

# **Application of Variance Analysis to Compare Characteristics of Various Types of Hard Coal**

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### Abstract

In Polish nomenclature many types and subtypes of coal can be found which differ between themselves by individual characteristics. However, it is often that is no easy to recognize them properly on the basis on, for example, chosen numerical data describing their features. In the paper, the variance analysis was used as the tool of comparing analysis for three chosen types of coal which were collected from three various hard coal mines located in Upper Silesia. There were coals of type 31, 34.2 and 35. Each of coals was first screened and then additionally divided into density fractions by means of zinc chloride aqueous solution. Such prepared material was then investigated because of several chosen features, like combustion heat, ash contents, sulfur contents, volatile parts contents and moisture. Together with mass it gave seven-dimensional vector describing each of chosen fractions for all three types of coals. Then, the full variance analysis was conducted with investigation of all assumptions required to its conduction. The results served to elaborate conclusions.

Keywords: hard coal, coal type, multidimensional statistical analysis, variance analysis, particle fractions

#### Introduction

The grained materials can be described by means of many features. Usually, the basic ones are particle size and its density, but for example in case of coal many other important ones can be found which influence on differentiation of coal types. Many papers in literature concern typology of coals, but rarely are supported with detailed statistical analysis. Recently, with development of information science, multidimensional statistical analyzes gained significant meaning. It is worthy to mention here multidimensional visualization methods which can be treated as modern analysis tools. Among such types of methods the observational tunnels method can be found [Jamróz and Niedoba, 2014; Niedoba 2013a], parallel axes method [Niedoba and Jamróz, 2013]], Kohonen maps [Jamróz and Niedoba, 2015a], relevance maps [Niedoba, 2015], Principal Component Method [Niedoba, 2014], multidimensional scaling [Jamróz, 2014a] or autoassociative neural networks [Jamróz, 2014b]. In most of cases these methods were used to identify type of coal on the basis of measuring data. For example, it was used to define coal usefulness to gasification process [Marciniak-Kowalska et al., 2014]. Comparison of efficiency of these methods can be found in [Jamróz and Niedoba, 2015b]. Furthermore, application of complex statistical analysis can be also found in [Brożek et al., 2015; Niedoba, 2013b; Niedoba and Jamróz, 2013; Surowiak, 2014; Surowiak

and Brożek, 2014a; 2014b; 2016]. In this paper the instrument was analysis of variance which assumptions can be found in [Krysicki et al., 2012; Tumidajski and Saramak, 2009]. In mineral processing it was used to evaluate flotation process [Agnew et al., 1995; Xiao and Vien, 2003] and recently also to recognize products of separation in fine coal jigs [Pięta, 2015].

The basic point of start in this paper is acceptance that grained material is characterized with multidimensional random variable  $W = [w_1, w_2, ..., w_n]$  $w_i$ ], where  $w_i$  (i = 1, ..., n) are researched material properties. For various types of coals the values of individual w, are significantly different. The purpose of the work is analysis of three types of coal properties (energetic coal, semi-coking coal and coking coal) and determination which of the investigated features allow to identify the type of coal. In papers [Jamróz and Niedoba, 2014; Marciniak-Kowalska et al., 2014] the identification of coal types by means of observational tunnels method was performed. On the basis of this analysis it was stated that three coal features, which were moisture, sulfur contents and volatile parts contents, were sufficient to identify the type of coal correctly.

### Experiment

In the paper, the comparison of properties for three types of coals was done by means of statis-

| Particle size | Туре 31 |                       | Туре 34.2    |                       | Туре 35 |        |
|---------------|---------|-----------------------|--------------|-----------------------|---------|--------|
| d [mm]        | anno)   | <i>s</i> <sup>2</sup> | atton (OTTA) | <i>s</i> <sup>2</sup> | anno)   | $s^2$  |
| 0.50-1.00     | 20.55   | 236.74                | 21.14        | 234.96                | 24.55   | 238.62 |
| 1.00-3.15     | 23.59   | 259.55                | 21.78        | 211.38                | 24.55   | 238.62 |
| 3.15-6.30     | 23.52   | 259.45                | 20.00        | 150.33                | 26.01   | 247.78 |
| 6.30-8.00     | 28.00   | 230.16                | 22.14        | 222.41                | 27.73   | 275.05 |
| 8.00-10.00    | 27.23   | 174.36                | 23.77        | 221.34                | 27.04   | 287.50 |
| 10.00-12.50   | 34.32   | 485.70                | 22.34        | 199.54                | 28.10   | 276.59 |
| 12.50-14.00   | 27.12   | 174.23                | 22.31        | 166.24                | 27.72   | 277.26 |
| 14.00-16.00   | 27.76   | 192.25                | 21.81        | 273.92                | 27.02   | 273.92 |
| 16.00-20.00   | 25.55   | 171.46                | 28.77        | 585.76                | 27.94   | 300.58 |

Tab. 1. Ash contents for various types of coal Tab. 1. Zawartość popiołu dla różnych typów wegli

Tab. 2. Results of Hartley's and Cochran's test for ash contents Tab. 2. Wyniki testów Hartleya i Cochrana dla zawartości popiołu

| <i>d</i> [mm] | H – Hartley's test | G – Cochran's test |
|---------------|--------------------|--------------------|
| 0.50–1.00     | 1.02               | 0.33               |
| 1.00-3.15     | 1.79               | 0.39               |
| 3.15-6.30     | 1.09               | 0.37               |
| 6.30-8.00     | 1.24               | 0.40               |
| 8.00-10.00    | 1.65               | 0.42               |
| 10.00-12.50   | 2.04               | 0.50               |
| 12.50-14.00   | 1.67               | 0.44               |
| 14.00-16.00   | 1.46               | 0.33               |
| 16.00-20.00   | 3.42               | 0.54               |

tical methods, such as analysis of variance and verification of hypotheses about mean value and variance. These coals were marked by numbers 31, 34.2 and 35, according to Polish nomenclature of coal types. To this purpose, coals were initially sieved into particle size fractions -1.00, -3.15,-6.30, -8.00, -10.00, -12.50, -14.00, -16.00 and -20.00 mm and then into density fractions by means of aqueous zinc chloride solution (1.30, 1.40, 1.50, 1.60, 1.70, 1.80 and 1.90 g/cm<sup>3</sup>). Each obtained size-density fraction was then investigated for such parameters as combustion heat, ash contents, sulfur contents, volatile parts contents, analytic moisture. All these features, together with mass of size-density fraction gave seven various features for each type of coal.

In the paper the assumption was made that for each type of coal, material being the part of the same particle–size fraction which is material of particle size d and density  $\rho$  is one measuring object for which ash contents, sulfur contents and moisture were measured. In this way the following projection was determined:

$$(d,\rho) \to (w_1, w_2, w_3) \tag{1}$$

where  $d_i$  is particle size in *i*<sup>th</sup> fraction,  $\rho_i - particle density in ith fraction, <math>w_{1-}$  ash contents,  $w_2 - sulfur$  contents,  $w_3 - moisture$ .

In such way the set of points  $(\rho j, w_1 j, w_2 j, w_3 j)$ , j = 1, ..., m was obtained.

#### Ash contents

As the first feature ash contents in coal was selected to the analysis. On the basis of conducted laboratory researches the results were obtained, which were presented in Table 1,

where:

$$\overline{w_{11}} = \frac{1}{n} \sum_{j=1}^{k} w_{11j}$$
 - is mean ash contents for coal,  
type 31 (2)

$$\frac{1}{w_{12}} = \frac{1}{n} \sum_{j=1}^{k} w_{12j} - \text{ is mean ash contents for coal,}$$
  
type 34.2 (3)

 $\overline{w_{13}} = \frac{1}{n} \sum_{j=1}^{k} w_{13j}$  - is mean ash contents for coal, type 35 (4)

 $s_i^2 = \frac{1}{n} \sum_{j=1}^k (w_{ij} - \overline{w_i})^2 \text{ for } i = 1, 2, 3 \text{ are variances for individual types of coals}$ (5)

To check if ash contents differs significantly for individual types of coals the verification of hy-

| <i>d</i> [mm] | F      |
|---------------|--------|
| 0.50-1.00     | 0.1182 |
| 1.00–3.15     | 0.0581 |
| 3.15-6.30     | 0.0423 |
| 6.30-8.00     | 0.2800 |
| 8.00-10.00    | 0.0990 |
| 10.00–12.50   | 0.6718 |
| 12.50–14.00   | 0.2562 |
| 14.00–16.00   | 0.2542 |
| 16.00–20.00   | 0.4122 |

#### Tab. 3. Results of F test for ash contents

Tab. 3. Wyniki testu F dla zawartości popiołu

Tab. 4. Sulfur contents for various types of coal Tab. 4. Zawartość siarki dla różnych typów węgli

| Particle size | Тур          | e 31                  | Туре         | 34.2                  | Тур  | e 35                  |
|---------------|--------------|-----------------------|--------------|-----------------------|------|-----------------------|
| <i>d</i> [mm] | atton (OTTA) | <i>s</i> <sup>2</sup> | atton (CONNA | <i>s</i> <sup>2</sup> | ന്ന  | <i>s</i> <sup>2</sup> |
| 0.50-1.00     | 1.18         | 0.15                  | 0.72         | 0.04                  | 0.60 | 0.01                  |
| 1.00-3.15     | 1.41         | 0.26                  | 0.62         | 0.06                  | 0.59 | 0.02                  |
| 3.15-6.30     | 1.23         | 0.21                  | 0.67         | 0.05                  | 0.60 | 0.01                  |
| 6.30-8.00     | 1.30         | 0.23                  | 0.63         | 0.04                  | 0.62 | 0.02                  |
| 8.00-10.00    | 0.65         | 0.08                  | 0.55         | 0.04                  | 0.94 | 0.03                  |
| 10.00-12.50   | 0.94         | 0.06                  | 0.38         | 0.01                  | 0.53 | 0.03                  |
| 12.50-14.00   | 1.20         | 0.27                  | 0.39         | 0.07                  | 0.62 | 0.15                  |
| 14.00-16.00   | 0.87         | 0.19                  | 0.72         | 0.26                  | 0.56 | 0.03                  |
| 16.00-20.00   | 1.04         | 0.27                  | 0.36         | 0.09                  | 0.79 | 0.13                  |

where  $\overline{w_{21}}$  are mean sulfur contents for individual types of (*i*=1, 2, 3) and  $s_i^2$  are variances

pothesis H<sub>0</sub>:  $\overline{w_{11}} = \overline{w_{12}} = \overline{w_{13}}$  (of equality of mean ash contents in individual types of coals) was performed for following fractions. To apply the analysis of variance, first the hypothesis of variance equality H<sub>0</sub>:  $s_1^2 = s_2^2 = s_3^2$  must be verified.

To this purpose, the Hartley's test H and Cochran's test G were applied, where:

$$H = \frac{\max(s_1^2, s_2^2, s_3^2)}{\min(s_1^2, s_2^2, s_3^2)}$$
(6)

$$G = \frac{\max(s_1^2, s_2^2, s_3^2)}{s_1^2 + s_2^2 + s_3^2}$$
(7)

The results were presented in Table 2.

For the significance level  $\alpha = 0.05$  the critical range for Hartley's test is range (6.94,  $+\infty$ ) and for Cochran's test the range (0.73,  $+\infty$ ). Because all results of conducted tests are outside of the critical range so the analysis of variance can be applied.

To verify the hypothesis of mean value the F– Snedecor test was applied [Dobosz, 2001; Krysicki et al., 2012; Tumidajski, 1997] of form:

$$F = \frac{(n-k)Q_G}{(n-k)Q_G}$$

 $F = \frac{1}{(k-1)Q_R} \text{ of Fisher-Snedecor distribution}$ with k-1, n-k of freedom degrees (8) where:  $Q_G = \sum_{i=1}^k (\overline{w_i} - \overline{w})^2 \cdot n_i$ 

$$Q_{R} = \sum_{i=1}^{k} \sum_{j=1}^{n_{i}} (w_{ij} - \overline{w_{i}})^{2}$$
where  $\overline{w} = \frac{1}{k} \overline{w_{i}}$ 
(10)

The results of *F* test were presented in Table 3. The critical range for significance level  $\alpha = 0.05$  is range (3.97,  $+\infty$ ). Because all values of *F* test are outside the critical range so the hypothesis of equality of mean ash contents for considered types of coals can be accepted for each fraction.

#### Sulfur contents

Next, the similar analysis was performer for sulfur contents in coal. Data was presented in Table 4.

Also in this case the hypotheses of equality of mean values H<sub>0</sub>:  $\overline{w_{21}} = \overline{w_{22}} = \overline{w_{23}}$  were checked. However, first the verification of equality of variances by means of Hartley's and Cochran's tests was done. The results of these tests were presented in Table 5.

Analyzing the results of tests is visible that starting from third fraction at least one test (by

(9)

| <i>d</i> [mm] | H – Hartley's test | G – Cochran's test |
|---------------|--------------------|--------------------|
| 0.50-1.00     | 9.35               | 0.75               |
| 1.00-3.15     | 14.77              | 0.76               |
| 3.15-6.30     | 8.33               | 0.64               |
| 6.30-8.00     | 11.95              | 0.70               |
| 8.00-10.00    | 2.69               | 0.54               |
| 10.00-12.50   | 6.44               | 0.62               |
| 12.50-14.00   | 3.61               | 0.55               |
| 14.00–16.00   | 7.64               | 0.54               |
| 16.00-20.00   | 3.04               | 0.55               |

Tab. 5. Results of Hartley's and Cochran's tests for sulfur contents Tab. 5. Wyniki testów Hartleya i Cochrana dla zawartości siarki

Tab. 6. Results of F test for sulfur contents (starting from third fraction)

Tab. 6. Wyniki testu F dla zawartości siarki (zaczynając od trzeciej klasy ziarnowej)

| <i>d</i> [mm] | F     |
|---------------|-------|
| 3.15-6.30     | 6.30  |
| 6.30-8.00     | 5.04  |
| 8.00–10.00    | 4.92  |
| 10.00–12.50   | 16.27 |
| 12.50–14.00   | 6.38  |
| 14.00–16.00   | 0.89  |
| 16.00–20.00   | 4.41  |

Tab. 7. Results of Cochran-Cox test for sulfur contents

| Tab. 7. Wyniki testu Cochrana-Coxa dla zawartości si | arki |
|--|------|
|--|------|

| <i>d</i> [mm] | H₀: ₩(((0) = ₩(((0) | H₀: ₩(((0)) = ₩(((0)) | $\mathbf{H}_{0}: \mathbf{W}((0) \neq \mathbf{W}((0))$ |
|---------------|---------------------|-----------------------|---|
| 0.50-1.00     | 2.60                | 3.40                  | 1.39  |
| 1.00-3.15     | 3.38                | 3.79                  | 0.26  |

significance test  $\alpha = 0.05$ ) allow to accept that variances for individual types of coals are equal and for these fractions the <u>*F*</u> test was performed, which results were presented in Table 6.

In all cases, except fraction (14.00, 16.00) the test result for significance level  $\alpha = 0.05$  is within the critical range, which is  $(3.97, +\infty)$ . That means that hypothesis about mean sulfur contents in individual fractions for various coal types should be rejected (except fraction (14.00, 16.00). Because for fractions (0.50–1.00) and (1.00, 3.15) is not possible to apply *F* test, the Cochran–Cox test was used instead [Dobosz, 2001; Krysicki et al., 2012; Tumidajski, 1997] of equality of two mean values. This test is conducted according to the formula

$$C = \frac{x_1 - x_2}{\sqrt{\frac{s_1^2}{n_1 - 1} + \frac{s_2^2}{n_2 - 1}}}$$
(11)

The hypotheses H<sub>0</sub>:  $\overline{w_{21}} = \overline{w_{22}}$ ; H<sub>0</sub>:  $\overline{w_{21}} = w_{23}$  and H<sub>0</sub>:  $\overline{w_{22}} = w_{23}$  were verified. For significance lev-

el  $\alpha = 0.05$  the critical range of C test is  $(-\infty, -1.94) \lor (1.94, +\infty)$ .

Results of Cochran–Cox test were presented in Table 7.

From the obtained results is possible to conclude that only hypothesis H<sub>0</sub>:  $w_{22} = w_{23}$  of equality of mean sulfur contents for coal, types 34.2 and 35 was verified correctly for both fractions.

#### Moisture

The next investigated coal feature is material's moisture, which results were given in Table 8.

As in previous cases, first the possibility of applying analysis of variance was verified by checking hypothesis of equality of variances by means of Hartley's and Cochran's tests. The results of both tests were presented in Table 9.

All values of test, apart from the fraction (12.50; 14.00) and (14.00; 16.00) were within the critical range of the test so is not possible to apply *F* test. However, this test was used for fractions (12.50; 14.00) and (14.00; 16.00)

| Particle size | Туре 31 |                       | Туре 34.2 |                       | Туре 35 |                       |
|---------------|---------|-----------------------|-----------|-----------------------|---------|-----------------------|
| <i>d</i> [mm] | anno)   | <i>s</i> <sup>2</sup> | ന്ന       | <i>s</i> <sup>2</sup> | MM (O)  | <i>s</i> <sup>2</sup> |
| 0.50-1.00     | 3.63    | 0.81                  | 0.68      | 0.01                  | 1.08    | 0.01                  |
| 1.00-3.15     | 3.02    | 0.61                  | 0.68      | 0.03                  | 1.02    | 0.001                 |
| 3.15-6.30     | 3.00    | 0.38                  | 0.99      | 0.03                  | 1.36    | 0.03                  |
| 6.30-8.00     | 3.00    | 0.22                  | 1.04      | 0.01                  | 1.22    | 0.02                  |
| 8.00-10.00    | 2.92    | 0.40                  | 1.16      | 0.02                  | 1.36    | 0.03                  |
| 10.00-12.50   | 2.44    | 0.90                  | 1.12      | 0.01                  | 1.38    | 0.01                  |
| 12.50-14.00   | 2.95    | 0.41                  | 1.01      | 0.22                  | 1.41    | 0.01                  |
| 14.00-16.00   | 3.04    | 0.09                  | 0.88      | 0.04                  | 1.34    | 0.01                  |
| 16.00-20.00   | 3.30    | 0.57                  | 1.18      | 0.01                  | 1.41    | 0.03                  |

Tab. 8. Moisture for various types of coals Tab. 8. Wilgotność dla różnych typów węgli

where  $\overline{w_{3k}}$  is mean moisture for  $k^{\text{th}}$  coal type and  $s_k^2$  is variance

| Tab. 9. Results of Hartley's and Cochran's tests for moisture |  |
|---|--|
| Tab. 9. Wyniki testów Hartleya i Cochrana dla wilgotności     |  |

| <i>d</i> [mm] | H – Hartley's test | G – Cochran's test |
|---------------|--------------------|--------------------|
| 0.50-1.00     | 0.50               | 0.97               |
| 1.00-3.15     | 89.90              | 0.95               |
| 3.15-6.30     | 14.73              | 0.87               |
| 6.30-8.00     | 20.47              | 0.87               |
| 8.00-10.00    | 15.97              | 0.89               |
| 10.00-12.50   | 97.57              | 0.97               |
| 12.50-14.00   | 45.06              | 0.64               |
| 14.00–16.00   | 5.80               | 0.63               |
| 16.00–20.00   | 67.90              | 0.93               |

 Tab. 10. Results of F test for moisture (fractions (12.50, 14.00) and (14.00, 16.00))

 Tab. 10. Wyniki testu F dla wilgotności (klasy (12.50, 14.00) oraz (14.00, 16.00))

| <i>d</i> [mm] | F     |
|---------------|-------|
| 12.50–14.00   | 29.52 |
| 14.00–16.00   | 66.21 |

Tab. 11. Results of Cochran-Cox test for moisture

| <i>d</i> [mm] | $\mathbf{H}_{0}: \mathfrak{M}(\mathbb{C}) \neq \mathfrak{M}(\mathbb{C})$ | $\mathbf{H}_{0}: \mathcal{U}((0) = \mathcal{U}((0))$ | H₀: ₩(((0)= ₩(((0) |
|---------------|--|--|--------------------|
| 0.50-1.00     | 8.01   | 6.91   | 6.96               |
| 1.00-3.15     | 7.18   | 6.27   | 6.49               |
| 3.15-6.30     | 7.71   | 6.24   | 3.75               |
| 6.30-8.00     | 9.82   | 8.81   | 0.90               |
| 8.00-10.00    | 6.66   | 5.82   | 2.16               |
| 10.00-12.50   | 4.97   | 2.73   | 4.20               |
| 16.00-20.00   | 6.82   | 5.96   | 2.78               |

and the obtained results were presented in Table 10.

Because on significance level  $\alpha = 0.05$  the test results for both fractions are within critical range which was the range (3.97,  $+\infty$ ), so the hypothesis of equality of mean moisture in various coal types should be rejected. For the remaining fractions the Cochran–Cox test of equality of two mean values was used. The results were presented in Table 11.

Only for fraction (6.30; 8.00) in cases of coal types 34.2 and 35 the result of test of equality of moisture lied outsider the critical range for significance level  $\alpha = 0.05$ , which was the range (1.94,

 $+\infty$ ). The other results contain within this range. So, it can be stated that the hypothesis of equality of mean moisture for various coal types should be rejected.

#### Conclusions

Analyzing the obtained results it can be stated that the results of moisture and sulfur contents are necessary to identify the type of coal, while ash contents does not differentiate significantly the coal type. However, because of the fact that for some fractions hypotheses of equality between mean sulfur contents and mean moisture were not rejected, in some cases another coal feature should be used, like volatile parts contents. The obtained results were completely in accordance with the results obtained by means of observational tunnels method, presented in [6, 11].

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Zastosowanie analizy wariancji do porównania charakterystyk różnego typu węgli kamiennych W polskiej nomenklaturze istnieje wiele typów i podtypów węgla, które różnią się między sobą różnymi cechami. Jednakże, często nie jest łatwo rozpoznać je na podstawie, na przykład, wybranej zmiennej numerycznej opisującej ich cechy. W artykule zastosowano analizę wariancji jako narzędzia porównawczego dla trzech typów węgli kamiennych, które zostały pobrane z trzech kopalni zlokalizowanych na Górnym Śląsku. Były to węgle typów 31, 34.2 oraz 35. Każdy z węgli został najpierw przesiany a następnie dodatkowo rozdzielony na frakcje gęstościowe przy użyciu wodnego roztworu chlorku cynku. Tak przygotowany materiał został następnie zbadany ze względu na kilka wybranych cech, takich jak ciepło spalania, zawartość popiołu, zawartość siarki, zawartość części lotnych oraz wilgotność. Wraz z masą dało to siedmiowymiarowy wektor opisujący każdą z wybranych frakcji dla wszystkich trzech typów węgli. Następnie, przeprowadzono pełną analizę wariancji z badaniem wszystkich założeń wymaganych do jej przeprowadzenia. Wyniki posłużyły do opracowania wniosków.

Słowa klucze: węgiel kamienny, typ węgla, wielowymiarowa analiza statystyczna, analiza wariancji, klasy ziarnowe