter h_n is often data dependent, see [52, 10, 3, 48, 50, 49]. An interesting approach for bandwidth selection based on FFT transform has been presented in [24]. Such algorithms are applicable also when the measurements are randomly noisy, thanks to their smoothing attribute. The choice of bandwidth parameter h_n is deciding of accuracy of the estimation particularly for derivatives. The choice of the kernel function is not discussed here. The numerical experiments and tests done by authors show that this is not a critical matter in detection of abrupt changes.
There are two useful types of kernels, in multidimensional estimation:

– radial kernel:

$$\mathbf{K}\left(\mathbf{u}^T\mathbf{u}\right) = c \cdot \sqrt{\mathbf{u}^T\mathbf{u}} \tag{6}$$

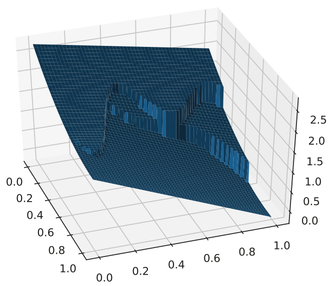
with, e.g., Euclidean norm, and

– product kernel:

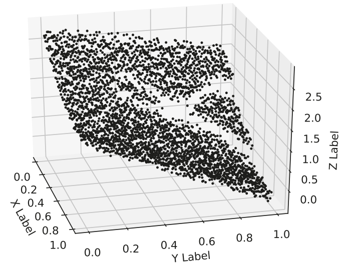
$$\mathbf{K}(\mathbf{x},\mathbf{u},h_n) = \prod_{p=1}^{d} K\left(\frac{|x_p - u_p|}{h_n}\right) = \mathbf{K}\left(\frac{||\mathbf{x}-\mathbf{u}||}{h_n}\right) \tag{7}$$

where $||\cdot||$ is L_1 norm.

The radial kernel is computationally more efficient. But product kernels are often chosen for their simplicity and computational efficiency, especially in the need of differentiation of functions. So we apply in our method the product kernel (7).



Original regression function defined by eq. (15)-(17)



Noisy measurements of the regression function

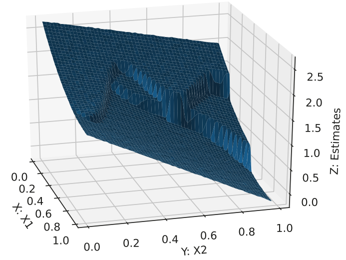


Figure 1. 3-D function, noisy measurements and regression function estimate

The k-th derivative with respect to x_j can be estimated by the formula:

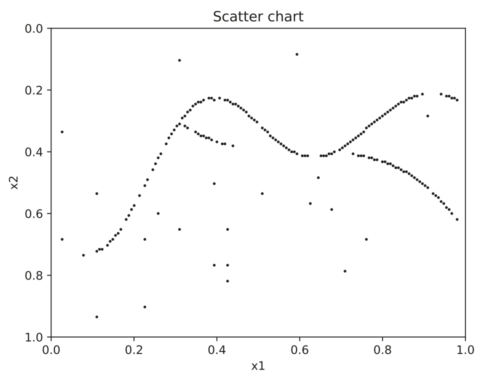
$$\hat{R}^{(k)}_{x_j}(\mathbf{x}) = h_n{}^{-d} \sum_{i=1}^{n} y_i \int_{D_i} \frac{\partial^k}{\partial x_j^k} \mathbf{K}\left(\frac{||\mathbf{x}-\mathbf{u}||}{h_n}\right) d\mathbf{u} \tag{8}$$

Because of product type kernel used, finding the derivative is easy and relies on differentiation of only the one function component in the kernel, with respect to specified coordinate. Next, the two-dimensional case will be described and analyzed in detail.
The model under consideration is defined by:

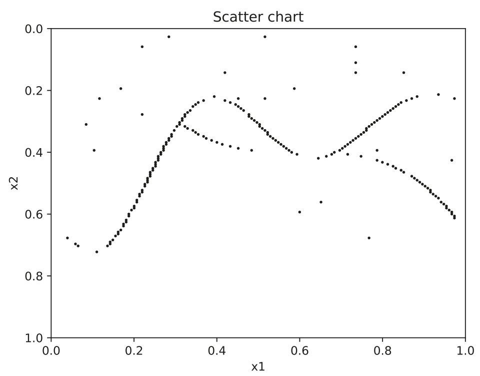
$$y_i = R([x_1, x_2]_i) + \varepsilon_i, \quad i = 1, \ldots, n \tag{9}$$

where $[x_1, x_2]_i$ is $2d$-vector of variables x_1 and x_2. The $2d$ Parzen kernel estimator of function R is defined by:

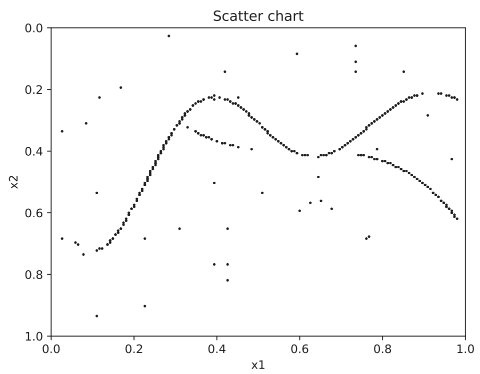
$$\hat{R}([x_1,x_2]) = h_n{}^{-2} \sum_{i=1}^{n} y_i \cdot \\ \cdot \int_{D_i} K\left(\frac{x_1-u_1}{h_n}\right) \cdot K\left(\frac{x_2-u_2}{h_n}\right) du_1 du_2 \tag{10}$$



Vertical projection 1. Derivative relative to x1 coordinate



Vertical projection 2. Derivative relative to x2 coordinate



Vertical projection - concatenated set of estimates

Figure 2. Vertical projections of edge curve. Estimation using noisy measurements

Applying the cosine kernel (4) we obtain the following estimation algorithm:

$$\hat{R}([x_1,x_2]) = \frac{\pi^2}{16}\cdot h_n{}^{-2}\sum_{i=1}^{n} y_i \cdot \int_{D_i} \cos\left(\frac{\pi(x_1-u_1)}{2h_n}\right) I_{(-1,1)}\left(\frac{x_1-u_1}{h_n}\right)\cdot \cos\left(\frac{\pi(x_2-u_2)}{2h_n}\right) I_{(-1,1)}\left(\frac{x_2-u_2}{h_n}\right) du_1 du_2. \qquad (11)$$

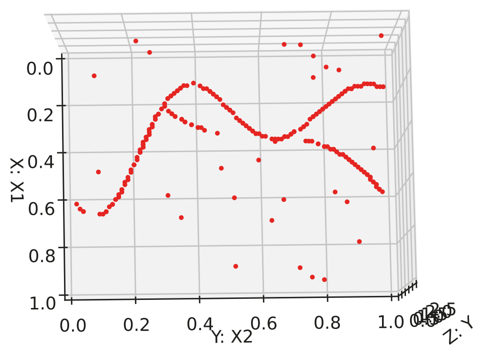
Basing on the fact $|u_k - x_k| \le 1$, whenever $x_k - h_n \le u_k \le x_k + h_n, k = 1,2$ and trigonometric identity $\sin(a) - \sin(b) = 2\sin((a-b)/2)\cos((a+b)/2)$ we can compute the integral in (11) analytically and obtain:

$$\hat{R}([x_1,x_2]) = \frac{\pi^2}{4}\sum_{i=1}^{n} y_i \cdot \prod_{k=1}^{2} \sin\left(\frac{\pi(x_{k,i}-x_{k,i+1})}{4h_n}\right)\cdot\cos\left(\frac{\pi(2x_k-x_{k,i}-x_{k,i+1})}{4h_n}\right). \qquad (12)$$

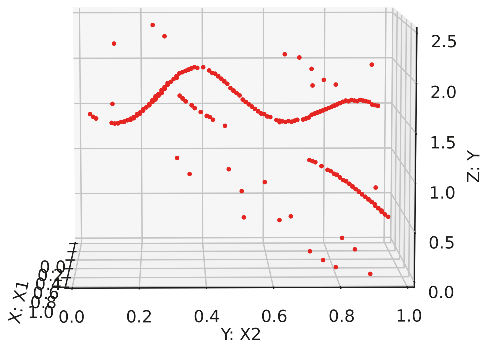
The estimators of the partial derivatives with respect to coordinates x_1 and x_2, respectively are given by:

$$\frac{\partial}{\partial x_1}\hat{R}([x_1,x_2]) = -\frac{\pi^3}{8h^n}\sum_{i=1}^{n} y_i \cdot \prod_{k=1}^{2} \sin\left(\frac{\pi(x_{k,i}-x_{k,i+1})}{4h_n}\right) \cdot \sin\left(\frac{\pi(2x_1-x_{k,i}-x_{k,i+1})}{4h_n}\right)\cdot\cos\left(\frac{\pi(2x_2-x_{k,i}-x_{k,i+1})}{4h_n}\right). \qquad (13)$$

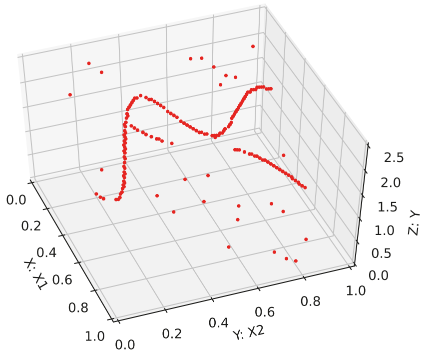
$$\frac{\partial}{\partial x_2}\hat{R}([x_1,x_2]) = -\frac{\pi^3}{8h^n}\sum_{i=1}^{n} y_i \cdot \prod_{k=1}^{2} \sin\left(\frac{\pi(x_{k,i}-x_{k,i+1})}{4h_n}\right) \cdot \cos\left(\frac{\pi(2x_1-x_{k,i}-x_{k,i+1})}{4h_n}\right)\cdot\sin\left(\frac{\pi(2x_2-x_{k,i}-x_{k,i+1})}{4h_n}\right). \qquad (14)$$



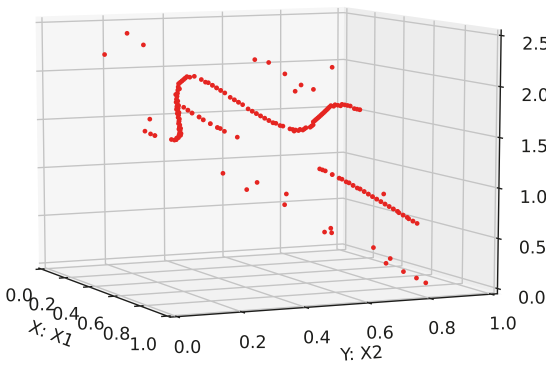
3-D edge curve - vertical view.



3-D edge curve - frontal view.



3-D edge curve - skew view 1.



3-D edge curve - skew view 2.

Figure 3. Edge curve viewed in 3-D scatter charts

The integrals in (11) and (12) could be easily calculated analytically. The experimenter chooses measurement points x_i from sub-regions D_i, i.e.: $x_i \in D_i$. A commonly used good choice is uniform grid of equidistant points.

4 Tests and simulations results

A series of tests was carried out using the function $R(\cdot)$ defined by:

$$R([x_1,x_2]) = \begin{cases} 1.5 - 0.75 \cdot x_1 + 1.25 \cdot (x_2 - 1)^2 \\ \quad \text{for } x_2 \leq L_1(x_1) \\ 1.5 - 1.25 \cdot x_1 + 1.25 \cdot (x_2 - 1)^2 \\ \quad \text{for } L_1(x_1) < x_2 \leq L_2(x_1) \\ 1.5 - 1.75 \cdot x_1 + 1.25 \cdot (x_2 - 1)^2 \\ \quad \text{for } x_2 > L_2(x_1) \end{cases} \tag{15}$$

where

$$L_1(x_1) = 0.2 + \frac{0.2625 + 0.225 \cdot \sin(12 \cdot x_1)}{(x_1 + 0.85)^2} \tag{16}$$

and

$$L_2(x_1) = 0.4 + 4.0 \cdot (x_1 - 0.6)^3 \tag{17}$$

Simulations were performed using artificially generated measurement pairs $\mathbf{x}, \mathbf{y}$ corrupted with random additive noise. Tested functions defined by (15) (16) (17) containing deep valleys and steep slopes is shown in the Figure 1. The goal is to establish the curve along edge of the fault. In Figure 1 one can see function without noise, next the noisy measurements and, in the second row, its nonparametric estimate (algorithm (10)). Figure 2 shows scatter plots of its partial derivatives obtained using algorithms (13) and (14), respectively, and concatenated set of estimates on the square $[0,1] \times [0,1]$. Figure 3 shows the scatter plots of estimated edge curve in $3d$-view. In the upper two rows we can see the vertical view analogous to the bottom row plot in Figure 2 and the horizontal direction view, the frontal view. In the next rows of Figure 3 two skew views can help to visualize the spatial shape of the edge curve.
The input contain the 300×300 set of measurements. They were corrupted with uniformly distributed random noise from the range $[-0.5, 0.5]$. The parameter h_n is deciding about the smoothing property of the algorithm, and its value was experimentally established as $h_n = 0.015$. Note, the larger h_n the larger the level of estimate flatness, which may lead to abrupt change detection failure. Otherwise, choice of too small h_n may cause narrow peaks in first derivative estimates leading to incorrect classification. Test program was written in python programming language. Automatic detection of the local maxima corresponding to function jumps of the first derivatives uses the $scipy.signal.find\_peaks$ function from the *SciPy* python library.

Conclusions and remarks

In this paper we have studied and solved the task of deciding whether the sudden and/or abrupt changes occurred in functions of two or more variables. We proposed a new algorithm based on the fixed-design nonparametric kernel regression estimation techniques. We developed the Parzen approach in multi-dimensional case, and our algorithm uses the estimation of the spatial derivatives of functions. The application to two-dimensional patterns is described in detail. The series of tests for abrupt function changes detection were performed on the sets of artificially generated measurements corrupted by the additive random noise. The validity of the proposed approach and its practical usefulness was verified in the simulation. From the simulation results one may conclude that the effectiveness of the proposed method improves when the magnitude of the jump increases. Presented methodology can be directly extended to the d-dimensional ($d > 2$) space and the hyper-curves-edge detection solution by applying the appropriate product-type kernels.

The newly proposed algorithm has been tested in the problem of detecting edge curves in two-dimensional patterns. The algorithm was validated in computer experiments. The future research will investigate possible further extensions of the proposed approach. Similar algorithms will be developed to solving the studied problem using the orthogonal series approach, see e.g., [42, 43].

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