

# Safety and Resilience Indicators of Critical Infrastructure Impacted by Operation Application to Port Oil Terminal Examination

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**ABSTRACT:** Modelling of operation process influence on safety of a critical infrastructure is presented. New safety and resilience indicators for a critical infrastructure are defined and procedures of their determination in the case of the created model are proposed. Next, this model is applied to safety and resilience analysis of the port oil terminal critical infrastructure impacted by its operation process and the results are compared to the indicators of this critical infrastructure without operation impacts.

## 1 INTRODUCTION

The safety and resilience indicators (Kołowrocki & Soszyńska-Budny, 2017) for critical infrastructure defined as a complex system in its operating environment that significant features are inside-system dependencies and outside-system dependencies (Lague et al., 2015) are crucial for its operators. A simple critical infrastructure safety model without considering outside impacts proposing safety indicators SafI1-SafI8 (Kołowrocki & Soszyńska-Budny, 2017, 2018a, 2019) can be generalized by linking it with the model of the critical infrastructure operation process (Kołowrocki & Soszyńska-Budny, 2017, 2018b). This way created joint impact model of the critical infrastructure related to its operation process can offer, additionally to the modified safety indicators SafI1-SafI8, two resilience indicators ResI1-ResI2 which are measures of the critical infrastructure operation impact on its safety and resilience to operation (Kołowrocki & Soszyńska-Budny, 2017, 2018b). The paper is devoted to development of this joint model of safety and operation process of critical infrastructure and its practical application to safety and resilience

examination of the port oil terminal critical infrastructure.

## 2 CRITICAL INFRASTRUCTURE IMPACTED BY ITS OPERATION PROCESS SAFETY MODEL

### 2.1 Critical Infrastructure operation process

We consider the critical infrastructure related to the operation process  $Z(t)$ ,  $t \in \langle 0, \infty \rangle$ , impacted in a various way at its operation states  $z_b$ ,  $b = 1, 2, \dots, v$ . We assume that the changes of the operation states of the critical infrastructure operation process  $Z(t)$  have an influence on the critical infrastructure safety structure and also on the safety of the critical infrastructure assets  $A_i$ ,  $i = 1, 2, \dots, n$ , (Kołowrocki & Soszyńska-Budny, 2011, 2017, 2018b).

The following critical infrastructure operation process parameters (OPP) can be identified either statistically using the methods given in (Kołowrocki, 2014; Kołowrocki & Soszyńska-Budny, 2011, 2017, 2018b) or evaluated approximately by experts:

- the number of operation states (OPP1)  $v$  ;

- the vector  $[p_b(0)]_{1 \times v}$  of the initial probabilities (OPP2)  $p_b(0) = P(Z(0) = z_b)$ ,  $b = 1, 2, \dots, v$ , of the critical infrastructure operation process  $Z(t)$  staying at particular operation states  $z_b$  at the moment  $t = 0$ ;
- the matrix  $[p_{bl}]_{1 \times v}$  of probabilities of transition (OPP3)  $p_{bl}$ ,  $b, l = 1, 2, \dots, v$ , of the critical infrastructure operation process  $Z(t)$  between the operation states  $z_b$  and  $z_l$ ;
- the matrix  $[M_{bl}]_{1 \times v}$  of mean values of conditional sojourn times (OPP4)  $M_{bl} = E[\theta_{bl}]$ ,  $b, l = 1, 2, \dots, v$ , of the critical infrastructure operation process  $Z(t)$  conditional sojourn times  $\theta_{bl}$  at the operation state  $z_b$  when the next state is  $z_l$ .

The main critical infrastructure operation process characteristic (OPC) that can be either calculated analytically using the above parameters of the operation process or evaluated approximately by experts (Kołowrocki & Soszyńska-Budny, 2011, 2017, 2018b) is the vector

$$[p_b]_{1 \times v} = [p_1, p_2, \dots, p_v], \quad (1)$$

of limit values of transient probabilities (OPC1)

$$p_b(t) = P(Z(t) = z_b), \quad t \in \langle 0, +\infty \rangle, \quad b = 1, 2, \dots, v, \quad (2)$$

of the critical infrastructure operation process  $Z(t)$  at the particular operation states  $z_b$ ,  $b = 1, 2, \dots, v$ .

## 2.2 Critical Infrastructure safety and resilience indicators

We denote the critical infrastructure conditional lifetime in the safety state subset  $\{u, u+1, \dots, z\}$ ,  $u = 1, 2, \dots, z$ , while its operation process  $Z(t)$ ,  $t \in \langle 0, \infty \rangle$ , is at the operation state  $z_b$ ,  $b = 1, 2, \dots, v$ , by  $[T^1(u)]^{(b)}$ ,  $u = 1, 2, \dots, z$ , and the conditional safety function of the critical infrastructure related to the operation process  $Z(t)$ ,  $t \in \langle 0, \infty \rangle$ , by the vector (Kołowrocki & Soszyńska-Budny, 2017, 2018b)

$$[\mathcal{S}^1(t, \cdot)]^{(b)} = [1, [\mathcal{S}^1(t, 1)]^{(b)}, \dots, [\mathcal{S}^1(t, z)]^{(b)}], \quad (3)$$

with the coordinates defined by

$$[\mathcal{S}^1(t, u)]^{(b)} = P([T^1(u)]^{(b)} > t | Z(t) = z_b) \quad (4)$$

for  $t \in \langle 0, \infty \rangle$ ,  $u = 1, 2, \dots, z$ ,  $b = 1, 2, \dots, v$ .

The safety function  $[\mathcal{S}^1(t, u)]^{(b)}$ ,  $u = 1, 2, \dots, z$ , is the conditional probability that the critical infrastructure related to the operation process  $Z(t)$ ,  $t \in \langle 0, \infty \rangle$ , lifetime  $[T^1(u)]^{(b)}$ ,  $u = 1, 2, \dots, z$ , in the safety state subset  $\{u, u+1, \dots, z\}$ ,  $u = 1, 2, \dots, z$ , is greater than  $t$ , while the critical infrastructure operation process  $Z(t)$  is at the operation state  $z_b$ .

Next, we denote the critical infrastructure related to the operation process  $Z(t)$ ,  $t \in \langle 0, \infty \rangle$ , unconditional lifetime in the safety state subset  $\{u, u+1, \dots, z\}$ ,  $u = 1, 2, \dots, z$ , by  $T^1(u)$ ,  $u = 1, 2, \dots, z$ , and the unconditional safety

function, the first safety indicator SafI1 (Kołowrocki & Soszyńska-Budny, 2017, 2018b) of the critical infrastructure related to the operation process  $Z(t)$ ,  $t \in \langle 0, \infty \rangle$ , by the vector

$$\mathcal{S}^1(t, \cdot) = [1, \mathcal{S}^1(t, 1), \dots, \mathcal{S}^1(t, z)], \quad (5)$$

with the coordinates defined by

$$\mathcal{S}^1(t, u) = P(T^1(u) > t) \quad (6)$$

for  $t \in \langle 0, \infty \rangle$ ,  $u = 1, 2, \dots, z$ .

In the case when the system operation time  $\theta$  is large enough, the coordinates of the unconditional safety function of the critical infrastructure related to the operation process  $Z(t)$ ,  $t \in \langle 0, \infty \rangle$ , defined by (6), are given by (Kołowrocki & Soszyńska-Budny, 2017, 2018b)

$$\mathcal{S}^1(t, u) \cong \sum_{b=1}^v p_b [\mathcal{S}^1(t, u)]^{(b)} \quad \text{for } t \geq 0, \quad u = 1, 2, \dots, z, \quad (7)$$

where  $[\mathcal{S}^1(t, u)]^{(b)}$ ,  $u = 1, 2, \dots, z$ ,  $b = 1, 2, \dots, v$ , are the coordinates of the critical infrastructure related to the operation process  $Z(t)$ ,  $t \in \langle 0, \infty \rangle$ , conditional safety functions defined by (3)-(4) and  $p_b$ ,  $b = 1, 2, \dots, v$ , are the critical infrastructure operation process  $Z(t)$ ,  $t \in \langle 0, \infty \rangle$ , limit transient probabilities at operation states  $z_b$ ,  $b = 1, 2, \dots, v$ , defined by (1).

Other safety indicators corresponding to SafI2-SafI8, defined in (Kołowrocki & Soszyńska-Budny, 2017, 2018b) are as follows:

- the risk function (SafI2)

$$r^1(t) = 1 - \mathcal{S}^1(t, r), \quad t \in \langle 0, \infty \rangle, \quad r \in \{1, 2, \dots, z\}, \quad (8)$$

- where  $r$  is the critical safety state;
- the graph of the critical infrastructure risk function  $r^1(t)$ ,  $t \in \langle 0, \infty \rangle$ , called the fragility curve (SafI3) (Ben, Gouldby, Shultz, Simm & Wibowo, 2010);
- the mean value of the critical infrastructure unconditional lifetime  $T^1(r)$  up to exceeding critical safety state  $r$  (SafI4) given by

$$\mu^1(r) = \int_0^\infty [\mathcal{S}^1(t, r)] dt \cong \sum_{b=1}^v p_b [\mu^1(r)]^{(b)}, \quad (9)$$

- where  $[\mu^1(r)]^{(b)}$  are the mean values of the critical infrastructure conditional lifetimes  $[T^1(r)]^{(b)}$  in the safety state subset  $\{r, r+1, \dots, z\}$  at the operation state  $z_b$ ,  $b = 1, 2, \dots, v$ , given by

$$[\mu^1(r)]^{(b)} = \int_0^\infty [\mathcal{S}^1(t, r)]^{(b)} dt, \quad b = 1, 2, \dots, v, \quad (10)$$

- and  $[\mathcal{S}^1(t, r)]^{(b)}$ ,  $b = 1, 2, \dots, v$ , are defined by (3)-(4) and  $p_b$  are given by (1);

- the standard deviation  $\sigma^1(r)$ , of the critical infrastructure lifetime in the safety state not worse than the critical state  $r$  (SafI5);
- the moment  $\tau^1$  of exceeding acceptable value of critical infrastructure risk function level  $\delta$  (SafI6);
- the intensities of degradation of the critical infrastructure / the intensities of critical infrastructure departure from the safety state subset  $\{u, u + 1, \dots, z\}$  (SafI7)

$$\lambda^1(t, u) = \frac{-dS^1(t, u)}{S^1(t, u)}, t \geq 0, u = 1, 2, \dots, z; \quad (11)$$

- the mean lifetimes

$$\begin{aligned} \bar{\mu}^1(u) &= \mu^1(u) - \mu^1(u + 1), u = 1, 2, \dots, z - 1, \\ \bar{\mu}^1(z) &= \mu^1(z). \end{aligned} \quad (12)$$

- of the critical infrastructure in the particular safety states (SafI8) where  $\mu^1(u)$ ,  $u = 1, 2, \dots, z$ , may be determined from (10) by substituting  $r = u$ .

To express the scale of influence of the operation process on the critical infrastructure safety, the following resilience indicators are defined:

- the coefficients of operation process impact on the critical infrastructure intensities of degradation (the coefficients of operation process impact on critical infrastructure intensities of departure from the safety state subset  $\{u, u + 1, \dots, z\}$ ) (ResI1), i.e. the coordinates of the vector

$$\rho^1(t, \cdot) = [0, \rho^1(t, 1), \dots, \rho^1(t, z)], t \geq 0, \quad (13)$$

- where

$$\lambda^1(t, u) = \rho^1(t, u) \cdot \lambda^0(t, u), t \geq 0, u = 1, 2, \dots, z, \quad (14)$$

- i.e.

$$\rho^1(t, u) = \frac{\lambda^1(t, u)}{\lambda^0(t, u)}, t \geq 0, u = 1, 2, \dots, z, \quad (15)$$

- and  $\lambda^0(t, u)$ , are the intensities of degradation of the critical infrastructure without operation process impact and  $\lambda^1(t, u)$ , are the intensities of degradation of the critical infrastructure with operation process impact,
- the indicator of critical infrastructure resilience to operation process impact (ResI2) defined by

$$RI^1(t, r) = \frac{1}{\rho^1(t, r)}, t \geq 0, \quad (16)$$

- where  $\rho^1(t, r)$ ,  $t \in \langle 0, +\infty \rangle$ , is the coefficients of operation process impact on the critical infrastructure intensities of degradation given by (15) for  $u = r$ .

### 3 APPLICATION

We consider the port oil terminal critical infrastructure impacted by its operation process placed at the Baltic seaside that is designated for receiving oil products from ships, storage and sending them by carriages or trucks. The terminal is described in details in (Kołowrocki & Soszyńska-Budny, 2019).

#### 3.1 Port oil terminal critical infrastructure assets

The considered terminal is composed of three parts  $A$ ,  $B$  and  $C$ , linked by the piping transportation system with the pier. The area in the neighborhood of the port oil piping transportation system is presented in Figs. 9-10 in (Kołowrocki & Soszyńska-Budny, 2019). The main technical assets of the port oil terminal critical infrastructure are:

- $A_1$  - port oil piping transportation system,
- $A_2$  - internal pipeline technological system,
- $A_3$  - supporting pump station,
- $A_4$  - internal pump system,
- $A_5$  - port oil tanker shipment terminal,
- $A_6$  - loading railway carriage station,
- $A_7$  - loading road carriage station,
- $A_8$  - unloading railway carriage station,
- $A_9$  - oil storage reservoir system.

The scheme of the asset  $A_1$ , the port oil piping transportation system is presented in Figure 11 in (Kołowrocki & Soszyńska-Budny, 2018a, 2019). The port oil transportation system is a series system composed of two series-parallel subsystems  $S_1$ ,  $S_2$ , each containing two pipelines and one series-“2 out of 3” subsystem  $S_3$  containing 3 pipelines. The subsystems  $S_1$ ,  $S_2$ ,  $S_3$  are forming a general series port oil piping transportation system safety structure presented in Fig. 1.

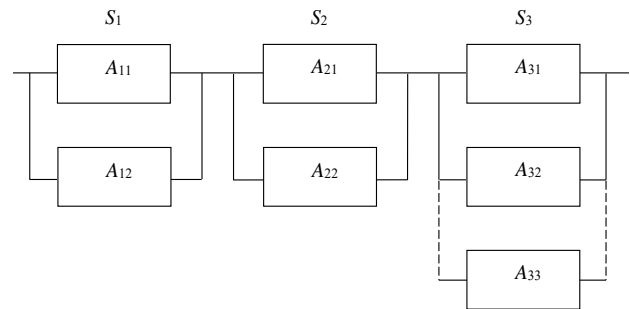


Figure 1. General scheme of the port oil piping transportation system safety structure

#### 3.2 Port oil terminal critical infrastructure safety parameters

After considering the comments and opinions coming from experts concerned with the port oil terminal critical infrastructure and its assets without any outside impacts, using (GMU Critical Infrastructure Safety Interactive Platform, 2018) the following safety parameters were fixed (Kołowrocki & Soszyńska-Budny, 2019):

- the number safety states (excluding safety state 0)  $z = 2$ ;

- three safety states 2, 1, 0;
- the critical safety state  $r = 1$ ;
- the risk function permitted level  $\delta = 0.05$ ;
- the mean values of the asset  $A_1$ , the port oil terminal critical infrastructure lifetimes in the safety state subsets  $\{1,2\}$ ,  $\{2\}$  :
- for safety state subset  $\{1,2\}$

$$\mu_1^0(1) = 63 \text{ years}, \quad (17)$$

- for safety state subset  $\{2\}$

$$\mu_1^0(2) = 46 \text{ years}; \quad (18)$$

- the mean values of the assets  $A_2 - A_9$  lifetimes in the safety state subsets  $\{1,2\}$ ,  $\{2\}$ , evaluated approximately by experts, are as follows:
- for safety state subset  $\{1,2\}$

$$\mu_i^0(1) = 80 \text{ years}, \quad i = 2,3,\dots,9, \quad (19)$$

- for safety state subset  $\{2\}$

$$\mu_i^0(2) = 50 \text{ years}, \quad i = 2,3,\dots,9. \quad (20)$$

From (Kołowrocki & Soszyńska-Budny, 2011), it follows that the intensities of assets departure from the safety states subset  $\{1,2\}$ , are:

- for asset  $A_1$

$$\lambda_1^0(1) = 0.015873, \quad \lambda_1^0(2) = 0.021739, \quad (21)$$

- for assets  $A_2 - A_9$

$$\lambda_i^0(1) = 0.0125, \quad \lambda_i^0(2) = 0.02, \quad i = 2,3,\dots,9. \quad (22)$$

### 3.3 Port oil terminal critical infrastructure safety indicators

Assuming that the oil terminal critical infrastructure was free of any outside impacts, its following safety indicators were determined (Kołowrocki & Soszyńska-Budny, 2019):

- the safety function  $S^0(t, \cdot)$ ;
- the expected values of the oil terminal critical infrastructure lifetimes in the safety state subsets  $\{1,2\}$ ,  $\{2\}$  :

$$\mu^0(1) \cong 8.63, \quad \mu^0(2) \cong 5.50 \text{ years}; \quad (23)$$

- the mean values of the oil terminal critical infrastructure lifetimes in the particular safety states:

$$\bar{\mu}^0(1) \cong 3.13, \quad \bar{\mu}^0(2) \cong 5.50 \text{ years}; \quad (24)$$

- the port oil terminal critical infrastructure risk function  $r^0(t)$ ;
- the moment when the oil terminal critical infrastructure risk function exceeds a permitted level  $\delta = 0.05$

$$\tau \cong 0.44 \text{ years}. \quad (25)$$

The oil terminal critical infrastructure intensities of ageing (SI7) are:

$$\lambda^0(1) = 0.115873, \quad \lambda^0(2) = 0.181739. \quad (26)$$

### 3.4 Parameters and characteristics of Port oil terminal critical infrastructure operation process

Operation of the asset  $A_1$ , the port oil piping transportation system is the main activity of the port oil terminal involving the remaining assets  $A_2 - A_9$  and determining their operation processes.

On the basis of the statistical data and expert opinions, it is possible to fix and to evaluate the following unknown basic parameters of the oil terminal critical infrastructure operation process:

- the number of operation process states (OPP1)

$$\nu = 7$$

and the operation process states:

- the operation state  $z_1$  - transport of one kind of medium from the terminal part B to part C using two out of three pipelines of the subsystem  $S_3$  of the asset  $A_1$  illustrated in Figure 2 and assets  $A_2, A_4, A_6, A_7, A_9$ ;

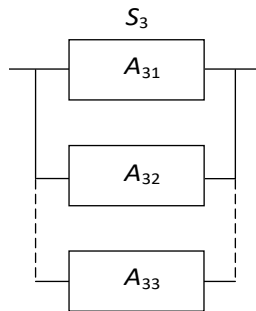


Figure 2. The scheme of the port oil piping transportation system at the operation state  $z_1$

- the operation state  $z_2$  - transport of one kind of medium from the terminal part C to part B using one out of three pipelines of the subsystem  $S_3$  of the asset  $A_1$  illustrated in Figure 3. and assets  $A_2, A_4, A_8, A_9$ ;

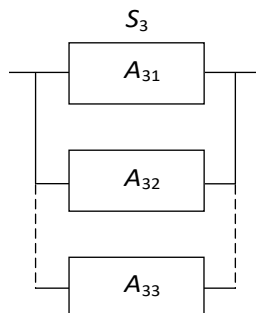


Figure 3. The scheme of the port oil piping transportation system at the operation state  $z_2$

- the operation state  $z_3$  - transport of one kind of medium from the terminal part B through part A

to pier using one out of two pipelines of the subsystem  $S_1$  and one out of two pipelines of the subsystem  $S_2$  of the asset  $A_1$  illustrated in Figure 4 and assets  $A_2, A_4, A_5, A_9$ ;

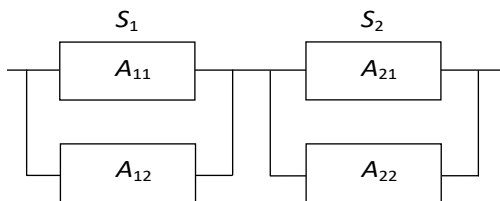


Figure 4. The scheme of the port oil piping transportation system at the operation state  $z_3$

- the operation state  $z_4$  – transport of one kind of medium from the pier through parts A and B to part C using one out of two pipelines of the subsystem  $S_1$ , one out of two pipelines in subsystem  $S_2$  and two out of three pipelines of the subsystem  $S_3$  of the asset  $A_1$  illustrated in Figure 5 and assets  $A_2, A_3, A_4, A_5, A_6, A_7, A_9$ ;

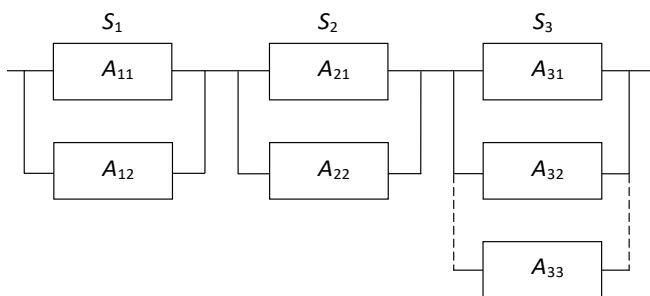


Figure 5. The scheme of the port oil piping transportation system at the operation state  $z_4$

- the operation state  $z_5$  – transport of one kind of medium from the pier through part A to B using one out of two pipelines of the subsystem  $S_1$  and one out of two pipelines of the subsystem  $S_2$  of the asset  $A_1$  illustrated in Figure 6. and assets  $A_2, A_3, A_4, A_5, A_9$ ;

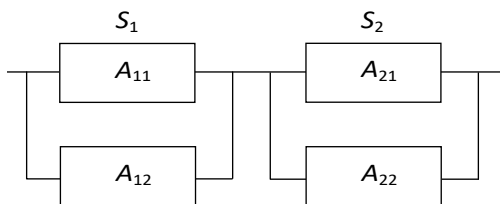


Figure 6. The scheme of port oil piping transportation system at the operation state  $z_5$

- the operation state  $z_6$  – transport of one kind of medium from the terminal part B to C using two out of three pipelines of the subsystem  $S_3$ , and simultaneously transport one kind of medium from the pier through part A to B using one out of two pipelines of the subsystem  $S_1$  and one out of two pipelines of the subsystem  $S_2$  of the asset  $A_1$  illustrated in Fig. 7 and assets  $A_2, A_3, A_4, A_5, A_6, A_7, A_9$ ;

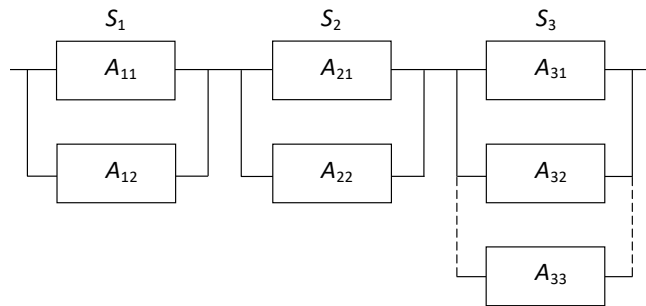


Figure 7. The scheme of the port oil piping transportation system at the operation state  $z_6$

- the operation state  $z_7$  – transport of one kind of medium from the terminal part B to C using one out of three pipelines of the subsystem  $S_3$ , and simultaneously transport second kind of medium from the terminal part C to B using one out of three pipelines of the subsystem  $S_3$  of the asset  $A_1$  illustrated in Figure 8 and assets  $A_2, A_4, A_6, A_7, A_8, A_9$ .

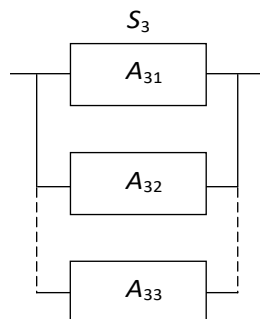


Figure 8. The scheme of the port oil piping transportation system at the operation states  $z_7$

The port oil terminal critical infrastructure operation process  $Z(t)$  characteristics are (Kołowrocki & Soszyńska-Budny, 2018b):

- the limit values of transient probabilities (OPC1) of the operation process  $Z(t)$  at the particular operation states  $z_b, b = 1, 2, \dots, 7$ :

$$p_1 = 0.395, p_2 = 0.060, p_3 = 0.003, p_4 = 0.002, \\ p_5 = 0.20, p_6 = 0.058, p_7 = 0.282. \quad (27)$$

### 3.5 Parameters of operation process impact on port oil terminal critical infrastructure safety

The coefficients of the operation process impact on the port oil terminal critical infrastructure intensities of ageing at the operation states  $z_b, b = 1, 2, \dots, 7$ , are as follows [GMU Safety interactive Platform]:

- for asset  $A_1$

$$[\rho_i^1(1)]^{(b)} = 1.10, [\rho_i^1(2)]^{(b)} = 1.10, b = 1, 2, 7, i = 1, \\ [\rho_i^1(1)]^{(b)} = 1.20, [\rho_i^1(2)]^{(b)} = 1.20, b = 3, 5, i = 1, \\ [\rho_i^1(1)]^{(b)} = 1.30, [\rho_i^1(2)]^{(b)} = 1.30, b = 4, 6, i = 1$$

(28)

- for asset  $A_2$

$$\begin{aligned}
[\rho_i^1(1)]^{(b)} &= 1.10, [\rho_i^1(2)]^{(b)} = 1.10, b = 1,2,7, i = 2, \\
[\rho_i^1(1)]^{(b)} &= 1.20, [\rho_i^1(2)]^{(b)} = 1.20, b = 3,5, i = 2, \\
[\rho_i^1(1)]^{(b)} &= 1.30, [\rho_i^1(2)]^{(b)} = 1.30, b = 4,6, i = 2; (29)
\end{aligned}$$

– for asset  $A_3$

$$\begin{aligned}
[\rho_i^1(1)]^{(b)} &= 1, [\rho_i^1(2)]^{(b)} = 1, b = 1,2,3,7, i = 3, \\
[\rho_i^1(1)]^{(b)} &= 1.20, [\rho_i^1(2)]^{(b)} = 1.20, b = 5, i = 3, \\
[\rho_i^1(1)]^{(b)} &= 1.30, [\rho_i^1(2)]^{(b)} = 1.30, b = 4,6, i = 3; (30)
\end{aligned}$$

– for asset  $A_4$

$$\begin{aligned}
[\rho_i^1(1)]^{(b)} &= 1.10, [\rho_i^1(2)]^{(b)} = 1.10, b = 1,2,7, i = 4, \\
[\rho_i^1(1)]^{(b)} &= 1.20, [\rho_i^1(2)]^{(b)} = 1.20, b = 3,5, i = 4, \\
[\rho_i^1(1)]^{(b)} &= 1.30, [\rho_i^1(2)]^{(b)} = 1.30, b = 4,6, i = 4; (31)
\end{aligned}$$

– for asset  $A_5$

$$\begin{aligned}
[\rho_i^1(1)]^{(b)} &= 1, [\rho_i^1(2)]^{(b)} = 1, b = 1,2,7, i = 5, \\
[\rho_i^1(1)]^{(b)} &= 1.20, [\rho_i^1(2)]^{(b)} = 1.20, b = 3,5, i = 5, \\
[\rho_i^1(1)]^{(b)} &= 1.30, [\rho_i^1(2)]^{(b)} = 1.30, b = 4,6, i = 5; (32)
\end{aligned}$$

– for asset  $A_6$

$$\begin{aligned}
[\rho_i^1(1)]^{(b)} &= 1, [\rho_i^1(2)]^{(b)} = 1, b = 2,5, i = 6, \\
[\rho_i^1(1)]^{(b)} &= 1.10, [\rho_i^1(2)]^{(b)} = 1.10, b = 1,7, i = 6, \\
[\rho_i^1(1)]^{(b)} &= 1.20, [\rho_i^1(2)]^{(b)} = 1.20, b = 3, i = 6, \\
[\rho_i^1(1)]^{(b)} &= 1.30, [\rho_i^1(2)]^{(b)} = 1.30, b = 4,6, i = 6;
\end{aligned}$$

(33)

– for asset  $A_7$

$$\begin{aligned}
[\rho_i^1(1)]^{(b)} &= 1, [\rho_i^1(2)]^{(b)} = 1, b = 2,3,5, i = 7, \\
[\rho_i^1(1)]^{(b)} &= 1.10, [\rho_i^1(2)]^{(b)} = 1.10, b = 1,7, i = 7, \\
[\rho_i^1(1)]^{(b)} &= 1.30, [\rho_i^1(2)]^{(b)} = 1.30, b = 4,6, i = 7; (34)
\end{aligned}$$

– for asset  $A_8$

$$\begin{aligned}
[\rho_i^1(1)]^{(b)} &= 1, [\rho_i^1(2)]^{(b)} = 1, b = 1,3,4,5,6, i = 8, \\
[\rho_i^1(1)]^{(b)} &= 1.10, [\rho_i^1(2)]^{(b)} = 1.10, b = 2,7, i = 8;
\end{aligned}$$

(35)

– for asset  $A_9$

$$\begin{aligned}
[\rho_i^1(1)]^{(b)} &= 1.10, [\rho_i^1(2)]^{(b)} = 1.10, b = 1,2,7, i = 9, \\
[\rho_i^1(1)]^{(b)} &= 1.20, [\rho_i^1(2)]^{(b)} = 1.20, b = 3,5, i = 9, \\
[\rho_i^1(1)]^{(b)} &= 1.30, [\rho_i^1(2)]^{(b)} = 1.30, b = 4,6, i = 9. (36)
\end{aligned}$$

### 3.6 Parameters of port oil terminal critical infrastructure safety

We assume that the port oil terminal critical infrastructure assets  $A_i$ ,  $i = 1,2,\dots,9$ , at the critical infrastructure operation process  $Z(t)$  states  $z_b$ ,  $b = 1,2,\dots,7$ , conditional safety functions

$$[S_i^1(t,\cdot)]^{(b)} = [1, [S_i^1(t,1)]^{(b)}, [S_i^1(t,2)]^{(b)}], t \geq 0, (37)$$

$$b = 1,2,\dots,7, i = 1,2,\dots,9,$$

are exponential with the coordinates

$$[S_i^1(t,u)]^{(b)} = \exp[-[\lambda_i^1(u)]^{(b)} t], t \geq 0, (38)$$

$$u = 1,2, b = 1,2,\dots,7, i = 1,2,\dots,9,$$

where

$$[\lambda_i^1(u)]^{(b)} = [\rho_i^1(u)]^{(b)} \cdot \lambda_i^0(u), u = 1,2, (39)$$

$$b = 1,2,\dots,7, i = 1,2,\dots,9,$$

and

$$[\rho_i^1(u)]^{(b)}, u = 1,2, b = 1,2,\dots,7, i = 1,2,\dots,9,$$

are the coefficients of operation process impact on the intensities of degradation of the port oil critical infrastructure assets  $A_i$ ,  $i = 1,2,\dots,9$ , at the operation states  $z_b$ ,  $b = 1,2,\dots,7$ , defined by (28)-(36) and

$$\lambda_i^0(u), u = 1,2, i = 1,2,\dots,9,$$

are the intensities of degradation of the port oil critical infrastructure assets without the operation process impact, defined by (21)-(22).

Under the assumption (39), considering (28)-(36) and (21)-(22), it follows that the intensities of assets departure from the safety states subset  $\{1,2\}$ ,  $\{2\}$ , with operation impact on their safety are:

– for asset  $A_1$

$$\begin{aligned}
[\lambda_i^1(1)]^{(b)} &= 0.017460, [\lambda_i^1(2)]^{(b)} = 0.023913, \\
b &= 1,2,7, i = 1, \\
[\lambda_i^1(1)]^{(b)} &= 0.019048, [\lambda_i^1(2)]^{(b)} = 0.026087, \\
b &= 3,5, i = 9, \\
[\lambda_i^1(1)]^{(b)} &= 0.020635, [\lambda_i^1(2)]^{(b)} = 0.028261, \\
b &= 4,6, i = 9; (40)
\end{aligned}$$

– for asset  $A_2$

$$\begin{aligned}
[\lambda_i^1(1)]^{(b)} &= 0.01375, [\lambda_i^1(2)]^{(b)} = 0.022, \\
b &= 1,2,7, i = 2, \\
[\lambda_i^1(1)]^{(b)} &= 0.015, [\lambda_i^1(2)]^{(b)} = 0.024, b = 3,5, i = 2, \\
[\lambda_i^1(1)]^{(b)} &= 0.01625, [\lambda_i^1(2)]^{(b)} = 0.026, \\
b &= 4,6, i = 2; (41)
\end{aligned}$$

– for asset  $A_3$

$$\begin{aligned} [\lambda_i^1(1)]^{(b)} &= 0.0125, [\lambda_i^1(2)]^{(b)} = 0.02, \\ b &= 1,2,3,7, i = 3, \\ [\lambda_i^1(1)]^{(b)} &= 0.015, [\lambda_i^1(2)]^{(b)} = 0.024, b = 5, i = 3, \\ [\lambda_i^1(1)]^{(b)} &= 0.01625, [\lambda_i^1(2)]^{(b)} = 0.026, \\ b &= 4,6, i = 3; \end{aligned} \quad (42)$$

– for asset  $A_4$

$$\begin{aligned} [\lambda_i^1(1)]^{(b)} &= 0.01375, [\lambda_i^1(2)]^{(b)} = 0.022, \\ b &= 1,2,7, i = 4, \\ [\lambda_i^1(1)]^{(b)} &= 0.015, [\lambda_i^1(2)]^{(b)} = 0.024, b = 3,5, i = 4, \\ [\lambda_i^1(1)]^{(b)} &= 0.01625, [\lambda_i^1(2)]^{(b)} = 0.026, b = 4,6, \\ i &= 4; \end{aligned} \quad (43)$$

– for asset  $A_5$

$$\begin{aligned} [\lambda_i^1(1)]^{(b)} &= 0.0125, [\lambda_i^1(2)]^{(b)} = 0.02, b = 1,2,7, i = \\ &5, \\ [\lambda_i^1(1)]^{(b)} &= 0.015, [\lambda_i^1(2)]^{(b)} = 0.024, b = 3,5, i = 5, \\ [\lambda_i^1(1)]^{(b)} &= 0.01625, [\lambda_i^1(2)]^{(b)} = 0.026, \\ b &= 4,6, i = 5; \end{aligned} \quad (44)$$

– for asset  $A_6$

$$\begin{aligned} [\lambda_i^1(1)]^{(b)} &= 0.0125, [\lambda_i^1(2)]^{(b)} = 0.02, b = 2,5, i = 6, \\ [\lambda_i^1(1)]^{(b)} &= 0.01375, [\lambda_i^1(2)]^{(b)} = 0.022, b = 1,7, i = \\ &6, \\ [\lambda_i^1(1)]^{(b)} &= 0.015, [\lambda_i^1(2)]^{(b)} = 0.024, b = 3, i = 6, \\ [\lambda_i^1(1)]^{(b)} &= 0.01625, [\lambda_i^1(2)]^{(b)} = 0.026, \\ b &= 4,6, i = 6; \end{aligned} \quad (45)$$

– for asset  $A_7$

$$\begin{aligned} [\lambda_i^1(1)]^{(b)} &= 0.0125, [\lambda_i^1(2)]^{(b)} = 0.02, b = 2,3,5, i = 7, \\ [\lambda_i^1(1)]^{(b)} &= 0.01375, [\lambda_i^1(2)]^{(b)} = 0.022, b = 1,7, i = \\ &7, \\ [\lambda_i^1(1)]^{(b)} &= 0.01625, [\lambda_i^1(2)]^{(b)} = 0.026, \\ b &= 4,6, i = 7; \end{aligned} \quad (46)$$

– for asset  $A_8$

$$\begin{aligned} [\lambda_i^1(1)]^{(b)} &= 0.0125, [\lambda_i^1(2)]^{(b)} = 0.02, b = 1,3,4,5,6, \\ i &= 8, \\ [\lambda_i^1(1)]^{(b)} &= 0.01375, [\lambda_i^1(2)]^{(b)} = 0.022, \\ b &= 2,7, i = 8; \end{aligned} \quad (47)$$

– for asset  $A_9$

$$\begin{aligned} [\lambda_i^1(1)]^{(b)} &= 0.01375, [\lambda_i^1(2)]^{(b)} = 0.022, b = 1,2,7, \\ i &= 9, \\ [\lambda_i^1(1)]^{(b)} &= 0.015, [\lambda_i^1(2)]^{(b)} = 0.024, b = 3,5, i = 9, \\ [\lambda_i^1(1)]^{(b)} &= 0.01625, [\lambda_i^1(2)]^{(b)} = 0.026, \\ b &= 4,6, i = 9. \end{aligned} \quad (48)$$

### 3.7 Prediction of safety and resilience characteristics of port oil terminal critical infrastructure

Considering that the coordinates of the conditional safety functions (37) for the port oil terminal critical infrastructure assets  $A_i$ ,  $i=1,2,\dots,9$ , are of the form (38) with the intensities of ageing at the operation states  $z_b$ ,  $b=1,2,\dots,7$ , given respectively by (40)-(48), as the oil terminal critical infrastructure is a three-state ( $z=2$ ) series system, then by Corollary 1 from (Kołowrocki & Soszyńska-Budny, 2019), they are given by:

$$[\mathbf{S}^1(t, \cdot)]^{(1)} = [1, [\mathbf{S}^1(t, 1)]^{(1)}, [\mathbf{S}^1(t, 2)]^{(1)}], t \geq 0,$$

where

$$\begin{aligned} [\mathbf{S}^1(t, 1)]^{(1)} &= \exp[-0.12371t], \\ [\mathbf{S}^1(t, 2)]^{(1)} &= \exp[-0.193913t]; \end{aligned} \quad (49)$$

$$[\mathbf{S}^1(t, \cdot)]^{(2)} = [1, [\mathbf{S}^1(t, 1)]^{(2)}, [\mathbf{S}^1(t, 2)]^{(2)}], t \geq 0,$$

where

$$\begin{aligned} [\mathbf{S}^1(t, 1)]^{(2)} &= \exp[-0.12246t], \\ [\mathbf{S}^1(t, 2)]^{(2)} &= \exp[-0.191913t]; \end{aligned} \quad (50)$$

$$[\mathbf{S}^1(t, \cdot)]^{(3)} = [1, [\mathbf{S}^1(t, 1)]^{(3)}, [\mathbf{S}^1(t, 2)]^{(3)}], t \geq 0,$$

where

$$\begin{aligned} [\mathbf{S}^1(t, 1)]^{(3)} &= \exp[-0.131548t], \\ [\mathbf{S}^1(t, 2)]^{(3)} &= \exp[-0.206087t]; \end{aligned} \quad (51)$$

$$[\mathbf{S}^1(t, \cdot)]^{(4)} = [1, [\mathbf{S}^1(t, 1)]^{(4)}, [\mathbf{S}^1(t, 2)]^{(4)}], t \geq 0,$$

where

$$\begin{aligned} [\mathbf{S}^1(t, 1)]^{(4)} &= \exp[-0.146885t], \\ [\mathbf{S}^1(t, 2)]^{(4)} &= \exp[-0.230261t]; \end{aligned} \quad (52)$$

$$[\mathbf{S}^1(t, \cdot)]^{(5)} = [1, [\mathbf{S}^1(t, 1)]^{(5)}, [\mathbf{S}^1(t, 2)]^{(5)}], t \geq 0,$$

where

$$[\mathcal{S}^1(t,1)]^{(5)} = \exp[-0.131548t], \quad \mu^1(1) \cong 7.89 \text{ years} \quad \mu^1(2) \cong 5.03 \text{ years}, \quad (59)$$

$$[\mathcal{S}^1(t,2)]^{(5)} = \exp[-0.206087t]; \quad (53)$$

$$[\mathcal{S}^1(t, \cdot)]^{(6)} = [1, [\mathcal{S}^1(t,1)]^{(6)}, [\mathcal{S}^1(t,2)]^{(6)}], t \geq 0,$$

where

$$[\mathcal{S}^1(t,1)]^{(6)} = \exp[-0.146885t],$$

$$[\mathcal{S}^1(t,2)]^{(6)} = \exp[-0.230261t]; \quad (54)$$

$$[\mathcal{S}^1(t, \cdot)]^{(7)} = [1, [\mathcal{S}^1(t,1)]^{(7)}, [\mathcal{S}^1(t,2)]^{(7)}], t \geq 0,$$

where

$$[\mathcal{S}^1(t,1)]^{(7)} = \exp[-0.12496t],$$

$$[\mathcal{S}^1(t,2)]^{(7)} = \exp[-0.195913t]. \quad (55)$$

Hence, applying (10), the expected values of the port oil terminal critical infrastructure lifetimes in the safety state subsets  $\{1,2\}$ ,  $\{2\}$ , at the operation states  $z_b$ ,  $b = 1,2,\dots,7$ , respectively are:

$$[\mu^1(1)]^{(1)} \cong 8.08, [\mu^1(2)]^{(1)} \cong 5.16 \text{ years},$$

$$[\mu^1(1)]^{(2)} \cong 8.17, [\mu^1(2)]^{(2)} \cong 5.21 \text{ years},$$

$$[\mu^1(1)]^{(3)} \cong 7.60, [\mu^1(2)]^{(3)} \cong 4.85 \text{ years},$$

$$[\mu^1(1)]^{(4)} \cong 6.81, [\mu^1(2)]^{(4)} \cong 4.34 \text{ years},$$

$$[\mu^1(1)]^{(5)} \cong 7.60, [\mu^1(2)]^{(5)} \cong 4.85 \text{ years},$$

$$[\mu^1(1)]^{(6)} \cong 6.81, [\mu^1(2)]^{(6)} \cong 4.34 \text{ years},$$

$$[\mu^1(1)]^{(7)} \cong 8.00, [\mu^1(2)]^{(7)} \cong 5.10 \text{ years}. \quad (56)$$

From the results (27) and (49)-(55), applying (7), the port oil terminal critical infrastructure unconditional safety function (SafI1) is given by

$$\mathcal{S}^1(t, \cdot) = [1, \mathcal{S}^1(t,1), \mathcal{S}^1(t,2)], t \geq 0,$$

where

$$\mathcal{S}^1(t,1) = 0.395\exp[-0.12371t] + 0.060\exp[-0.12246t]$$

$$+ 0.003\exp[-0.131548t] + 0.002\exp[-0.146885t]$$

$$+ 0.200\exp[-0.131548t] + 0.058\exp[-0.146885t]$$

$$+ 0.282\exp[-0.12496t] \quad (57)$$

$$\mathcal{S}^1(t,2) = 0.395\exp[-0.193913t] + 0.060\exp[-0.191913t] +$$

$$0.003\exp[-0.206087t] + 0.002\exp[-0.230261t] +$$

$$0.200\exp[-0.206087t] + 0.058\exp[-0.230261t] +$$

$$0.282\exp[-0.195913t] \quad (58)$$

Considering (27) and (56) and applying (9) for  $r = u$ , the expected values and standard deviations (SafI 4 -SafI5) of the port oil terminal critical infrastructure lifetimes in the safety state subsets  $\{1,2\}$ ,  $\{2\}$ , respectively are:

$$\sigma^1(1) \cong 7.91, \quad \sigma^1(2) \cong 5.05 \text{ years}, \quad (60)$$

and further, by (12), it follows that the mean values of the oil terminal critical infrastructure lifetimes in the particular safety states are:

$$\bar{\mu}^1(1) \cong 2.86, \quad \bar{\mu}^1(2) \cong 5.03 \text{ years}. \quad (61)$$

As the critical safety state is  $r = 1$ , then by (8) and (57), the port oil terminal critical infrastructure risk function (SafI2), is given by

$$r^1(t) = 1 - \mathcal{S}^1(t,1) \text{ for } t \geq 0. \quad (62)$$

The graph of the risk function  $r^1(t)$  of the oil terminal critical infrastructure is (SafI3).

From (3.16) in (Kołowrocki & Soszyńska-Budny, 2017) and (62), the moment when the oil terminal critical infrastructure risk function exceeds a permitted level  $\delta = 0.05$  (SafI6), is

$$\tau^1 = (r^1)^{-1}(\delta) \cong 0.404 \text{ years}. \quad (63)$$

Applying (11), the oil terminal critical infrastructure intensities of ageing (SafI7) are:

$$\lambda^1(t,1) \cong 0.126743, \quad \lambda^2(t,2) \cong 0.198807. \quad (64)$$

Considering (26) and (64) and applying (15), the coefficients of the operation process impact on the oil terminal critical infrastructure intensities of ageing (ResI1), are:

$$\rho^1(t,1) \cong 1.094, \quad \rho^1(t,2) \cong 1.094. \quad (65)$$

Finally, by (16) and (65), the port oil terminal critical infrastructure resilience indicator (ResI2), i.e. the coefficient of the port oil terminal critical infrastructure resilience to the operation process impact, is

$$RI(t) = 1 / \rho^1(t,1) \cong 0.914 = 91\%.1 / \quad (66)$$

In the Table 1 given below, the inventory of basic safety and resilience indicators for the port oil piping transportation system is presented.

From this inventory it can be seen that operation process of the oil terminal critical infrastructure has significant impact on its safety and resilience.



Table 1. Inventory of basic safety, risk and resilience indicators of oil terminal critical infrastructure

Oil terminal critical infrastructure safety, risk and resilience indicators			
Impacts	mean value of lifetime up to exceeding critical safety state 1 in years $\mu(l)$	moment of exceeding acceptable risk level $\delta = 0.05$ in years	resilience indicator in range 0-1 $RI(1)$ $\tau$
without impacts	8.630	0.440	1.00
operation impact	7.890	0.404	0.914

#### 4 CONCLUSIONS

The safety and resilience indicators were determined for port oil terminal. Further research can be related with considering other impacts and solving the problems of critical infrastructure safety optimization and finding of optimal values of safety and resilience indicators. These results can help to mitigate critical infrastructure accident consequences and to enhance critical infrastructure resilience to operation and other impacts. This research can also result in the backgrounds for business continuity and cost-effectiveness analysis of critical infrastructures under operation and other impacts.

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